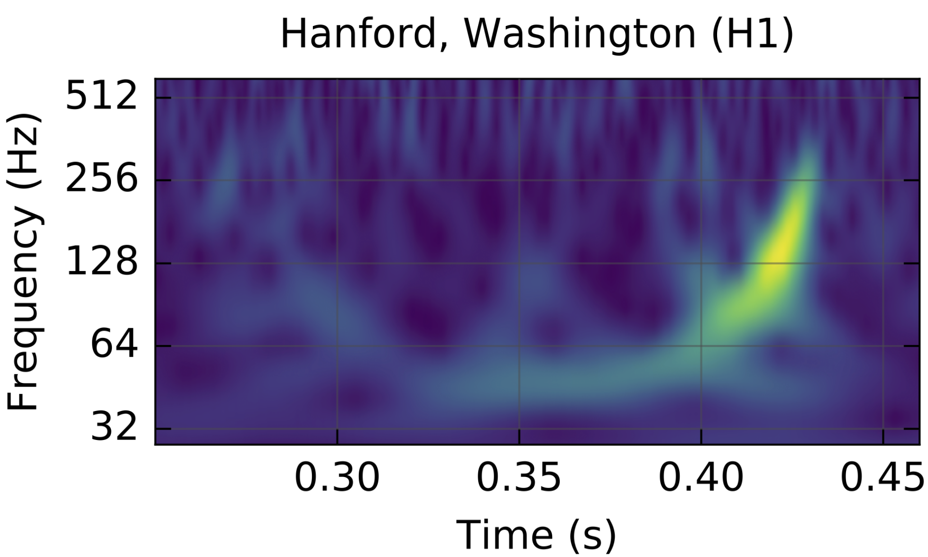
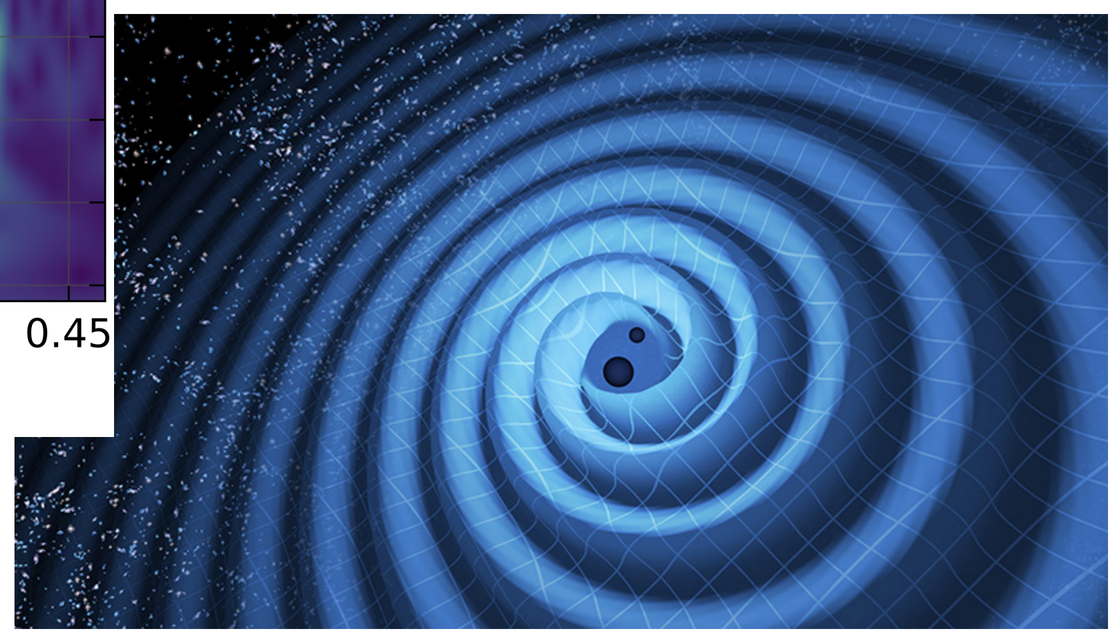
Michela Mapelli

Compact Object Astrophysics with Gravitational Waves

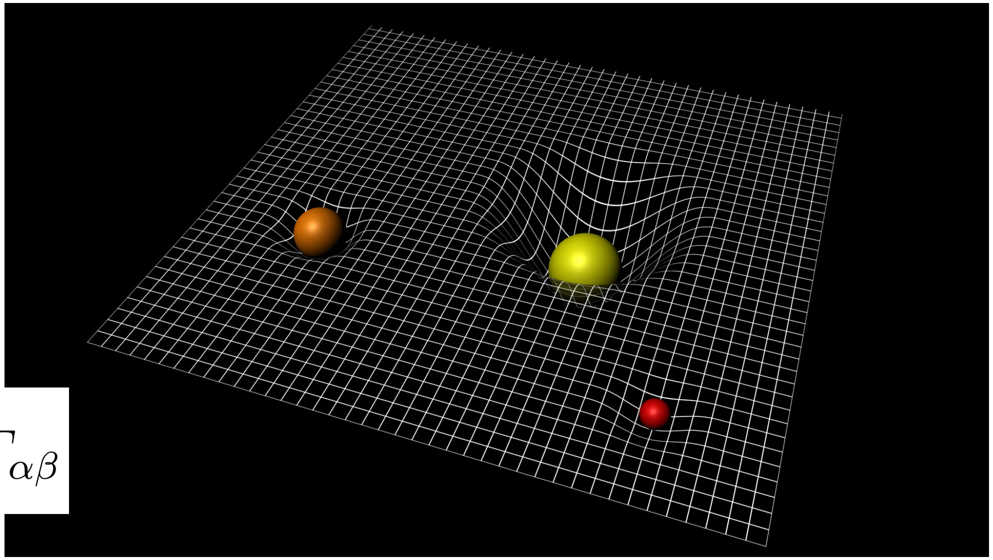




Bad Honnef, January 23rd 2026, 5th Graduate School on Astro-Particle Physics

Gravitational Wave (GW) Theory's Extreme Summary

How to derive gravitational waves from Einstein eqs?



1. if strong local curvature
   and rapidly moving sources:
   Numerical solution of fully nonlinear Einstein equations
   (e.g. during merger of two black holes)

$$R_{\alpha\beta} - \frac{1}{2} R\, g_{\alpha\beta} = \frac{8\,\pi\, G}{c^4} T_{\alpha\beta}$$

2. if weak field (almost flat curvature) and slow moving sources (v<<c):
   Analytic solution of LINEARIZED Einstein equations
   (e.g. two black holes orbiting about each other but still binary systems, not yet merging)

   Model the space-time metrics as flat $\eta_{\alpha\beta}$ + perturbation $h_{\alpha\beta}$

   Solving linearized Einstein's equation reduces to solving this simplified equation (WAVE EQUATION):

$$\Box\, \bar{h}_{\alpha\beta} = \frac{16\,\pi\, G}{c^4} T_{\alpha\beta}$$

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \sum_{i=1}^{d-1} \frac{\partial^2}{\partial {x_i}^2}$$

Summary:

By integrating equation $\Box \, \bar{h}_{\alpha\beta} = \frac{16\,\pi\,G}{c^4} T_{\alpha\beta}$

for distant and slowly moving sources, we obtain the solution:

$$\overline{h}^{ij}(t, \vec{x}) \sim \frac{2\,G}{r\,c^4} \frac{d^2}{dt^2} I^{ij}(t - r/c)$$

Distance source-observer

Second mass moment
(moment of inertia)

retarded time

$$I^{ij} = \int dx^3 \, \rho(t, \vec{x}) \, x^i \, x^j$$

→ **not all masses produce gravitational waves,** but only those with
1) non-zero QUADRUPOLE moment of mass
2) non-zero second time derivative of quadrupole moment of mass
(i.e. they are accelerating)

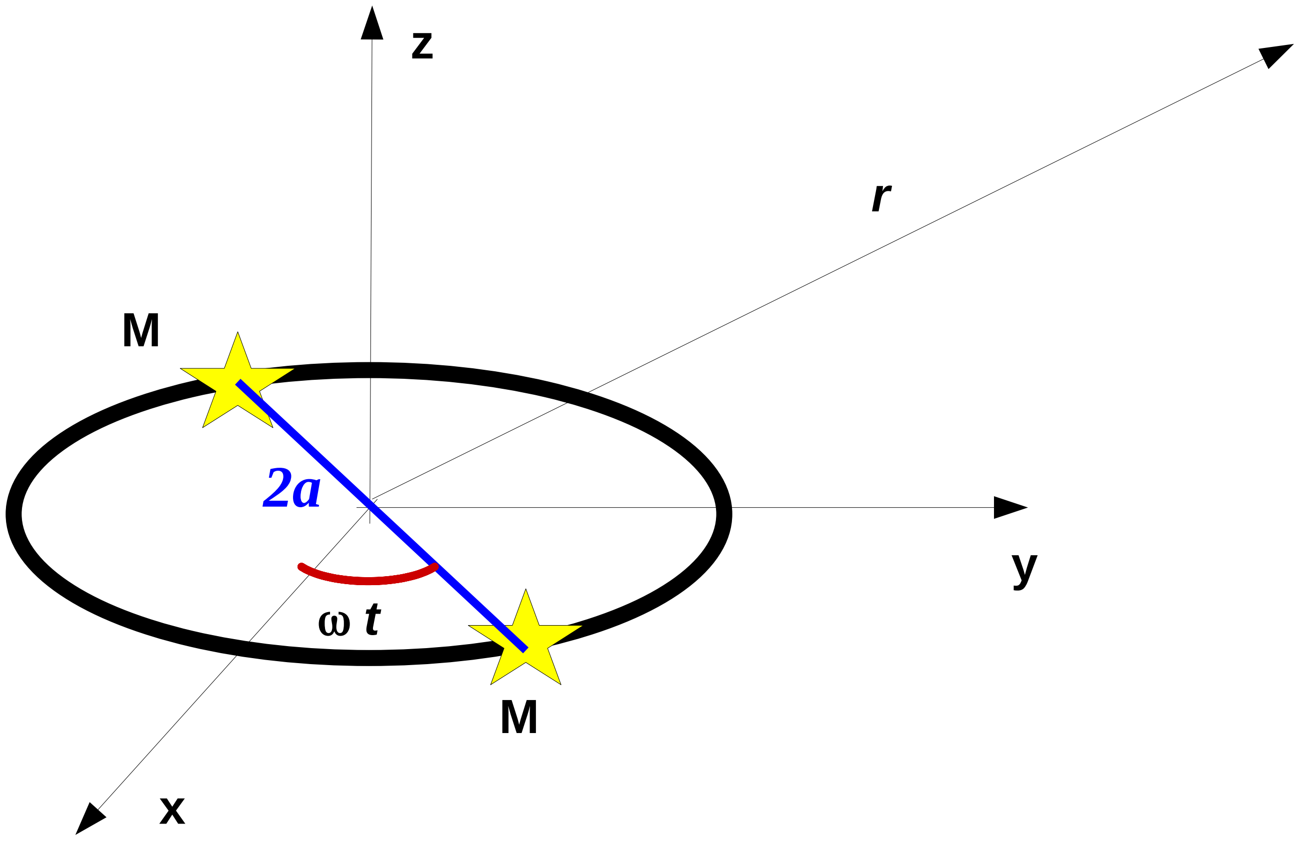
→ **for a gravitational wave to form, there must be an**
ASYMMETRY IN MASS DISTRIBUTION
and masses should be accelerating (e.g., circular motion)

Any time-varying non axi-symmetric distribution of masses produces GWs

Binary systems:

Now we restrict to the interesting case of a binary system

Two stars of <u>equal mass M</u> orbiting about each other with <u>CIRCULAR orbit</u> and relative distance (= semi-major axis) a



See Hartle, Gravity: An Introduction to Einstein's General Relativity

Binary systems:

For the assumed geometry: x(t) = *a* cos(ωt), y(t) = *a* sin(ωt), z(t) = 0

The second mass moment is

$$I^{xx} = 2\,M\,a^2\cos^2(\omega\,t) = M\,a^2\,\left[1+\cos(2\,\omega\,t)\right]$$
$$I^{xy} = 2\,M\,a^2\sin(\omega\,t)\,\cos\omega\,t = M\,a^2\,\sin(2\,\omega\,t)$$
$$I^{yy} = 2\,M\,a^2\sin^2(\omega\,t) = M\,a^2\,\left[1-\cos(2\,\omega\,t)\right]$$
$$I^{zz} = I^{zx} = I^{zy} = 0$$

By taking the second derivatives and considering retarded time:

$$\bar{h}^{ij} \sim -\frac{2}{r}\frac{G}{c^4}(2\,\omega)^2\,M\,a^2\begin{pmatrix} \overset{xx}{\cos(2\,\omega\,(t-r/c))} & \overset{xy}{\sin(2\,\omega\,(t-r/c))} & \overset{xz}{0} \\ \overset{yx}{\sin(2\,\omega\,(t-r/c))} & \overset{yy}{-\cos(2\,\omega\,(t-r/c))} & \overset{yz}{0} \\ \overset{zx}{0} & \overset{zy}{0} & \overset{zz}{0} \end{pmatrix}$$

Using Newton description of binary star:

$$\omega^2 = \frac{G\,2\,M}{a^3}$$

We find

$$\bar{h}^{ij} \sim -\frac{16\,G^2}{c^4}\frac{M^2}{r\,a}\begin{pmatrix} \cos(2\,\omega\,(t-r/c)) & \sin(2\,\omega\,(t-r/c)) & 0 \\ \sin(2\,\omega\,(t-r/c)) & -\cos(2\,\omega\,(t-r/c)) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Binary systems:

Propagation of the wave along the z axis
No oscillation in the direction of propagation

Gravitational waves for a circular, equal-mass binary system:

$$\bar{h}^{ij} \sim -\frac{16\,G^2}{c^4}\frac{M^2}{r\,a}\begin{pmatrix} \cos\left(2\,\omega\,(t-r/c)\right) & \sin\left(2\,\omega\,(t-r/c)\right) & 0 \\ \sin\left(2\,\omega\,(t-r/c)\right) & -\cos\left(2\,\omega\,(t-r/c)\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

GRAVITATIONAL WAVES from a BINARY STAR
with constant masses and semi-major axis are (almost) monochromatic
with frequency = 2 orbital frequency

$$\omega_{GW} = 2\,\omega_{orb} = 2\sqrt{\frac{G\,(m_1+m_2)}{a^3}}$$

Binary systems:

GRAVITATIONAL WAVE SEMI-AMPLITUDE:

$$h \sim \frac{8\,G^2}{c^4}\,\frac{M^2}{r\,a}$$

$$h \sim 10^{-21}\left(\frac{M}{M_\odot}\right)^2\left(\frac{1\,\mathrm{kpc}}{r}\right)\left(\frac{1\,R_\odot}{a}\right)$$

or in terms of period

$$P^2 = (2\,\pi)^2\,\frac{a^3}{G\,2\,M} \longrightarrow h \sim \frac{8\,(2\,\pi^2)^{1/3}\,G^{5/3}}{c^4}\,\frac{M^{5/3}}{r\,P^{2/3}}$$

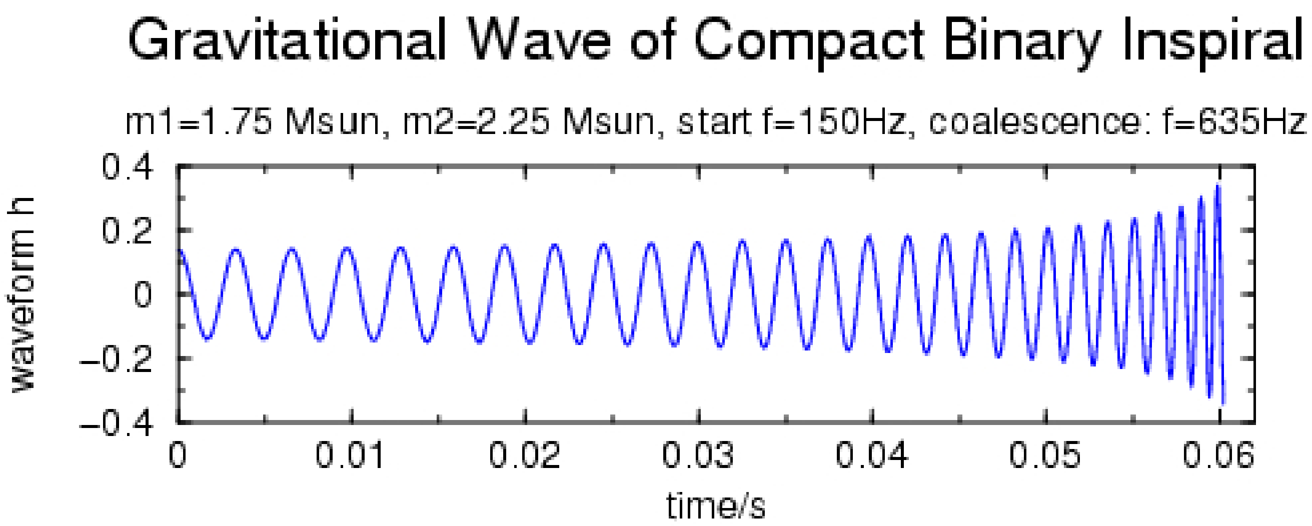
$$h \sim 10^{-21}\left(\frac{M}{M_\odot}\right)^{5/3}\left(\frac{1\,\mathrm{kpc}}{r}\right)\left(\frac{1\,\mathrm{h}}{P}\right)^{2/3}$$

Binary systems:

AMPLITUDE of GWs from binary systems:

$$h \sim \frac{8\,G^2}{c^4}\,\frac{M^2}{r\,a}$$

*** the bigger the amplitude (strain), the easier the detection**



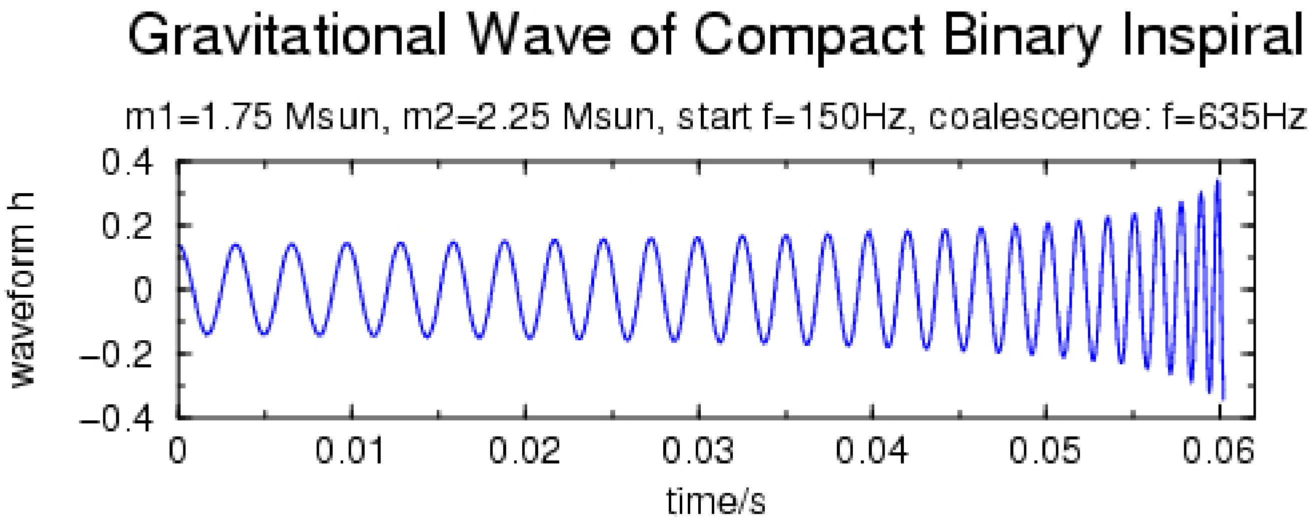
Binary systems:

AMPLITUDE of GWs from binary systems:

$$h \sim \frac{8\,G^2}{c^4}\frac{M^2}{r\,a}$$

*** the bigger the amplitude (strain), the easier the detection**

*** the farther the binary, the smaller the amplitude**



Binary systems:

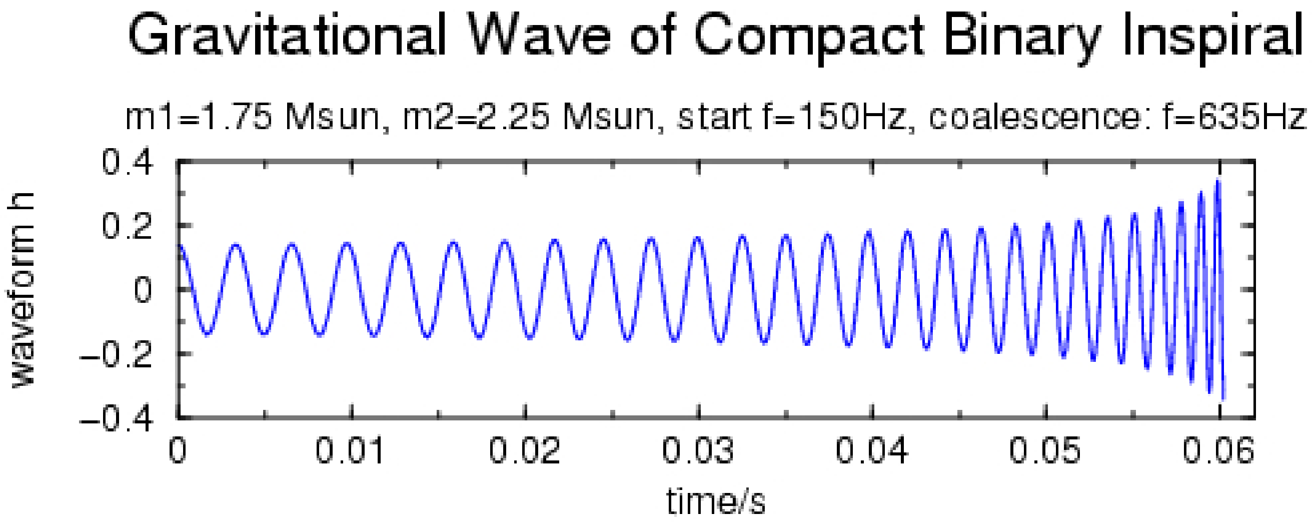
AMPLITUDE of GWs from binary systems:

$$h \sim \frac{8\,G^2}{c^4}\,\frac{M^2}{r\,a}$$

*** the bigger the amplitude (strain), the easier the detection**

*** the farther the binary, the smaller the amplitude**

*** the larger the masses, the larger the amplitude**



Binary systems:

AMPLITUDE of GWs from binary systems:

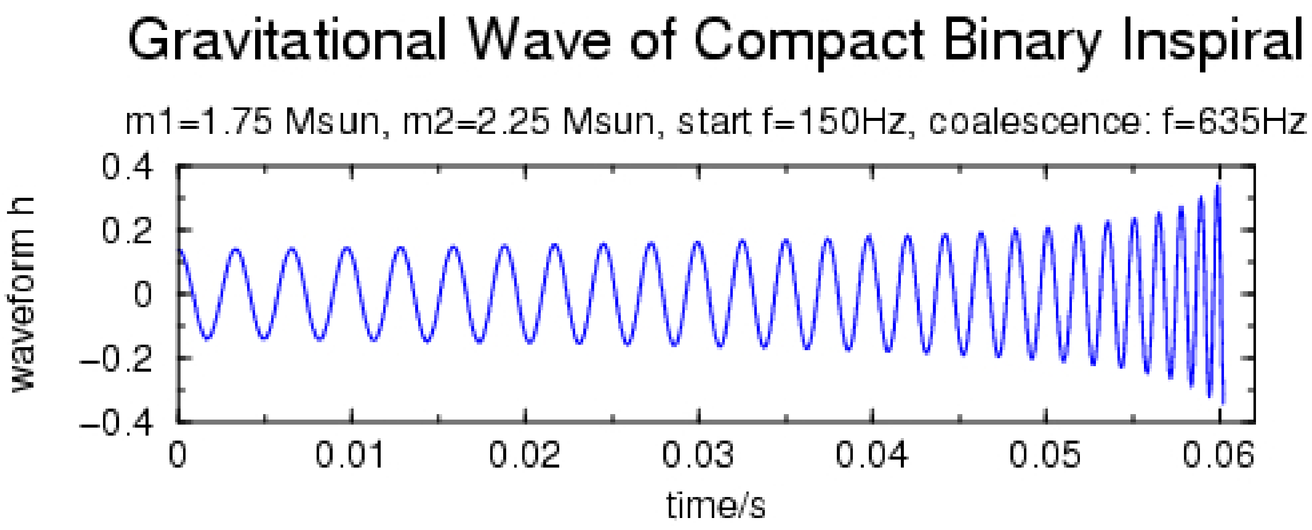
$$h \sim \frac{8\,G^2}{c^4}\,\frac{M^2}{r\,a}$$

*** the bigger the amplitude (strain), the easier the detection**

*** the farther the binary, the smaller the amplitude**

*** the larger the masses, the larger the amplitude**

*** the smaller the semi-major axis, the larger the amplitude**



Binary systems:

EMISSION of GWs implies LOSS of ORBITAL ENERGY:

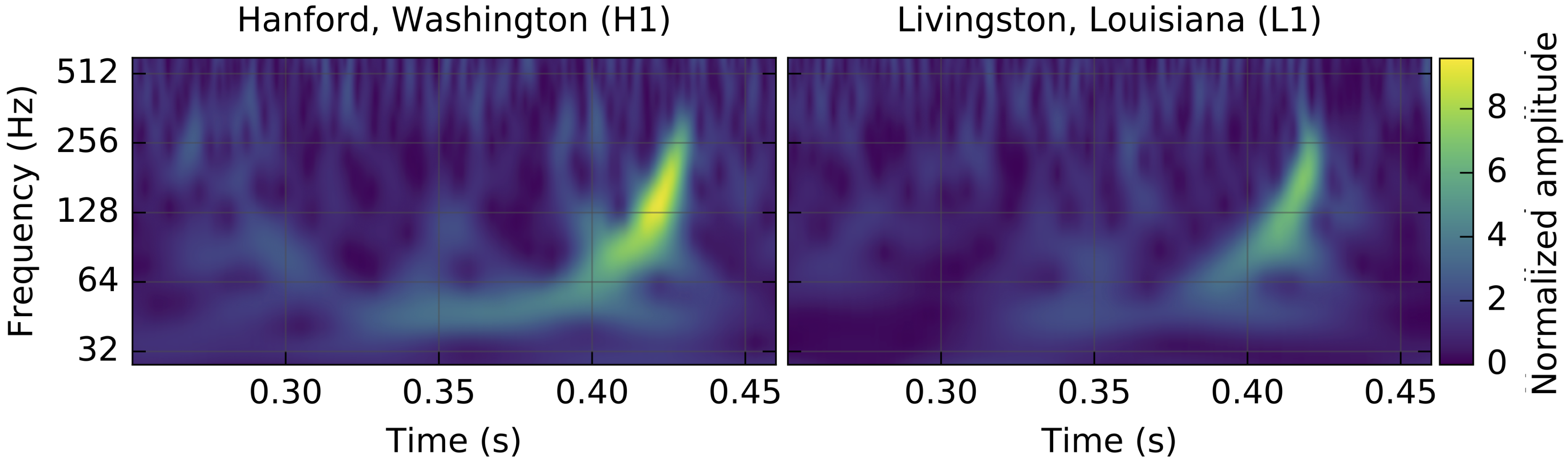
THE BINARY SHRINKS WHILE EMITTING GWs until it merges

$$E_{orb} = -\frac{G\,m_1\,m_2}{2\,a}$$

– If the binary shrinks ($a \rightarrow 0$), frequency becomes higher

$$\omega_{GW} = 2\,\omega_{orb} = 2\sqrt{\frac{G\,(m_1 + m_2)}{a^3}}$$

GWs are not monochromatic anymore when merger is close



Abbott et al. 2016

Binary systems:

EMISSION of GWs implies LOSS of ORBITAL ENERGY:

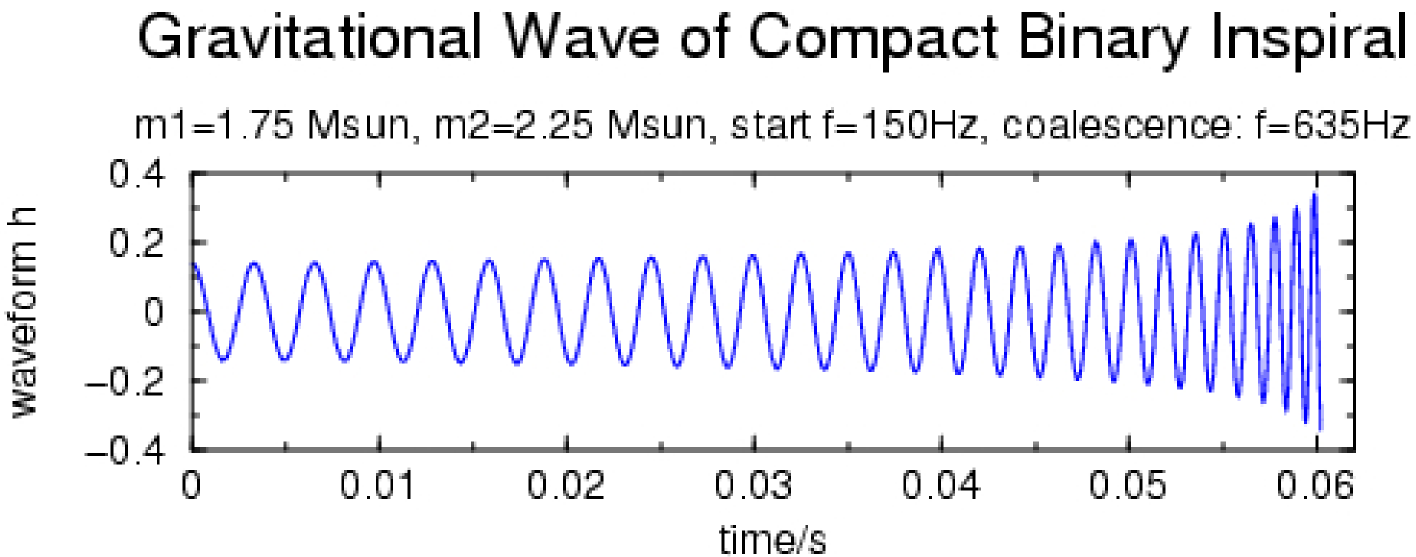
THE BINARY SHRINKS WHILE EMITTING GWs until it merges

$$E_{orb} = -\frac{G\,m_1\,m_2}{2\,a}$$

– If the binary shrinks ($a \rightarrow 0$), frequency becomes higher

– If the binary shrinks amplitude increases

$$h \propto \frac{1}{a}$$



Binary systems:

EMISSION of GWs implies LOSS of ORBITAL ENERGY:

Power radiated by GWs (circular binary):

From GR $$P_{GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{1}{a^5} m_1^2\, m_2^2\, (m_1 + m_2)$$

$$P_{GW} = \frac{dE_{orb}}{dt} = \frac{G\, m_1\, m_2}{2\, a^2}\frac{da}{dt}$$ From Kepler and Newton

$$\Rightarrow \quad \frac{da}{dt} = \frac{64}{5}\frac{G^3}{c^5} a^{-3}\, m_1\, m_2\, (m_1 + m_2)$$

Integrating differential equation:

$$t_{GW} = \frac{5}{256}\frac{c^5}{G^3}\frac{a^4}{m_1\, m_2\, (m_1 + m_2)}$$

Timescale for a system to merge by GW emission

Binary systems:

Timescale extremely long

$$t_{GW} = \frac{5}{256}\,\frac{c^5}{G^3}\,\frac{a^4}{m_1\, m_2\,(m_1 + m_2)}$$

EXERCISE: calculate tGW for 2 neutron stars with mass equal to the Sun mass (1 Msun) orbiting at the distance between Sun and Earth (1 AU)

If m1 = m2 = Msun, *a* = 1 AU, *eccentricity* = *0*

Binary systems:

Timescale extremely long

$$t_{GW} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4}{m_1 m_2 (m_1 + m_2)}$$

EXERCISE: calculate tGW for 2 neutron stars
with mass equal to the Sun mass (1 Msun)
orbiting at the distance
between Sun and Earth (1 AU)

If m1 = m2 = Msun, a = 1 AU, eccentricity = 0

→ tGW ~ 2 x 10^17 yr

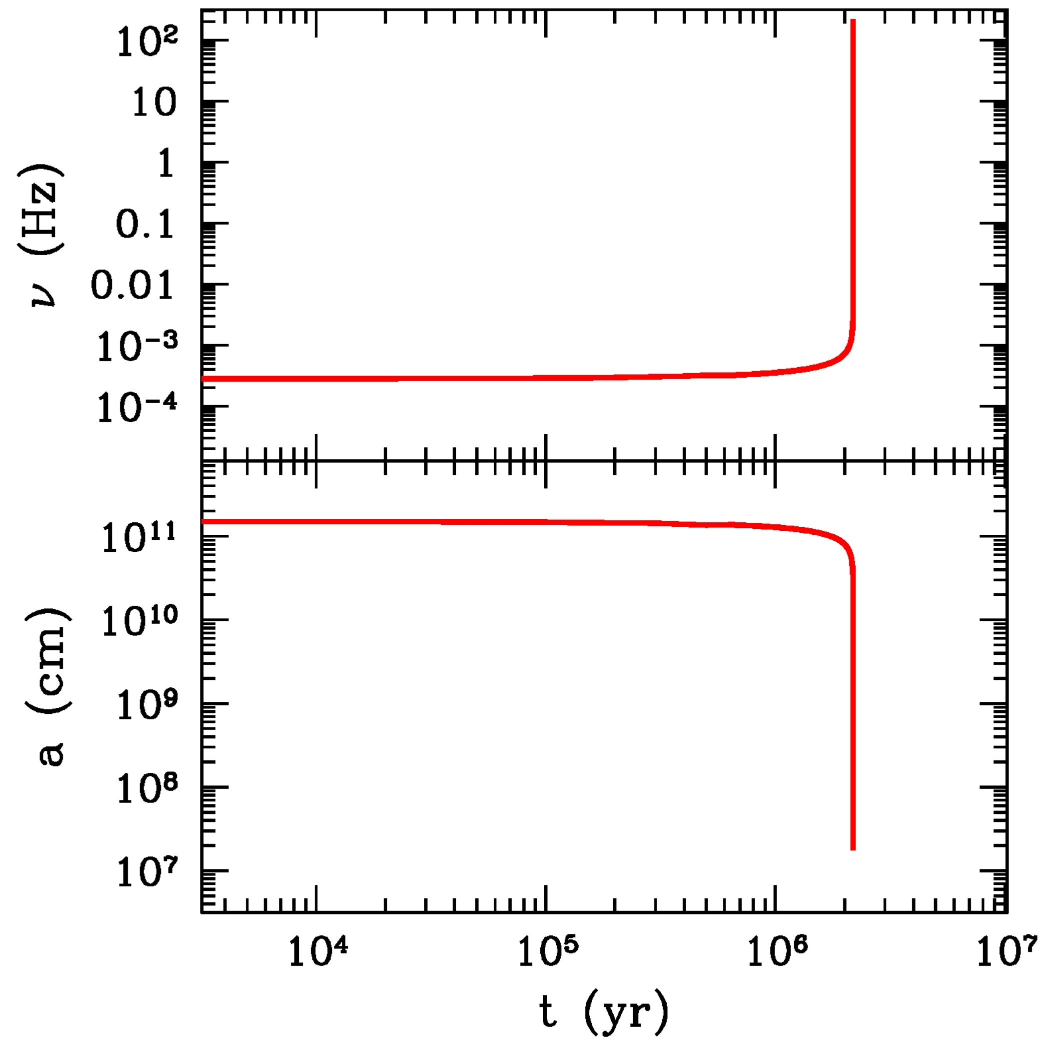
Life of the Universe ~ 13 x 10^9 yr

Binary systems:

$$\frac{da}{dt} = \frac{64}{5}\,\frac{G^3}{c^5}\,a^{-3}\,m_1\,m_2\,(m_1+m_2)$$

$$\longrightarrow \quad a(t) = a_0\left[1 - \frac{256/5\,G^3\,m_1\,m_2\,(m_1+m_2)\,t}{c^5\,a_0^4}\right]^{1/4}$$

m1 = m2 = 10 Msun
a_0 = 0.01 AU
ecc = 0



Note:
above formula neglects evolution of eccentricity via GW emission

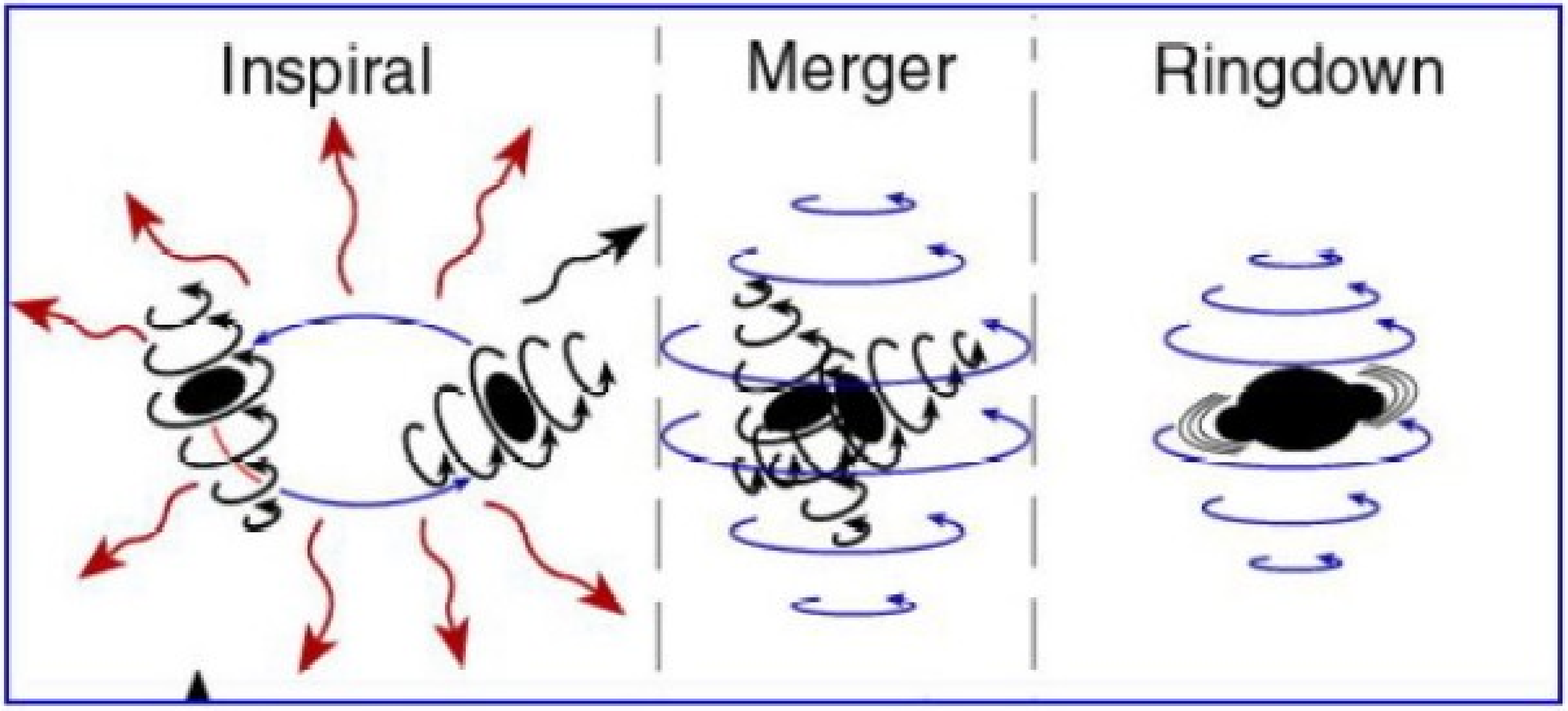
correct only if initial eccentricity is zero

Binary systems:

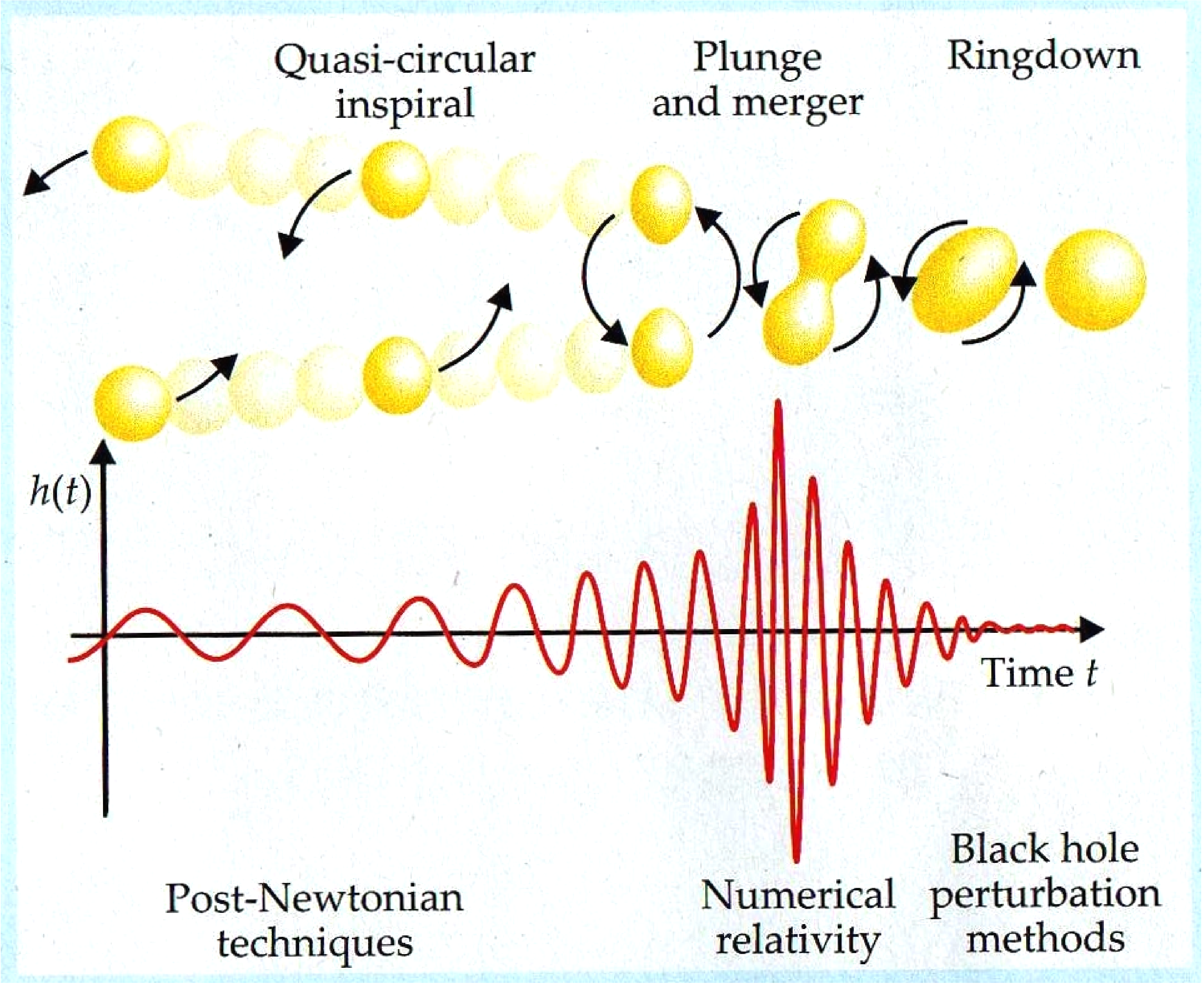
Previous equations are not always true!

Only before merger when binary can be considered Keplerian

i.e. only during inspiral



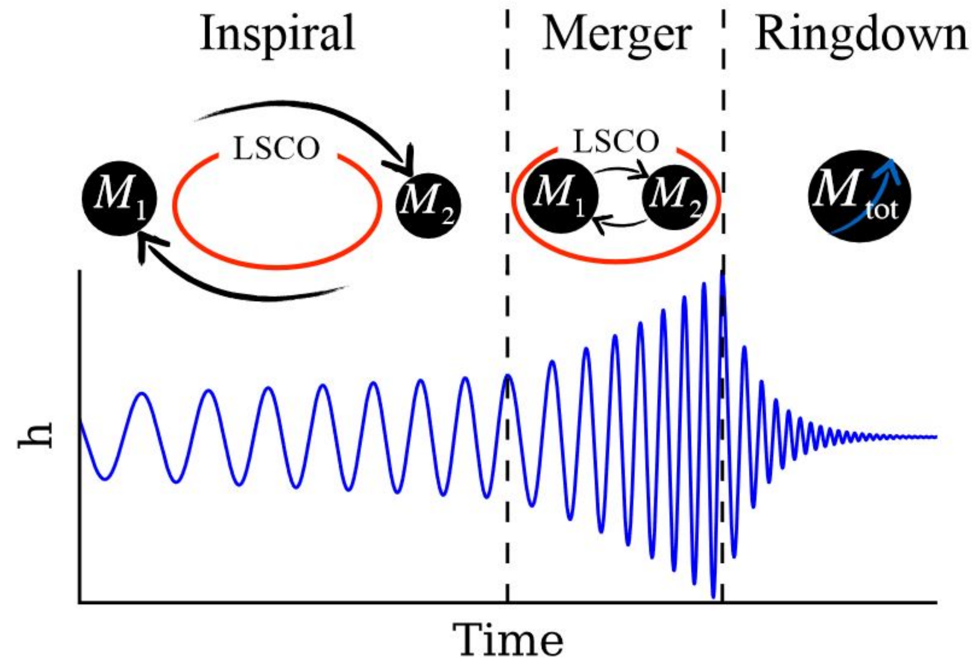
Binary systems:



Binary systems:

Simple way to estimate frequency at merger:
Last stable circular orbit around a non-spinning black hole

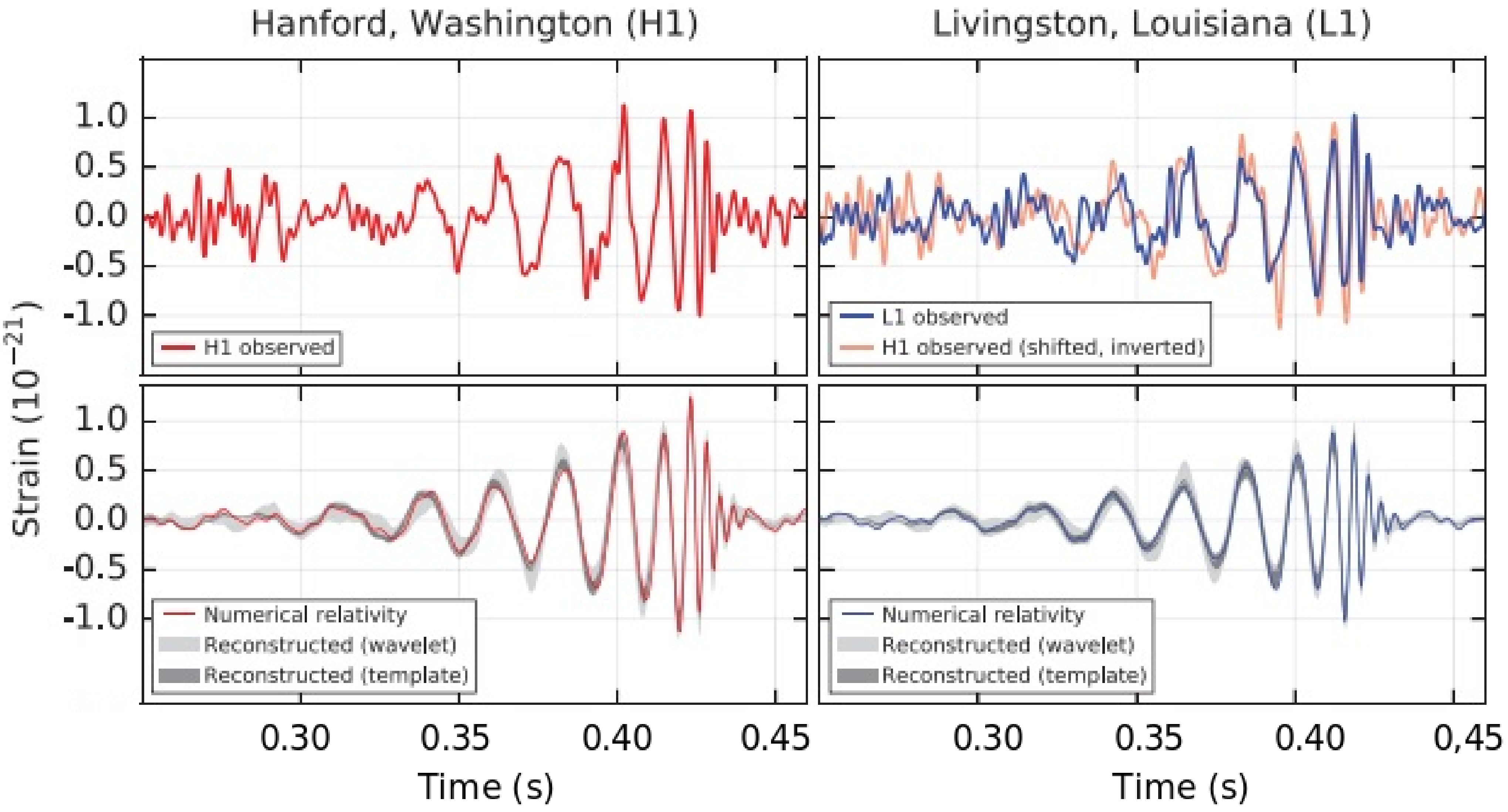
$$r_{\mathrm{LSCO}} = 6\,\frac{G\,(m_1 + m_2)}{c^2}$$



$$\omega_{\mathrm{GW,LSCO}} = 2\sqrt{\frac{G\,(m_1 + m_2)}{r_{\mathrm{LSCO}}^3}} = \frac{2\,c^3}{6^{3/2}\,G\,(m_1 + m_2)}$$

$$\boxed{\omega_{\mathrm{GW,LSCO}} = 460\,\mathrm{Hz}\,\frac{60\,M_{\mathrm{sun}}}{(m_1 + m_2)}}$$

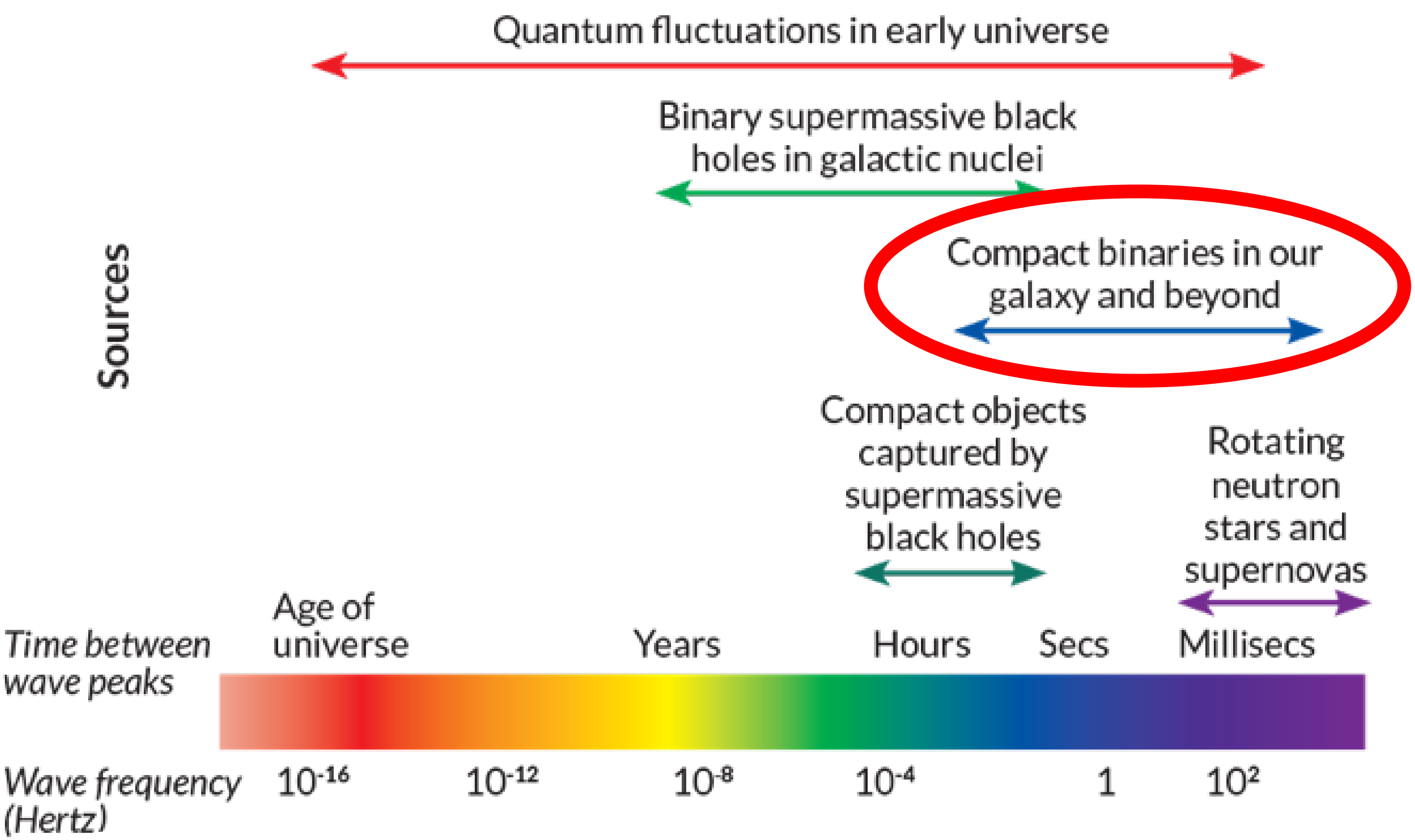
Binary systems:



Abbott et al. 2016

Binary systems:

What are the astrophysical objects with non-zero quadrupole?



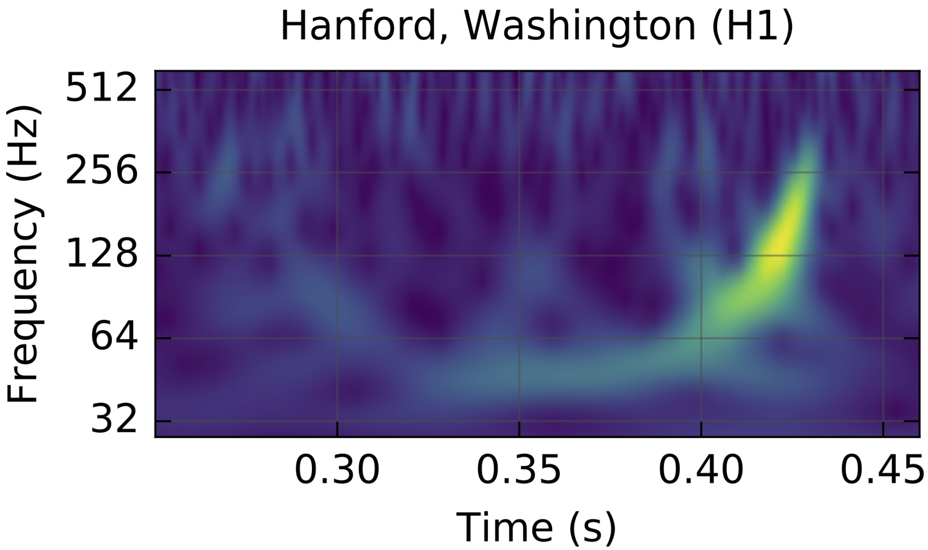
Compact binaries will be the focus of this course!

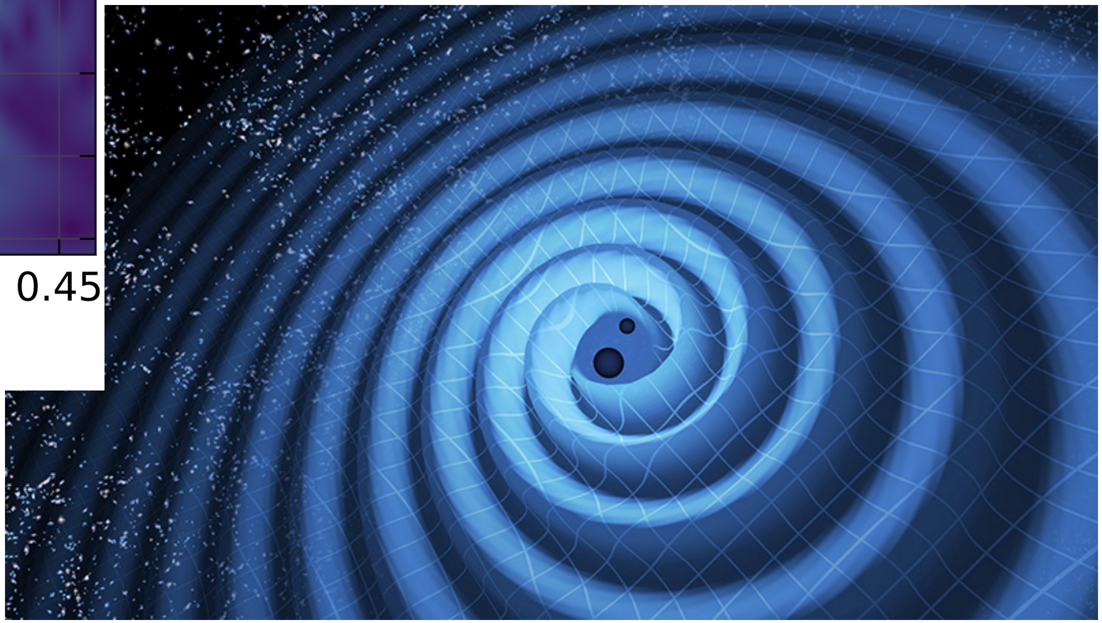
References:

*** James B. Hartle, Gravity: An Introduction to Einstein's General Relativity, Pearson**

*** Jolien D. E. Creighton & Warren G. Anderson, Gravitational-Wave Physics and Astronomy: An Introduction to Theory, Experiment and Data Analysis, Wiley Series in Cosmology**

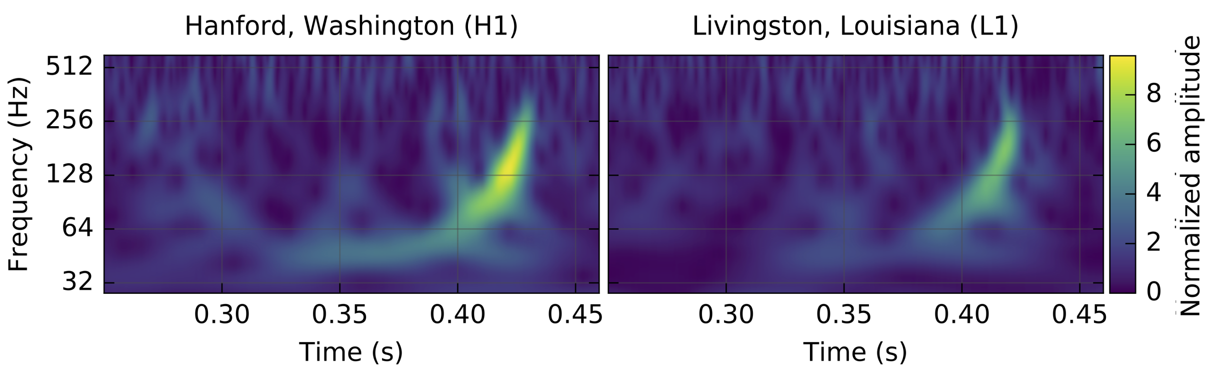
Observational Facts



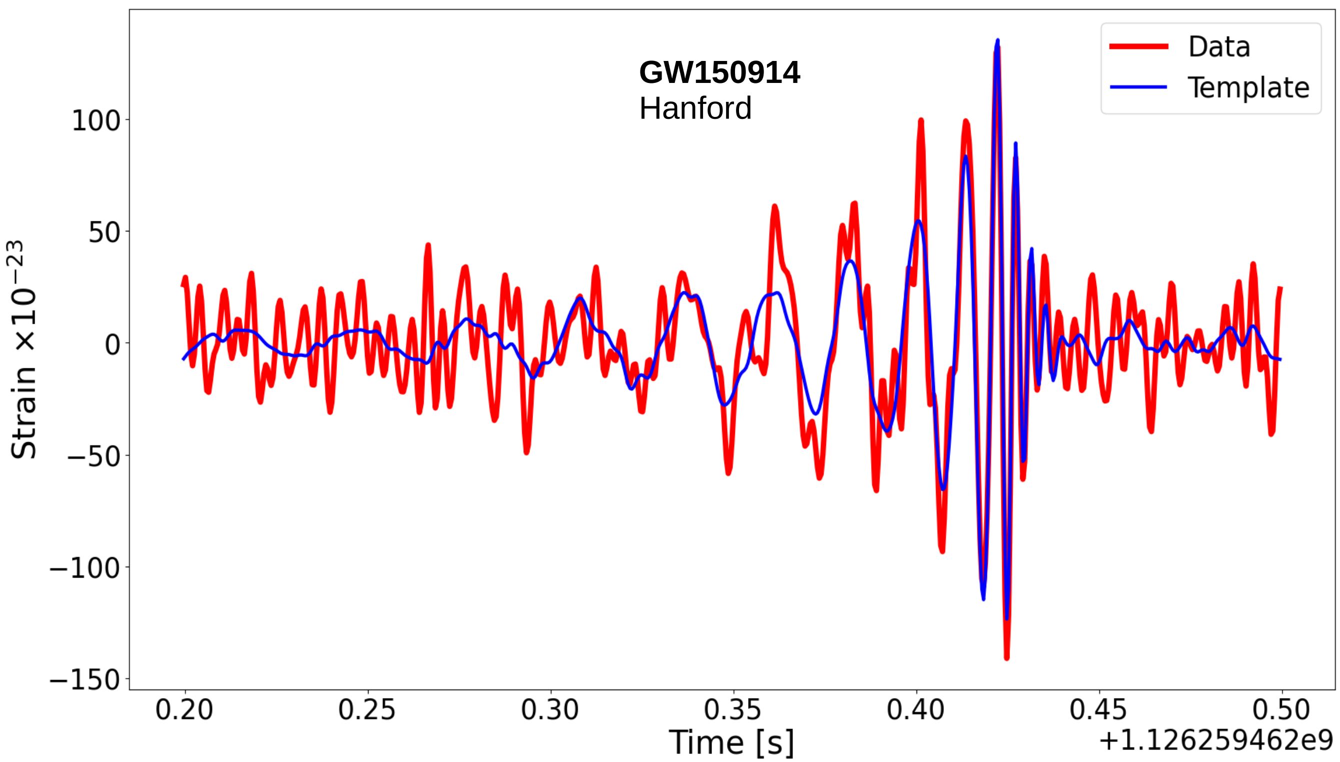


Observing runs:

GW150914: the first binary black hole (BBH)

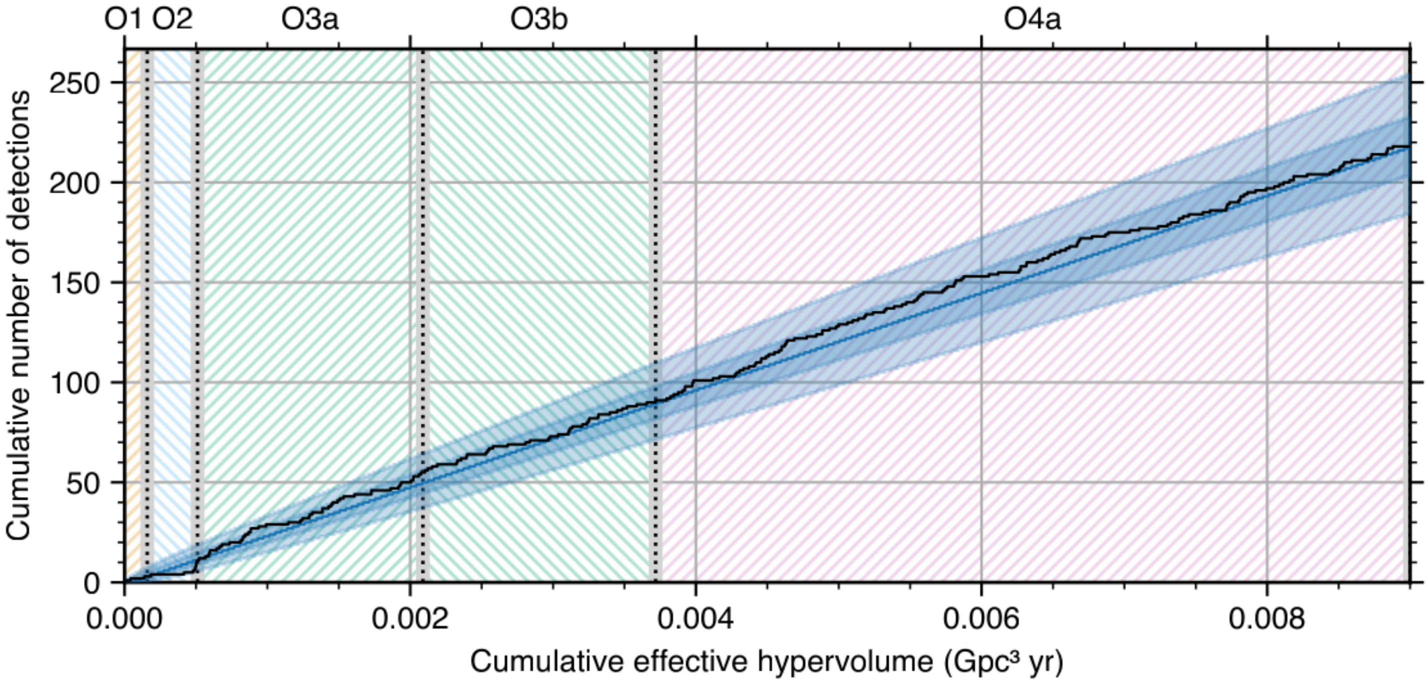


Abbott et al. 2016, PhRvL, 116, 1102



Observing runs:

https://dcc.ligo.org/P2400293/public

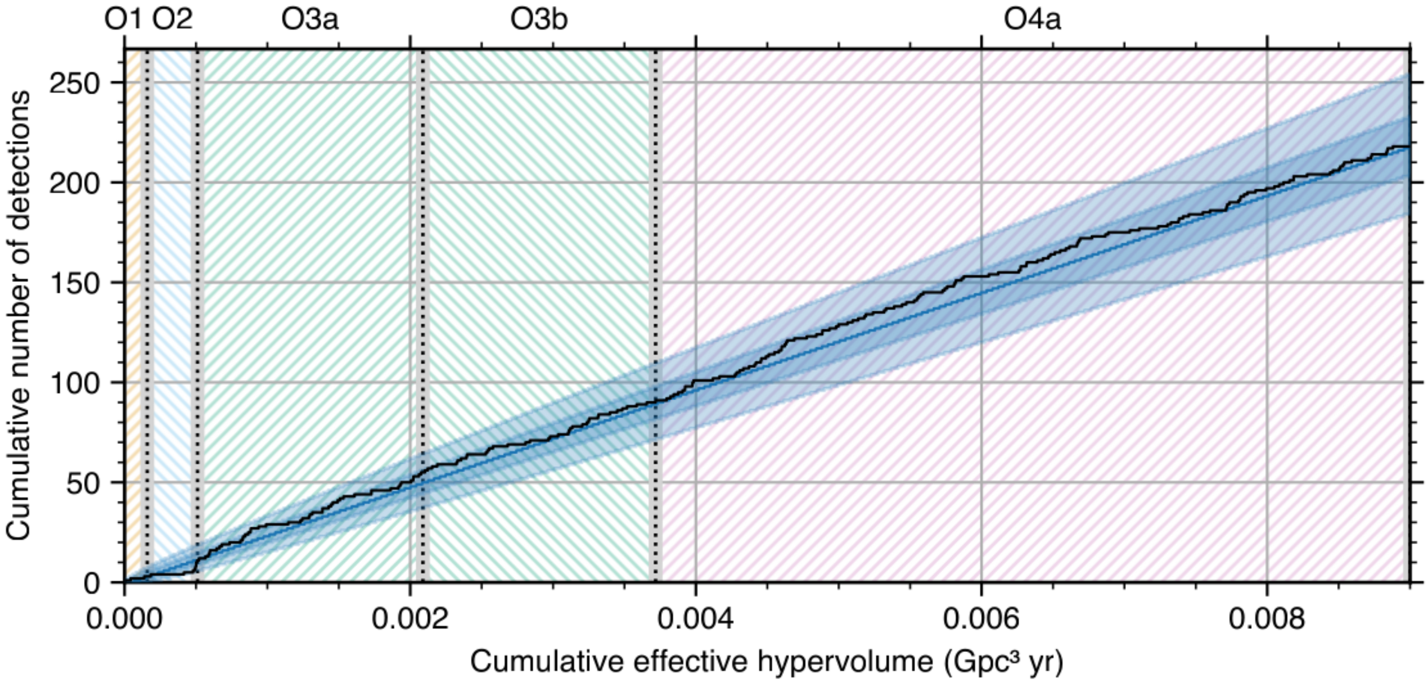


4 observing runs:
2015: O1
2017: O2
2019 – 2020: O3 (O3a+O3b)
2023 – 2025: O4 (O4a+O4b+O4c)

https://gracedb.ligo.org/

Observing runs:

https://dcc.ligo.org/P2400293/public



GWTC-4: O1 + O2 + O3 + O4a

218 GW event candidates
most of them BBHs

(Abbott et al. 2021, GWTC-2; Abbott et al. 2022, GWTC-2.1; Abbott et al. 2022, GWTC-3; Abac et al. 2025, GWTC-4)

O4c ended November 18 2025

https://gracedb.ligo.org/

Disclaimer: Catalog names

Observing runs:

2015: O1

2017: O2

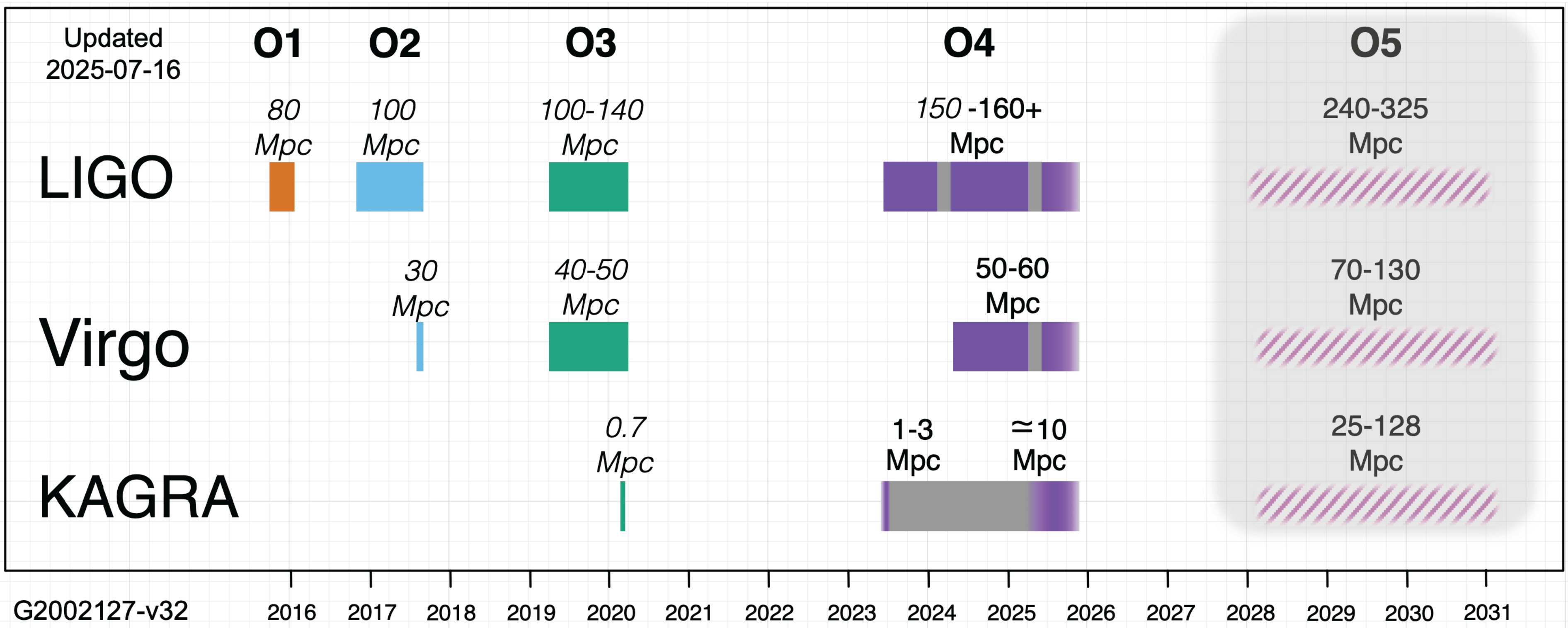
2019 – 2020: O3
- **O3a**
- **O3b**

2023 – 2025: O4
- **O4a**
- **O4b**
- **O4c**

Catalogs:
GWTC:= Gravitational Wave Transient Catalog

GWTC-1: O1+O2
GWTC-2: O1+O2+O3a
GWTC-3: O1+O2+O3a+O3b
GWTC-4: O4a

Observing runs:



$$d \sim d_{\rm BNS} \left(\frac{m_{\rm chirp}}{m_{\rm chirp,BNS}}\right)^{5/6} \sim 2\,{\rm Gpc} \left(\frac{d_{\rm BNS}}{130\,{\rm Mpc}}\right) \left(\frac{m_{\rm chirp}/m_{\rm chirp,BNS}}{25}\right)^{5/6}$$

See Abbott et al. 2019, Observing scenarios paper

https://dcc.ligo.org/LIGO-P1200087-V58/public

Detectors:

Advanced LIGO (Livingstone + Hanford, US)
Advanced Virgo (Pisa, Italy)

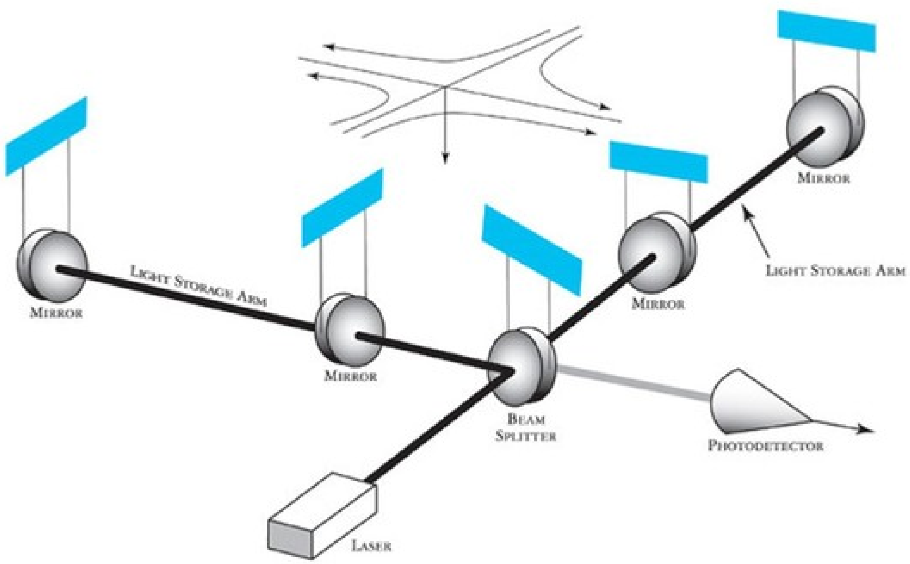
LIGO Lab/Virgo

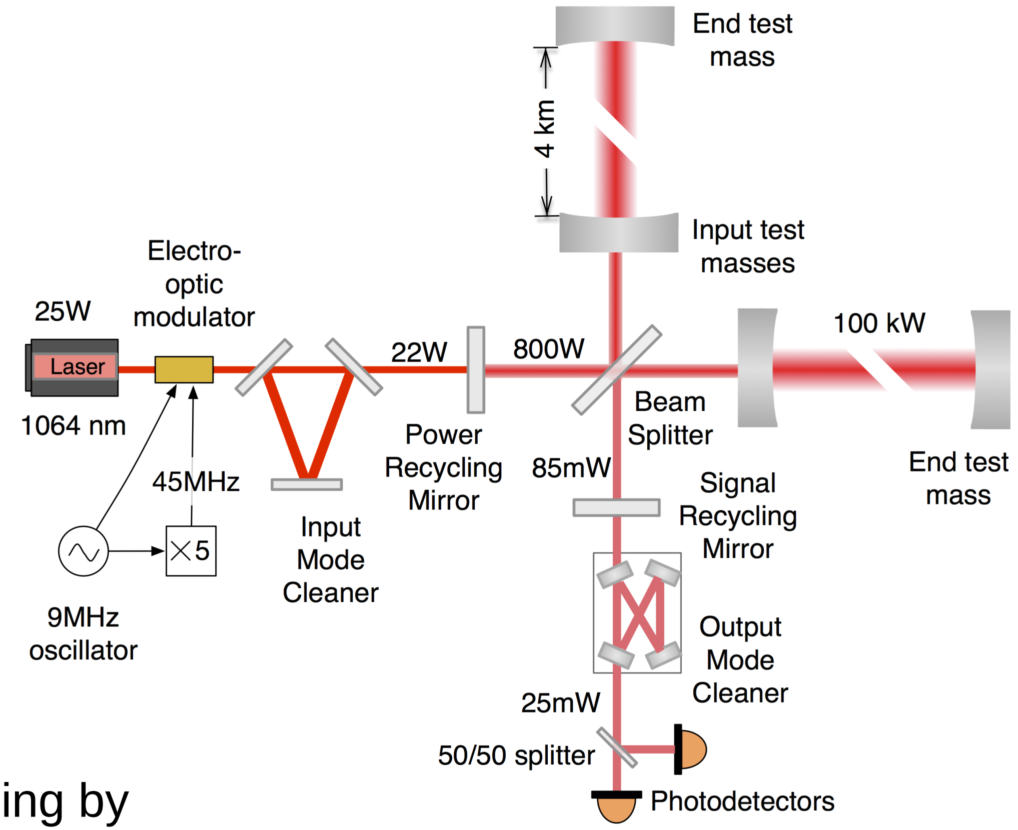
Michelson interferometers

Design started in the '90s
First science runs ~ 2007 (no detection)
Being upgraded in 2007 – 2015
First run LIGO advanced detectors 2015
First run Virgo advanced detector 2017

Frequency range:
~ 10 – 10'000 Hz
fine to observe mergers of compact binaries

Detectors:



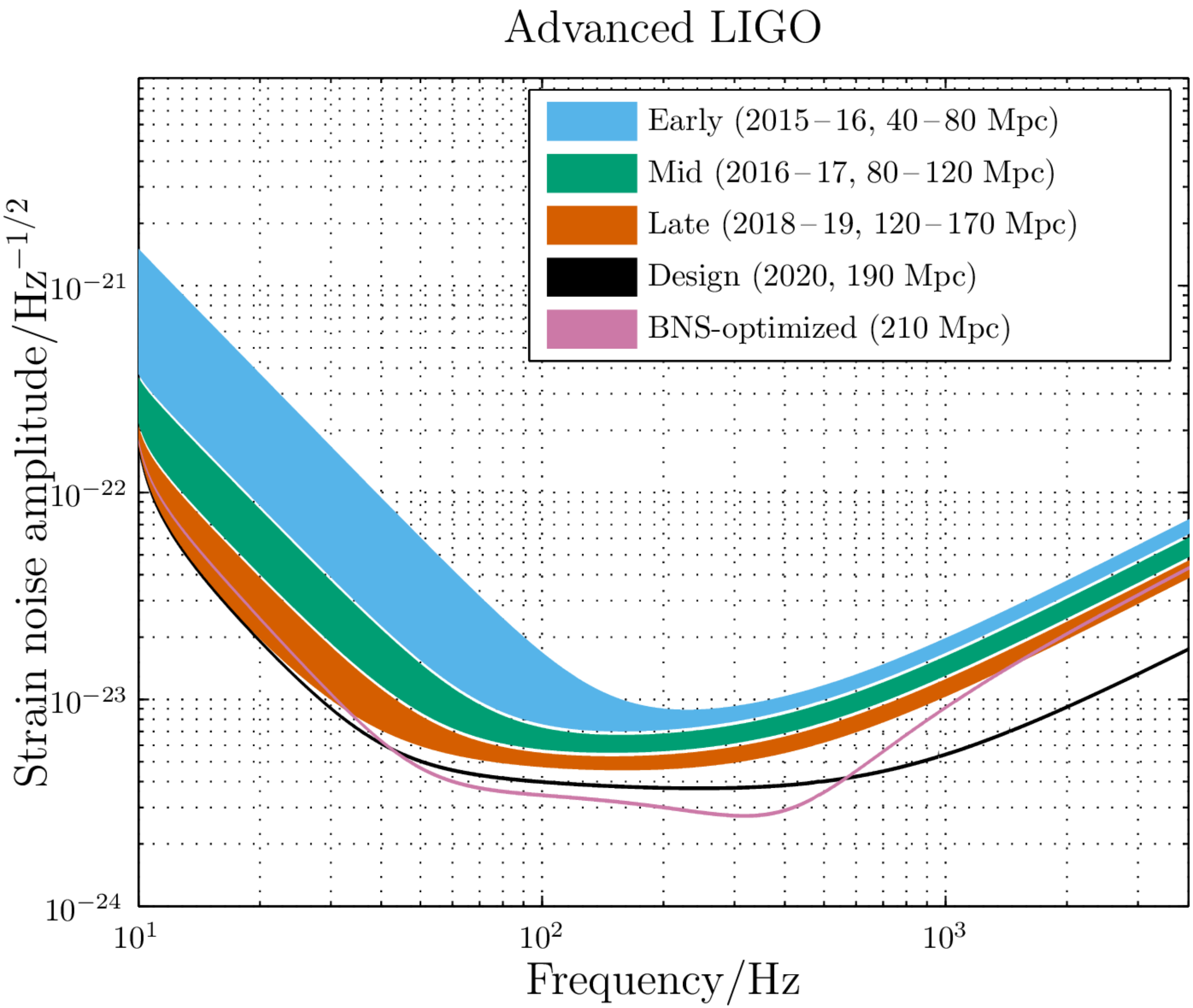


Destructive interference if no GWs are passing by

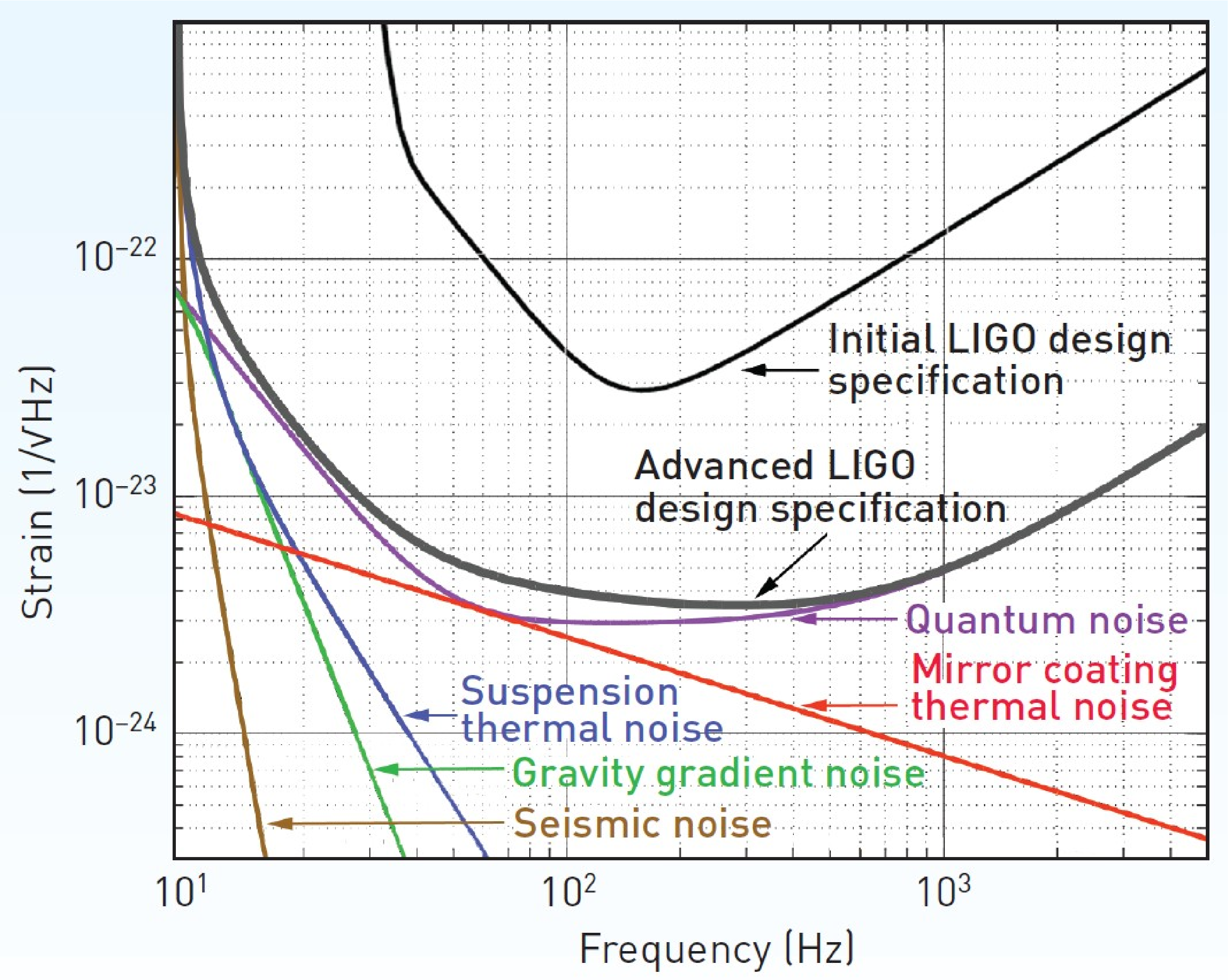
When GW passes through it, it changes the length of one arm wrt the other → constructive interference

https://ligo.caltech.edu/system/video_items/files/21/Einsteins_messengers_hi_res_Nov_17_MPEG720p.mp4?1447873693

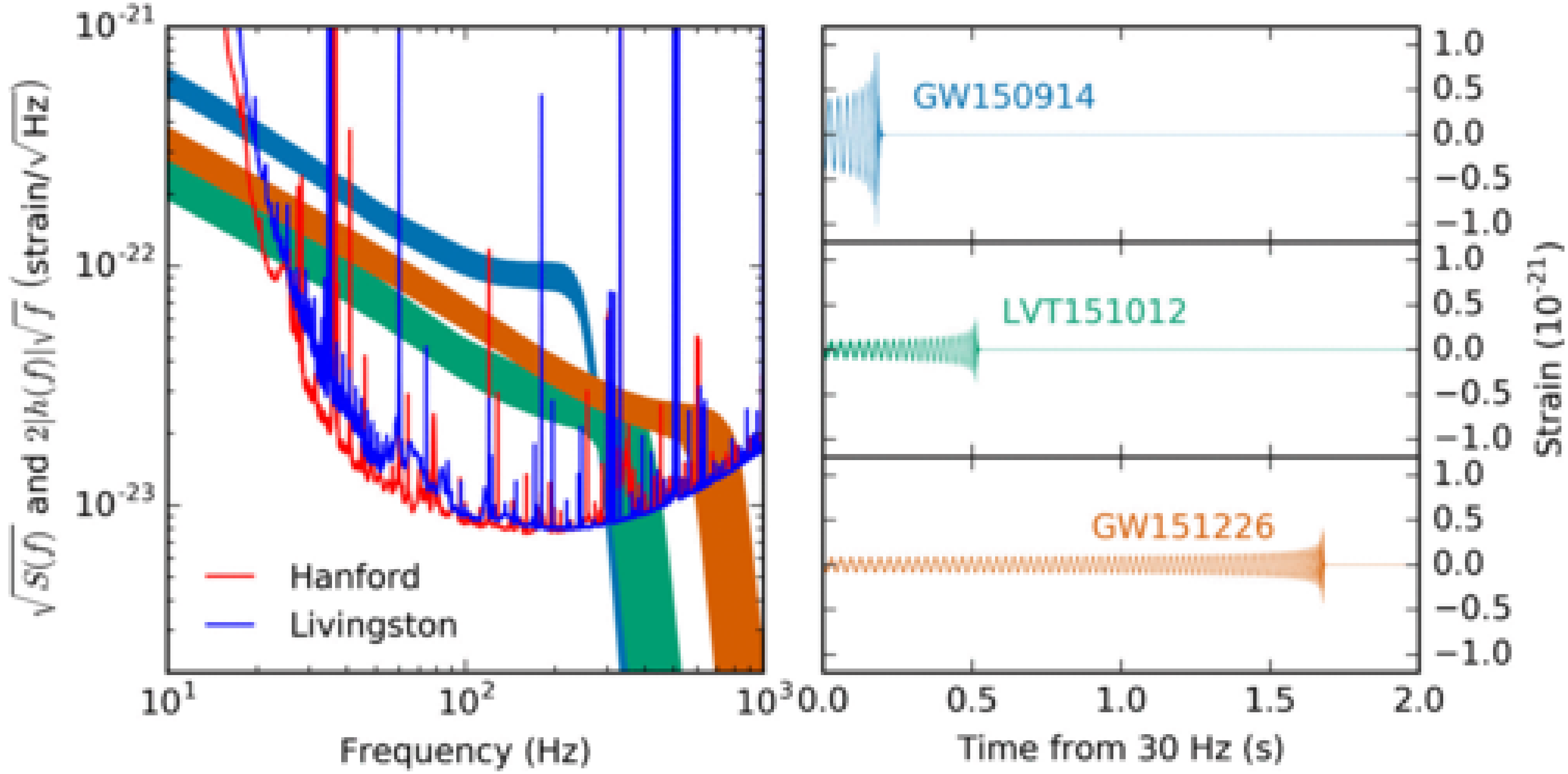
Sensitivity curve:



Main sources of noise at design:



Duration of signals in band:



Duration of signal depends on 2 things:

- its frequency, i.e. how much the signal overlap with the sensitivity band of LVK

- its amplitude, i.e. whether the strain is sufficiently large to observe it

Abbott et al. 2016, PRX, 6, 041015
https://journals.aps.org/prx/abstract/10.1103/PhysRevX.6.041015#fulltext

Observables:

15 Observables:

- 2 masses
- 6 spin components
- redshift of merger (or, to better say: luminosity distance)
- 2 sky positions (RA, DEC)
- reference time (e.g. merger time)
- polarization angle
- inclination of binary wrt observer
- phase of binary at reference time

+ eccentricity
+ tidal deformation

Observables:

Mass and Spins:

black holes (BHs) are uniquely defined by mass & spin (electric charge deemed to be negligible)

for neutron stars also radius and tidal deformability → equation of state

Masses:

If black hole (BH) is in binary system, we have 2 masses:

m_1, m_2 = Mass of first BH, Mass of second BH

but LIGO-Virgo measured two combinations of m_1, m_2 :

Chirp mass:

$$m_{\rm chirp} = \frac{(m_1\, m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

change of frequency during inspiral scales with it

Total mass:

$$M = (m_1 + m_2)$$

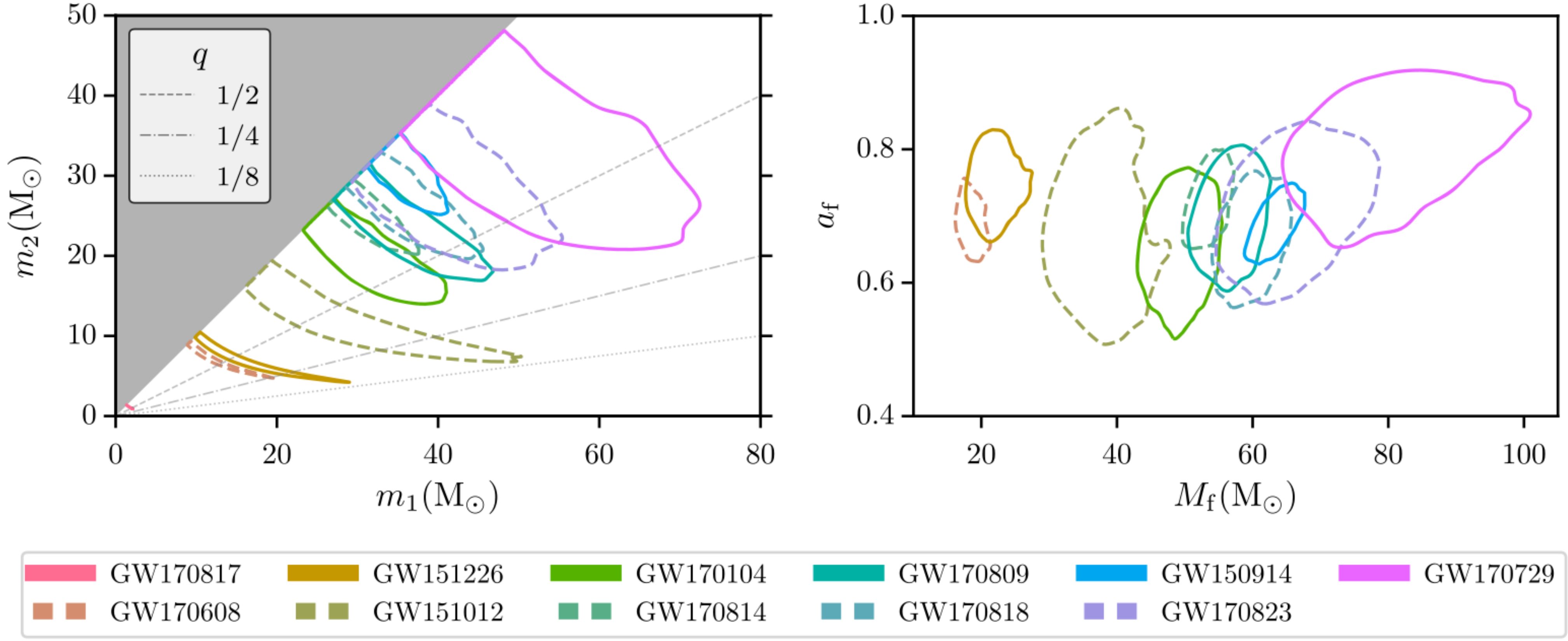
frequency at merger scales with it

Other relevant mass (for phase of GWs): mass ratio

$$q = \frac{m_2}{m_1}$$

Masses:

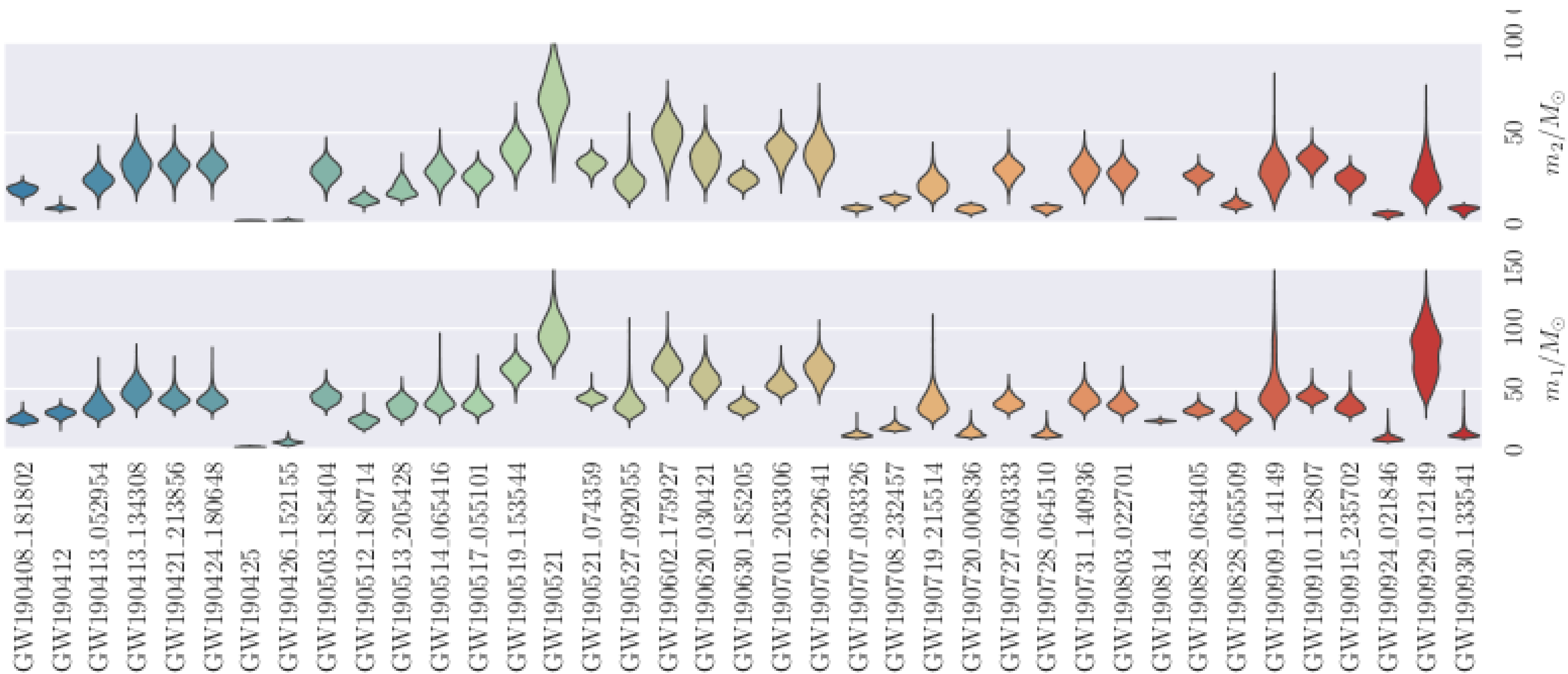
Masses in the first GW transient catalog (GWTC-1):



Abbott et al. 2019, GWTC1, https://ui.adsabs.harvard.edu/abs/2019PhRvX...9c1040A/

Masses:

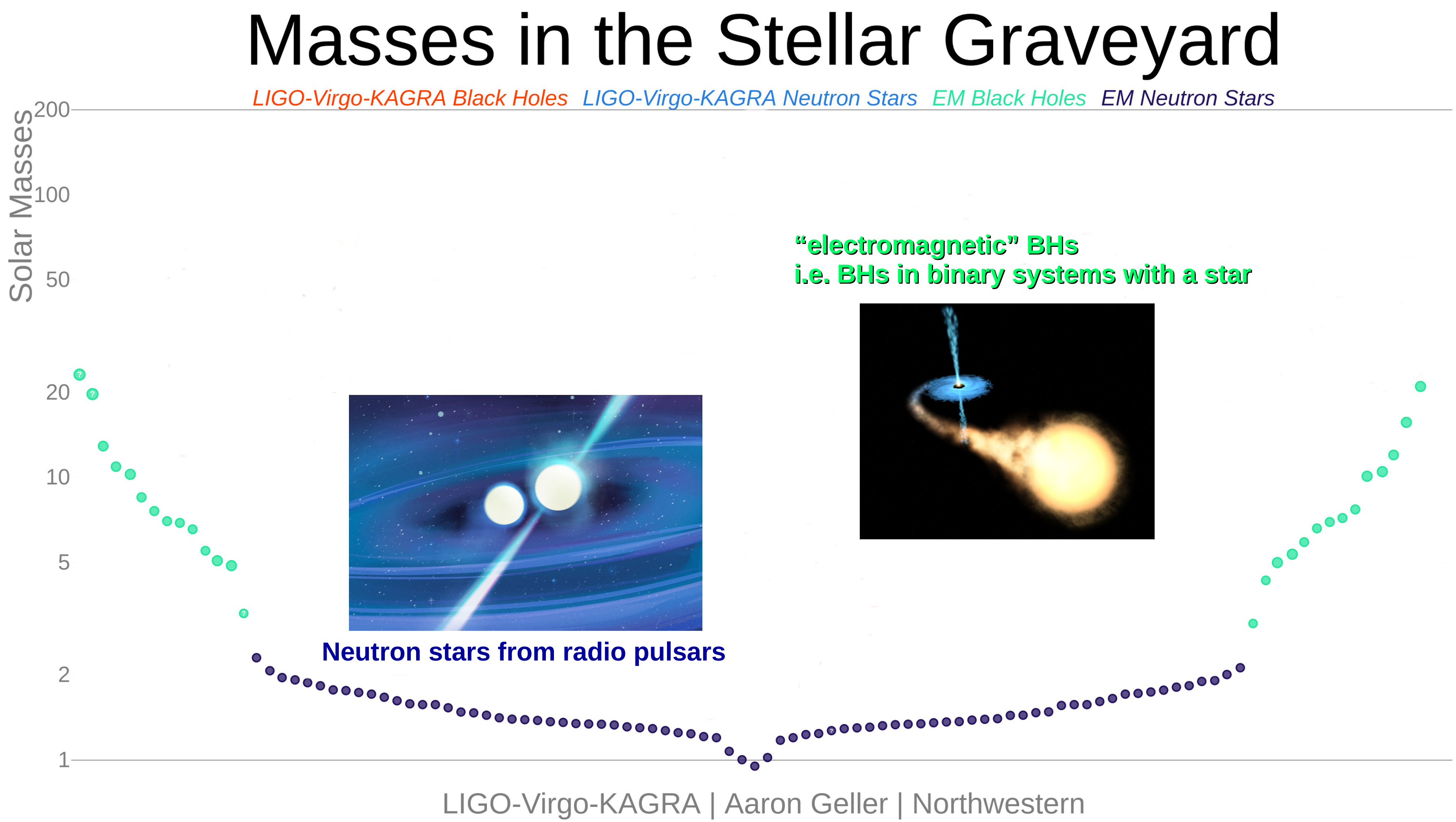
Masses in the second GW transient catalog (GWTC-2):



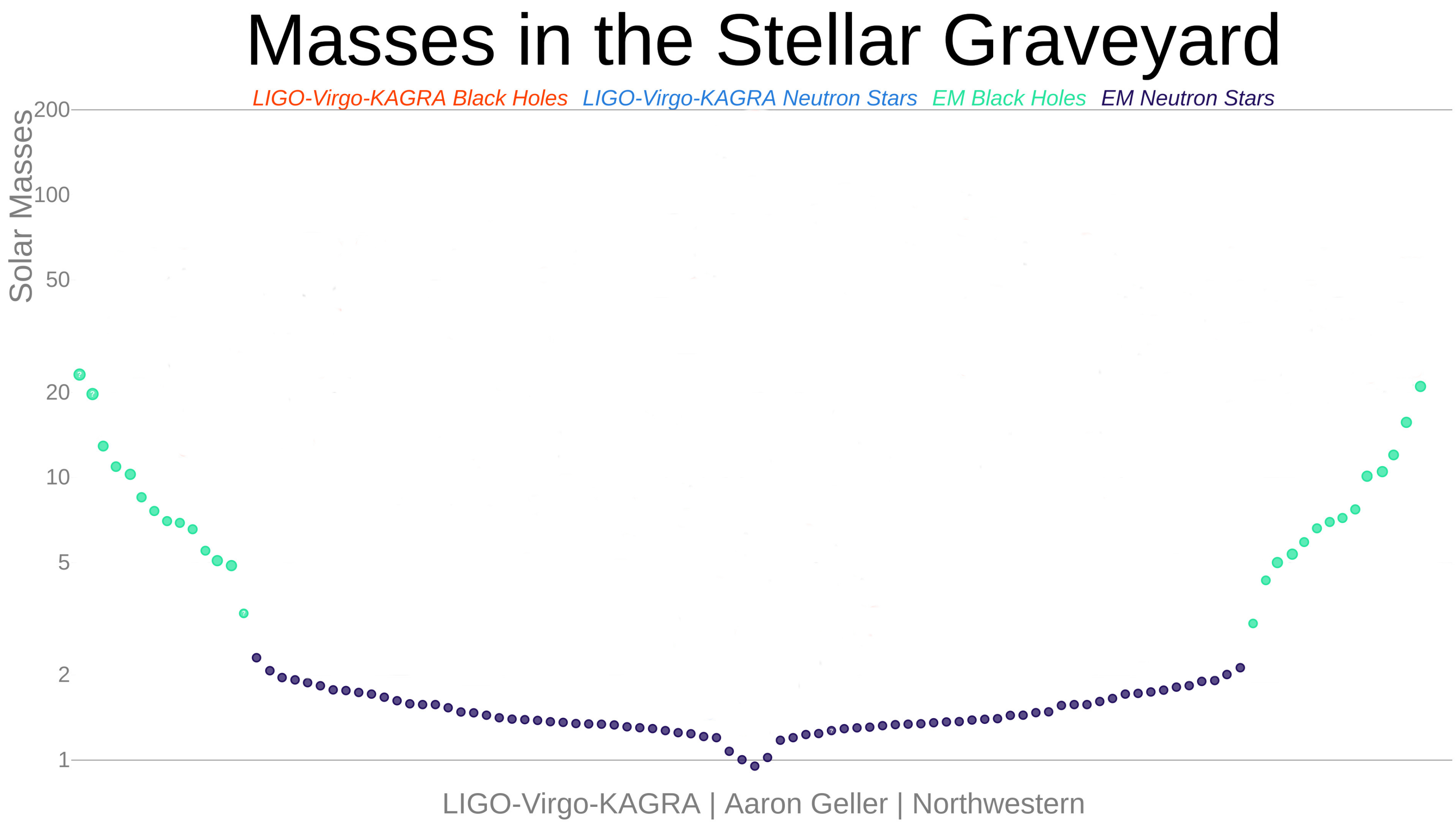
Masses of the 39 new event candidates in O3a

Abbott et al. 2020, GWTC-2, 2020, https://arxiv.org/abs/2010.14527

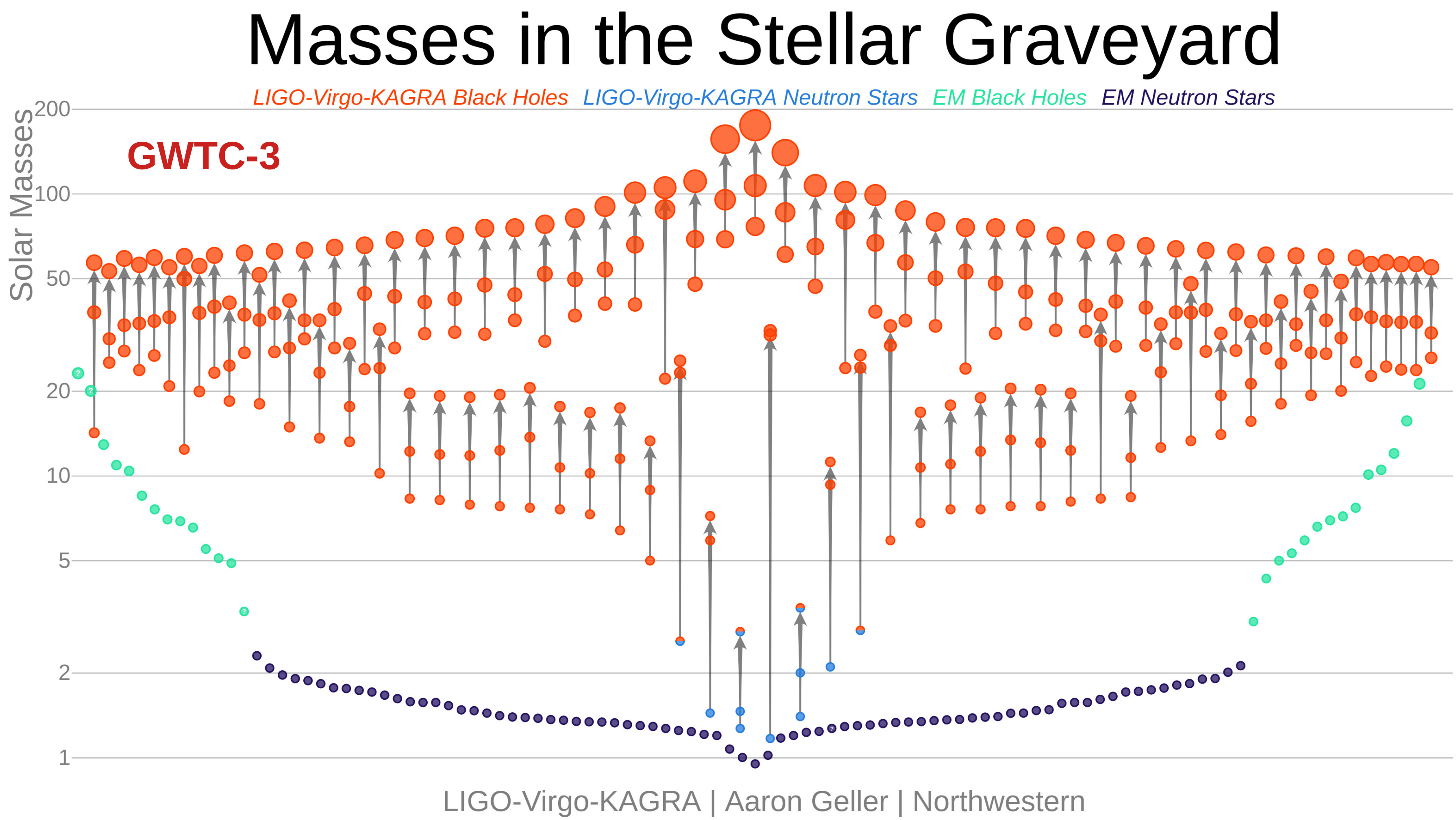
Masses:


Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
"electromagnetic" BHs
i.e. BHs in binary systems with a star
Neutron stars from radio pulsars
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Masses:

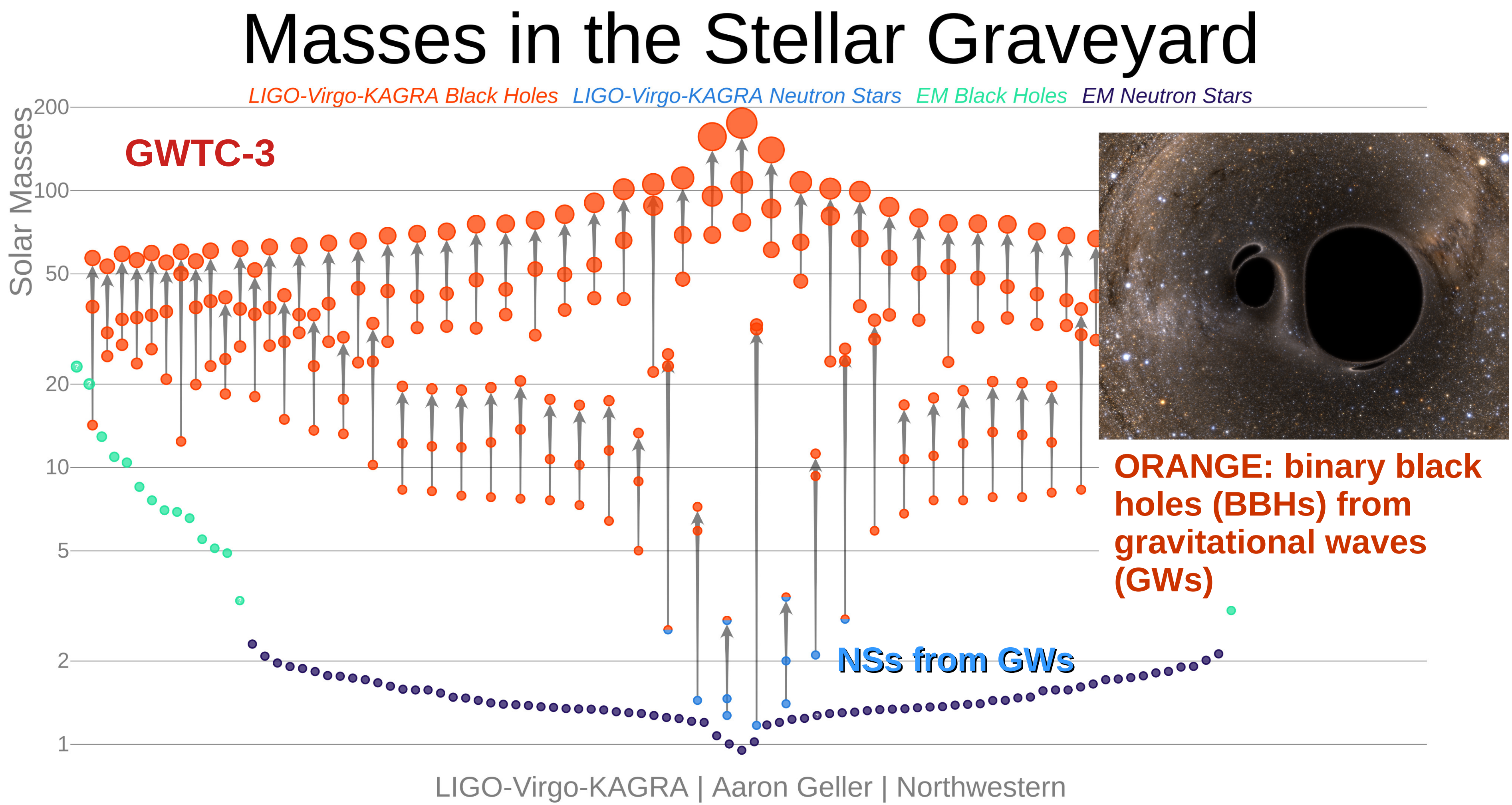

Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Masses:



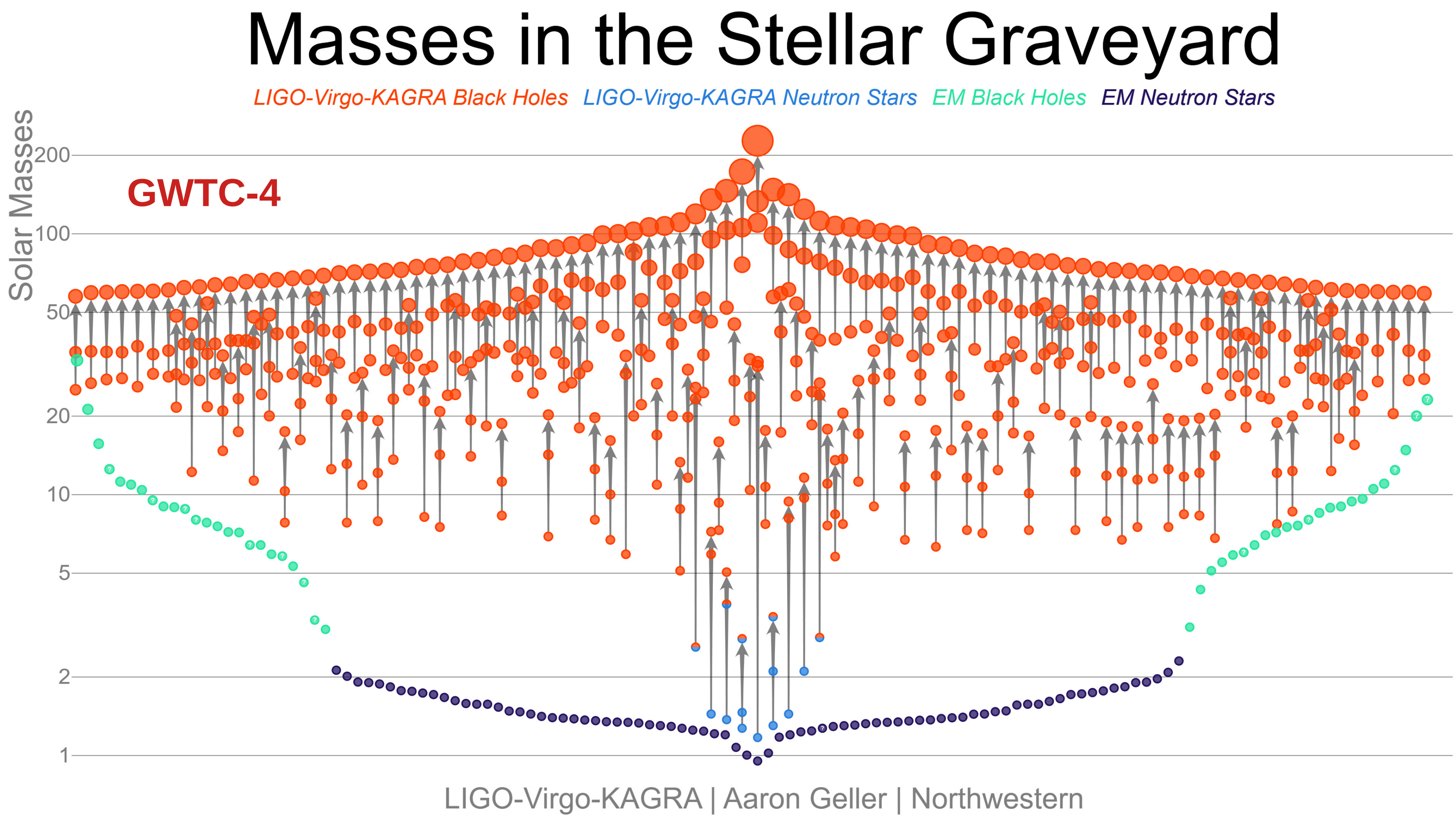
Abbott et al. 2023, GWTC-3

Masses:



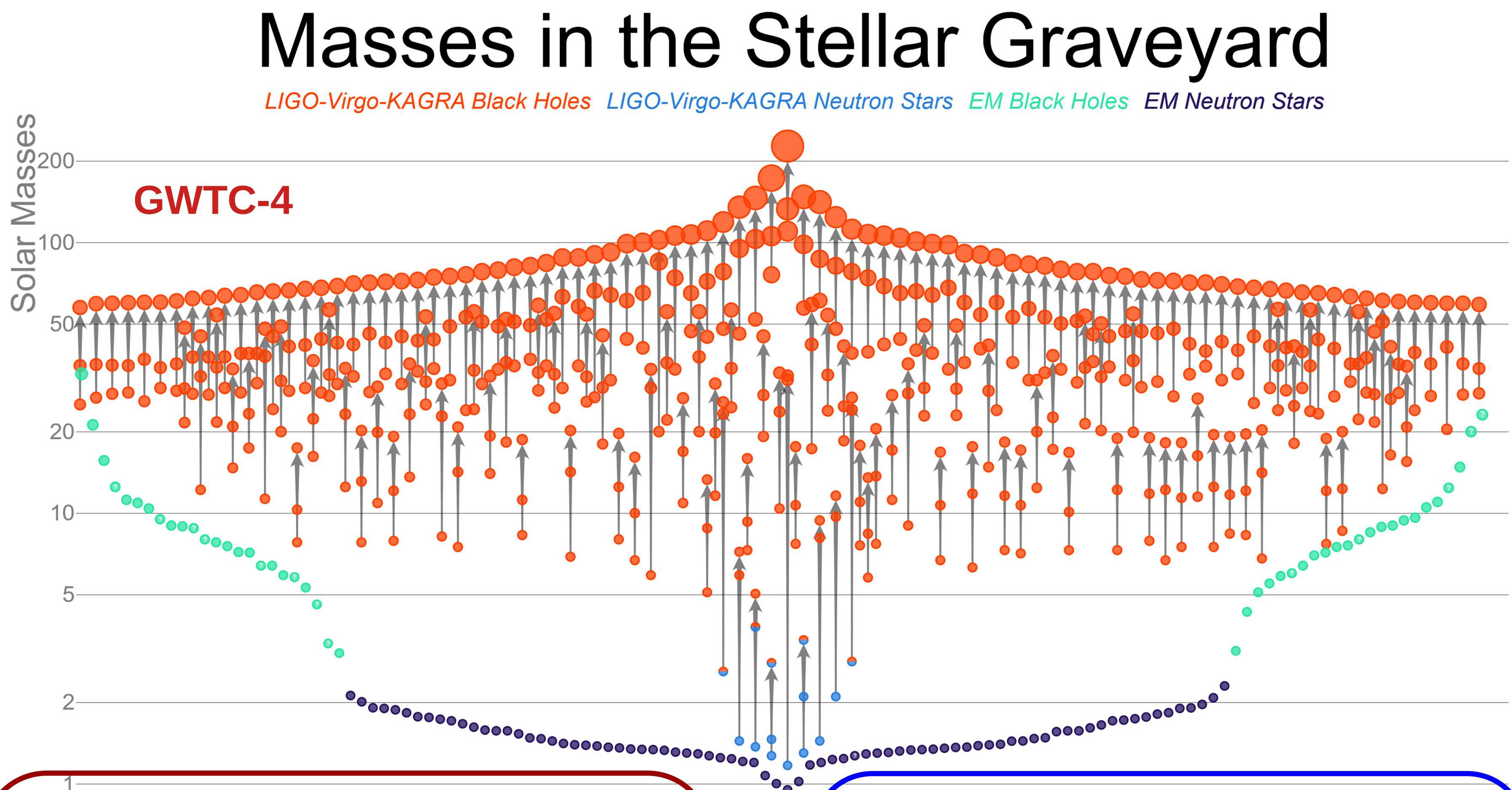
Abbott et al. 2023, GWTC-3

Masses:



GWTC-4 population paper

Masses:



Before LIGO – Virgo:

* No BBHs or BHNSs known

* A dozen stellar-origin BHs
  with dynamical mass measure

* No stellar-origin BHs with mass >20 M⊙

* A dozen BNSs but NO MERGER

Now:

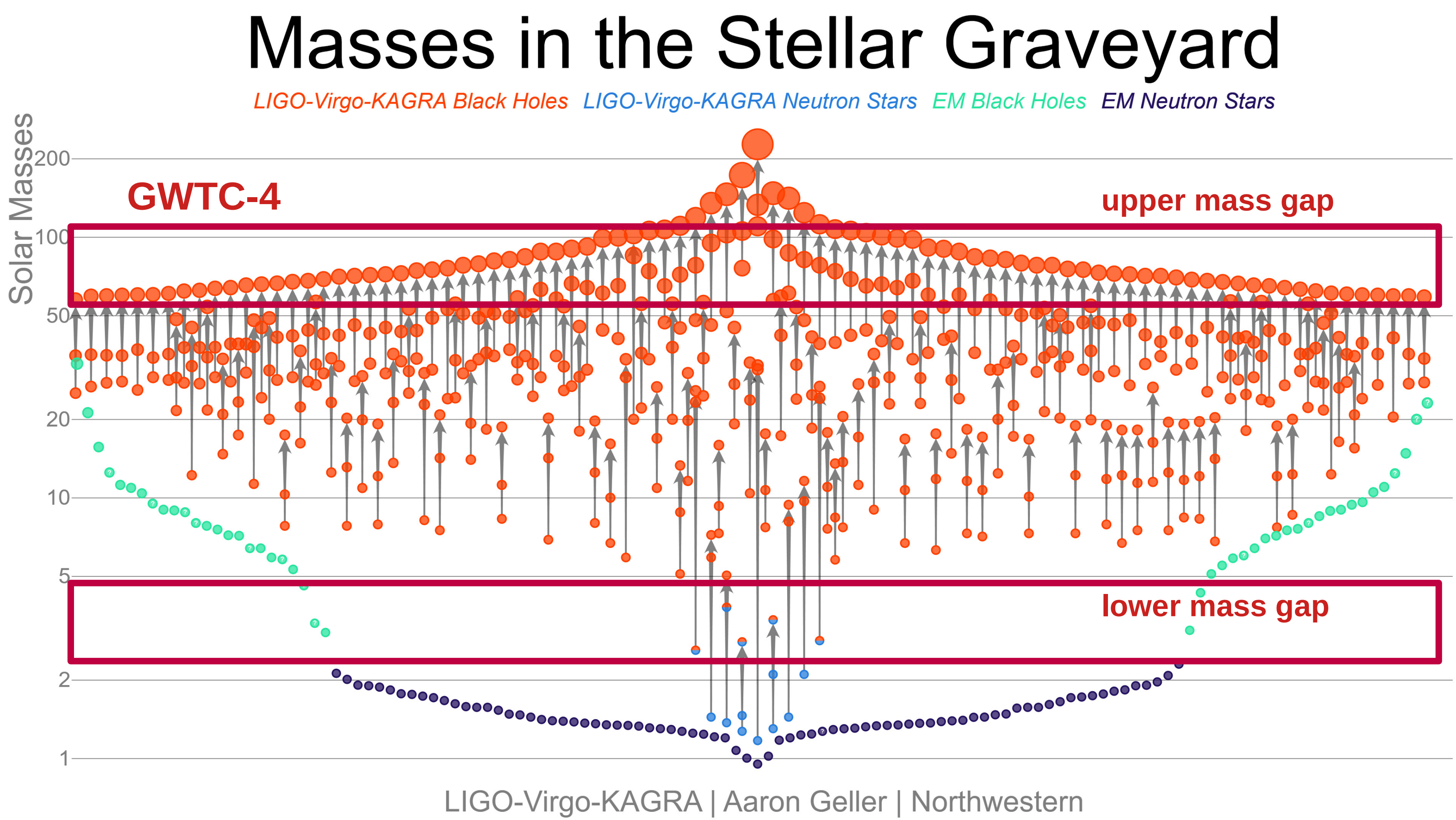
* >200 BBHs from gravitational waves

* 3 BHNS candidates

* 2 BNS mergers and confirmation of
  short GRB – BNS connection

* BH masses in the range ~ 3 – 250 M⊙

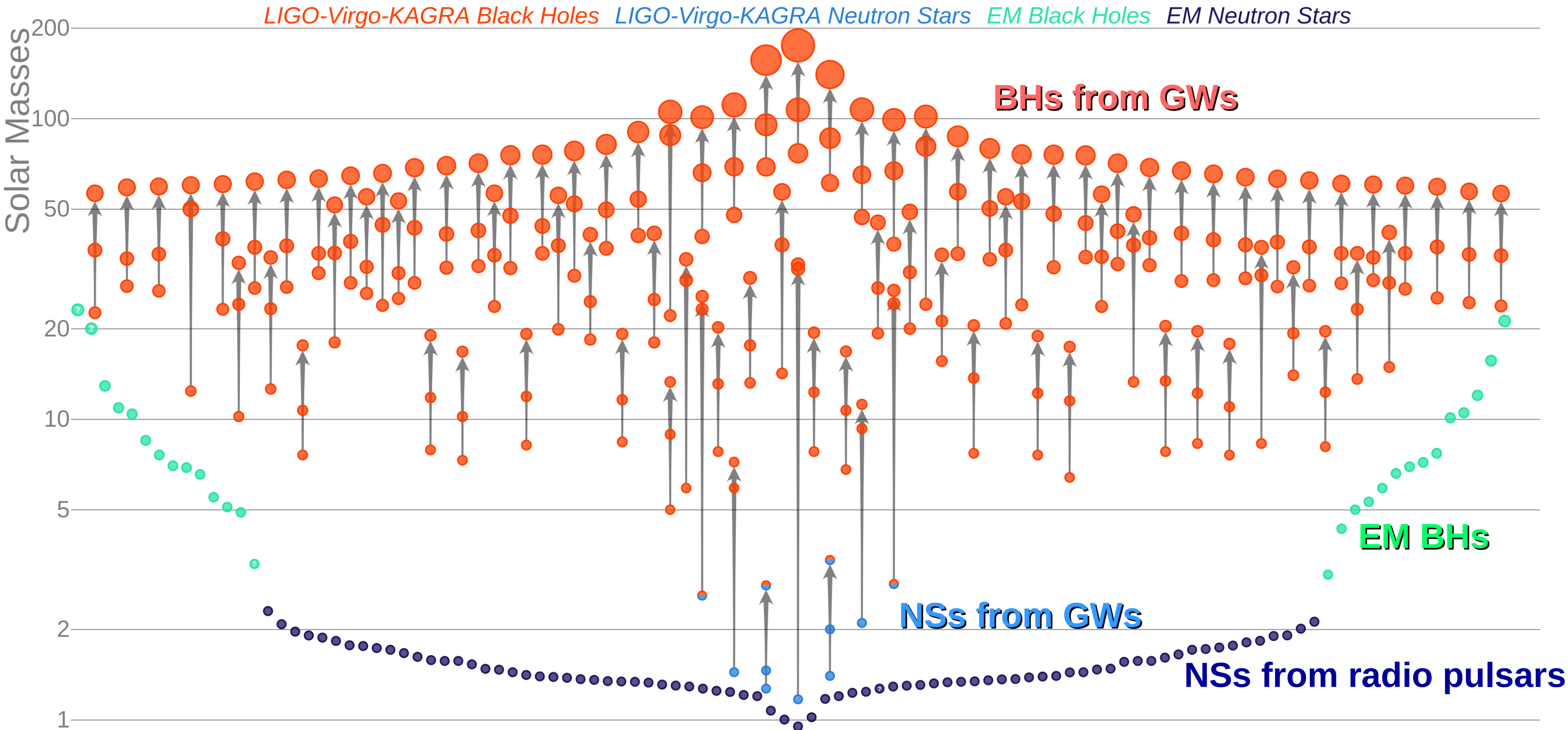
Masses:



GWTC-4 population paper

Masses:

Abbott et al. 2022, GWTC-3; credit: LIGO-Virgo-KAGRA | A. Geller | Northwestern



Before LIGO – Virgo:

* No BBHs or BHNSs known

* A dozen stellar-origin BHs with dynamical mass measure

* No stellar-origin BHs with mass >20 M⊙

* A dozen BNSs but NO MERGER

Now:

* 85 BBHs from gravitational waves

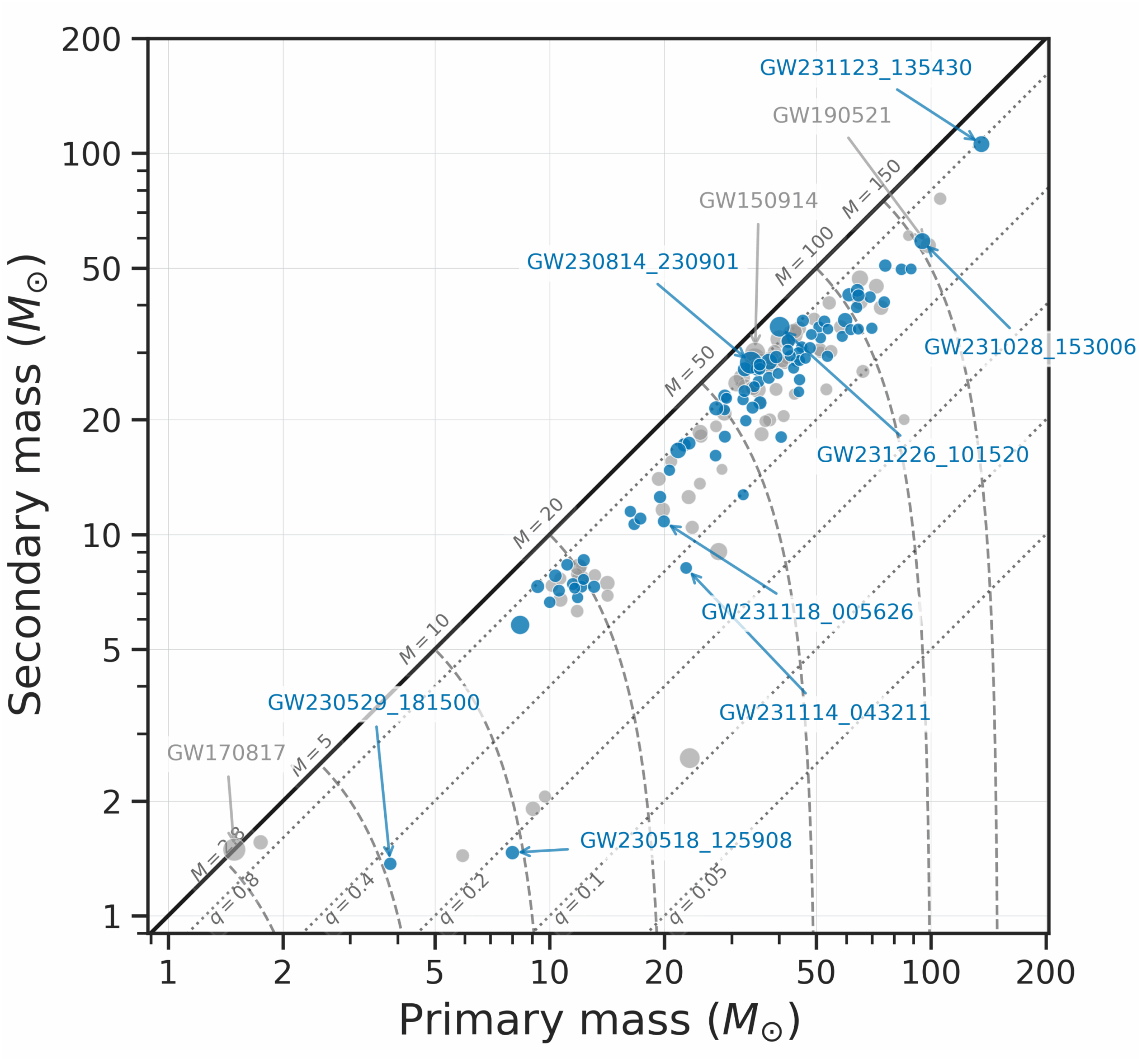
* 3 BHNS candidates

* 2 BNS mergers and confirmation of short GRB – BNS connection

* BH masses in the range ~ 3 – 200 M⊙

Masses:

https://ligo.org/detections/o4a-catalog/

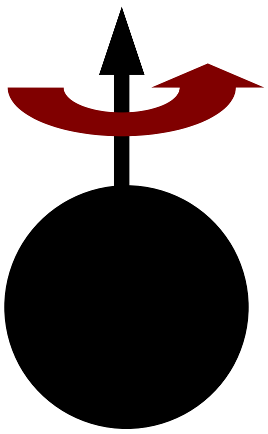


Spins:

$$\vec{\chi} = \frac{\vec{J}\,c}{G\,m_{\rm BH}^2}$$

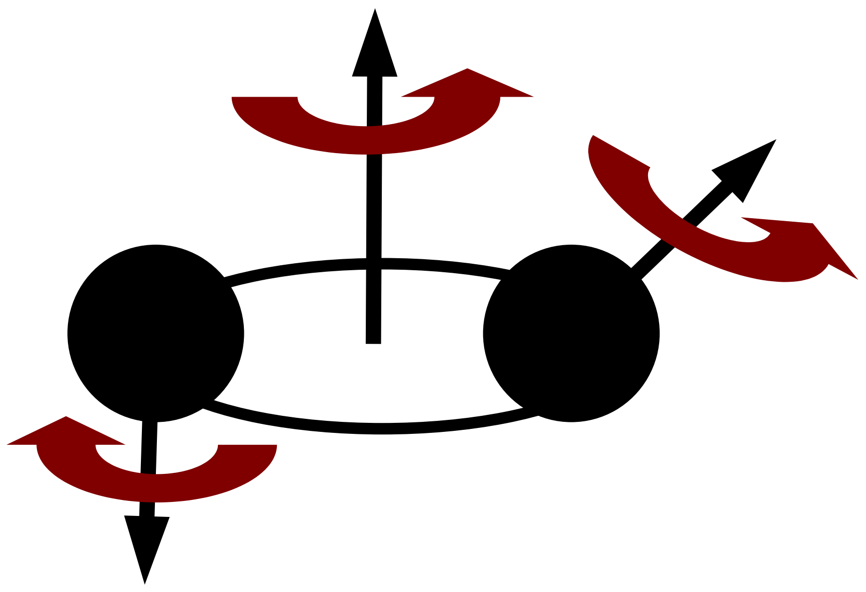
$\chi = 0 \rightarrow$ Schwarzschild BH

$\chi = 0.998 \rightarrow$ Maximally Rotating BH



If black hole is in binary
we have 6 spin components:

3 per each black hole



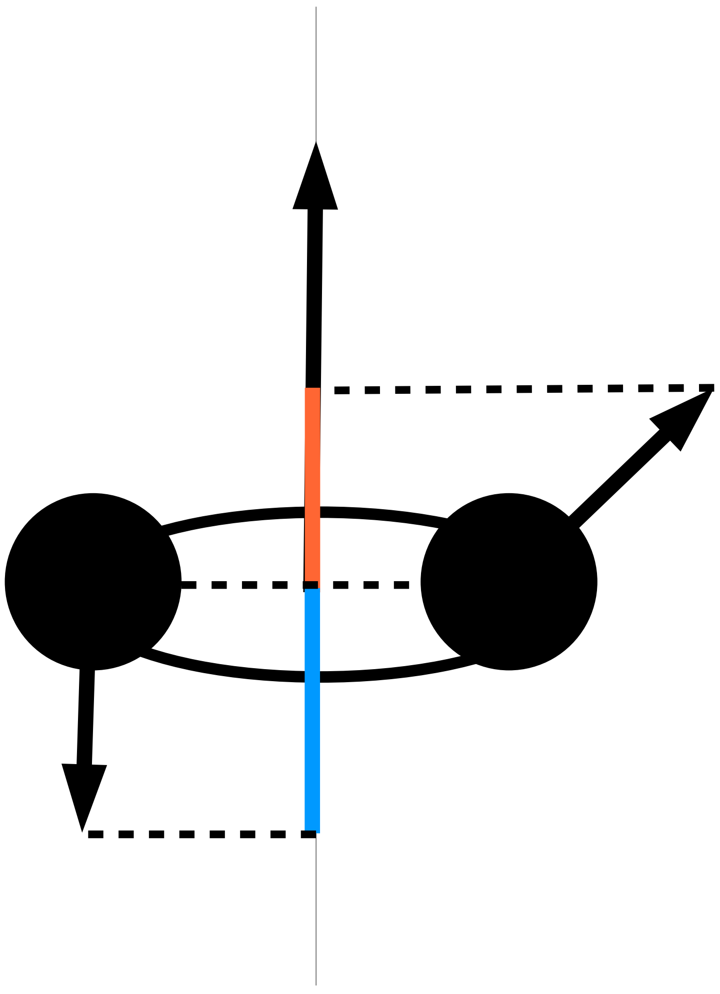
Spins:

LIGO – Virgo not enough to measure 6 spins, hence alternative params

1. EFFECTIVE SPIN:

$$\chi_{\rm eff} = \frac{(m_1\,\vec{\chi}_1 + m_2\,\vec{\chi}_2)}{(m_1 + m_2)} \cdot \frac{\vec{L}}{L}$$

$$-1 \leq \chi_{\rm eff} \leq 1$$



Spins:

LIGO – Virgo not enough to measure 6 spins, hence alternative params

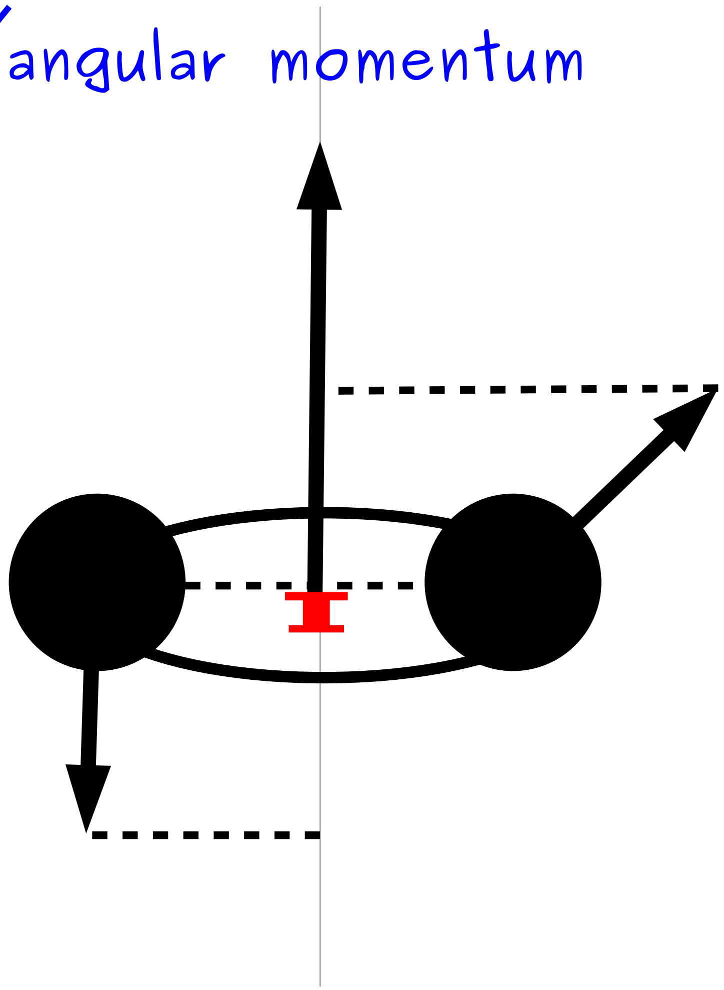
1. EFFECTIVE SPIN:

$$\chi_{\rm eff} = \frac{(m_1\,\vec{\chi}_1 + m_2\,\vec{\chi}_2)}{(m_1 + m_2)} \cdot \frac{\vec{L}}{L}$$

BBH orbital angular momentum

$$-1 \leq \chi_{\rm eff} \leq 1$$

Measured because affects phase of GWs
while orthogonal spin to binary ang. mom.
measures precession



Spins:

LIGO – Virgo not enough to measure 6 spins, hence alternative params

1. EFFECTIVE SPIN:

$$\chi_{\rm eff} = \frac{(m_1\,\vec{\chi}_1 + m_2\,\vec{\chi}_2)}{(m_1 + m_2)} \cdot \frac{\vec{L}}{L}$$

Components of spins perpendicular to BBH orbital angular momentum

2. Effective precession spin:

$$\chi_{\rm p} = \frac{1}{B_1\, m_1^2}\,\max\left(B_1\,\chi_{1,\perp},\; B_2\,\chi_{2,\perp}\right)$$

$$B_1 = 2 + 3\,\frac{q}{2} \qquad B_2 = 2 + \frac{3}{2\,q}$$

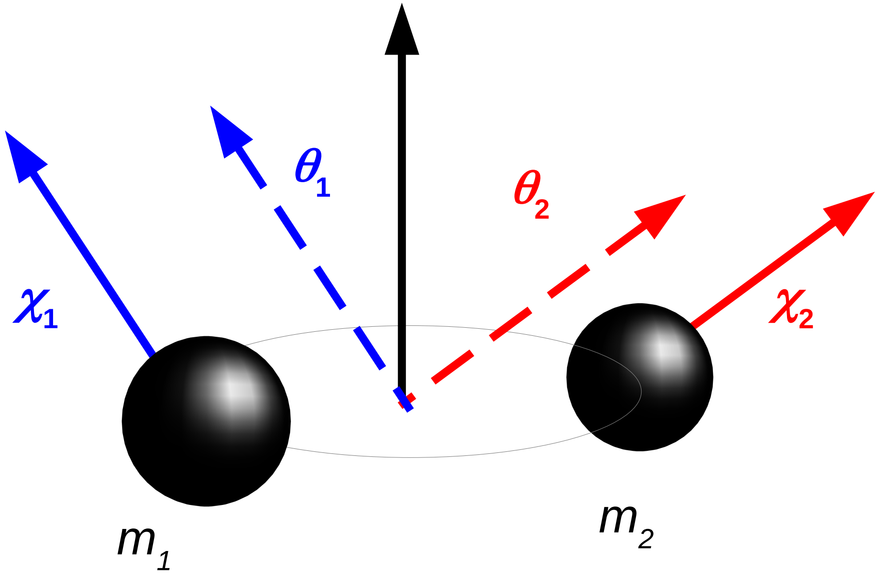
Spin components perpendicular to binary ang. mom.
measured by precession

Spins:

LIGO – Virgo not enough to measure 6 spins, hence alternative params

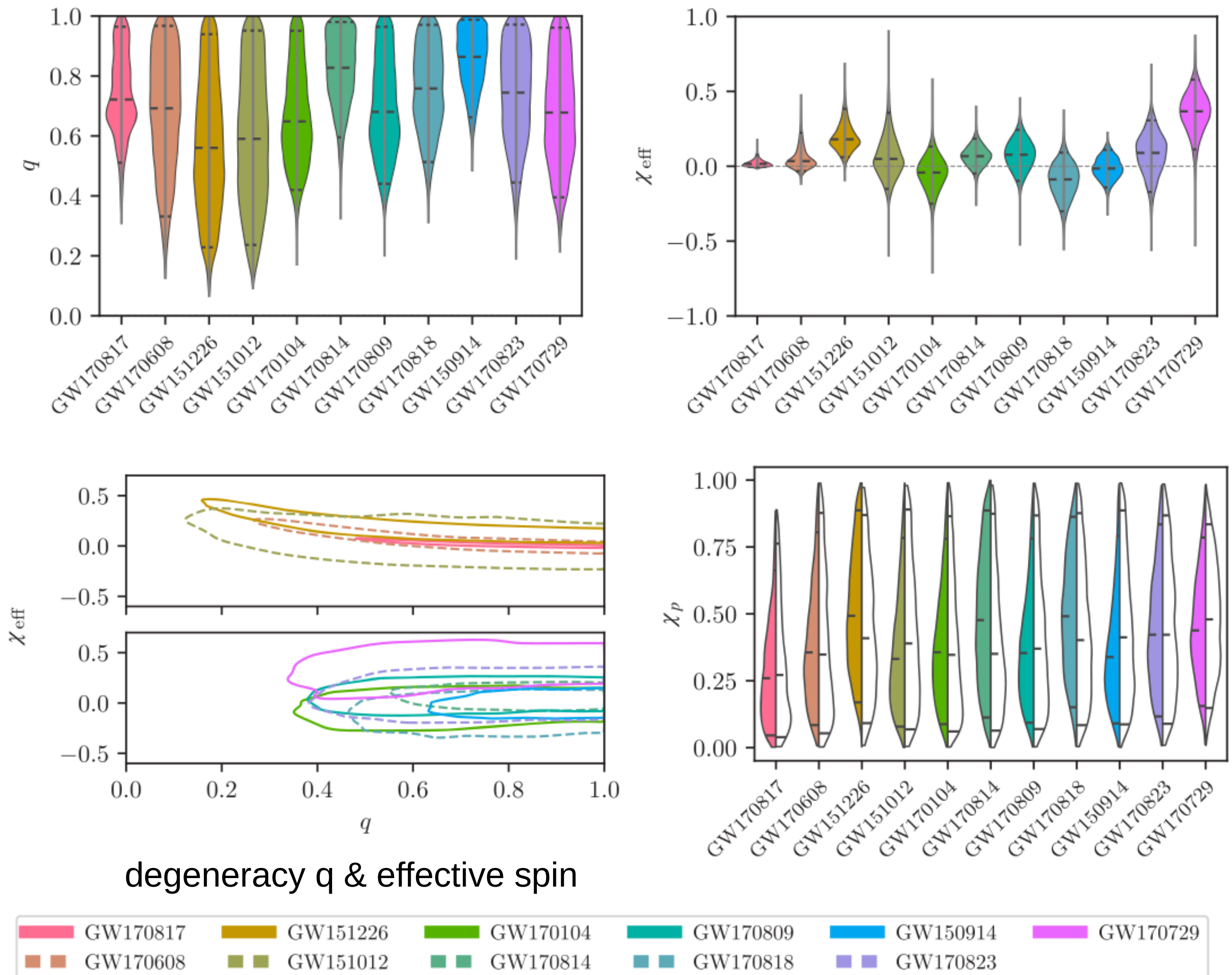
3. SPIN MAGNITUDE χ

4. Spin TILT $\cos\theta$



Spins:

Mass ratios and spins in GWTC-1:

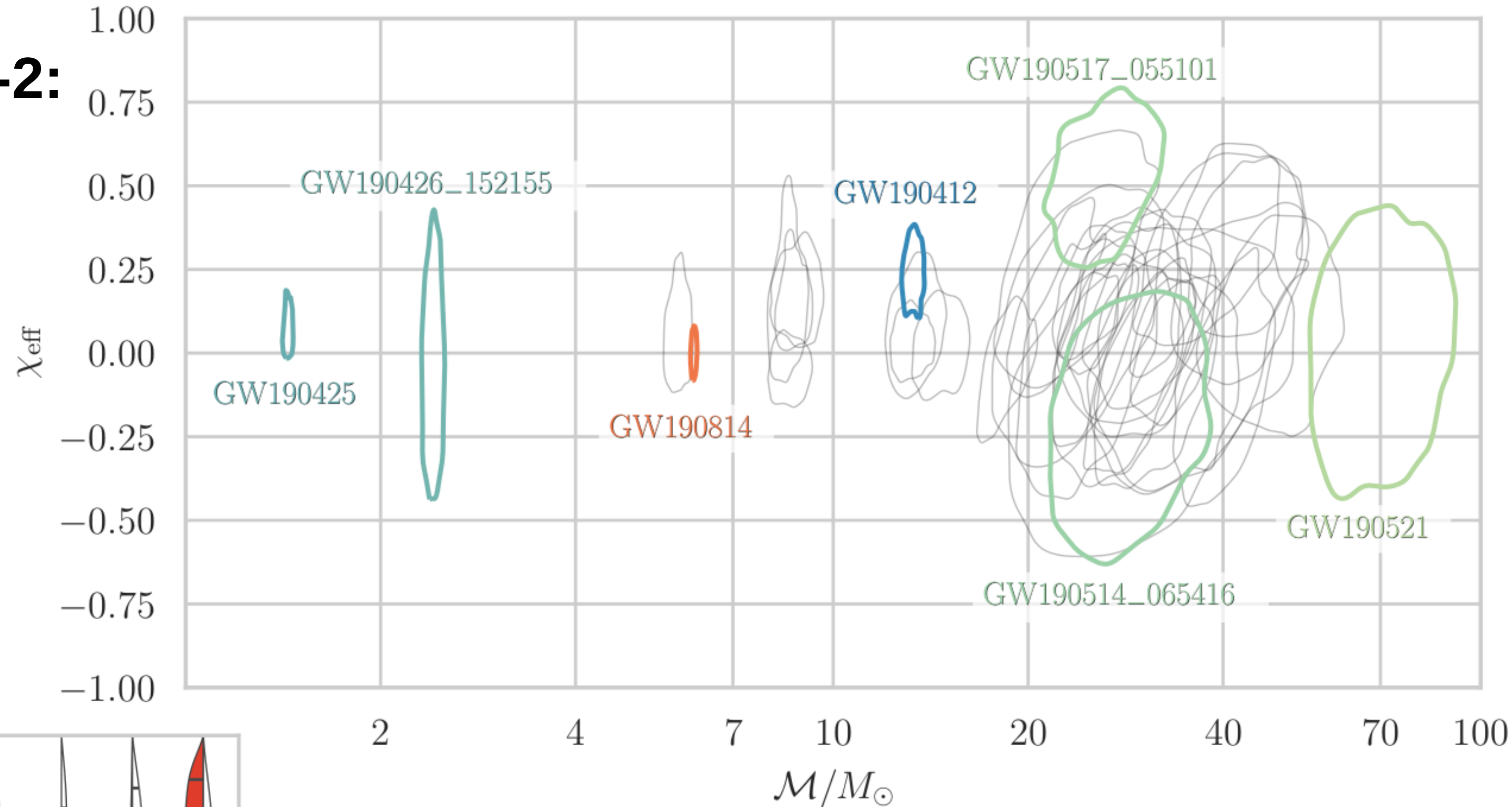


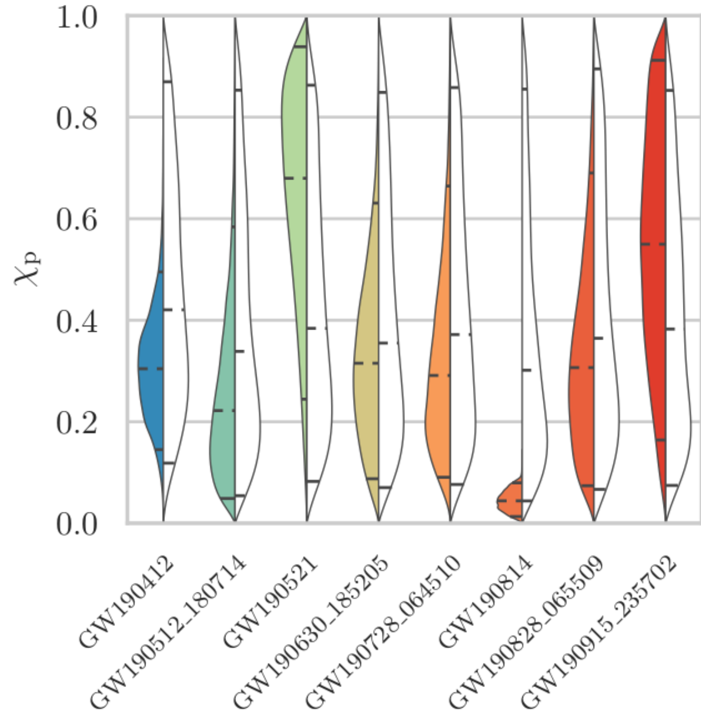
degeneracy q & effective spin

Abbott et al. 2019, GWTC1, https://ui.adsabs.harvard.edu/abs/2019PhRvX...9c1040A/

Spins:

Spins in GWTC-2:





MOSTLY VERY LOW VALUES OF χeff :

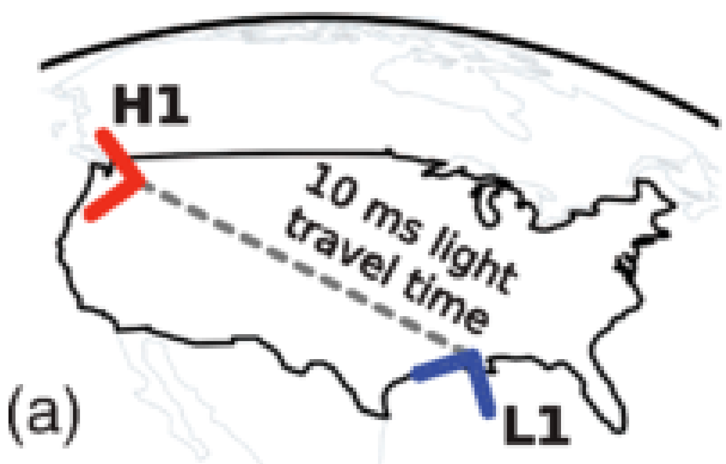
- either low spin magnitudes

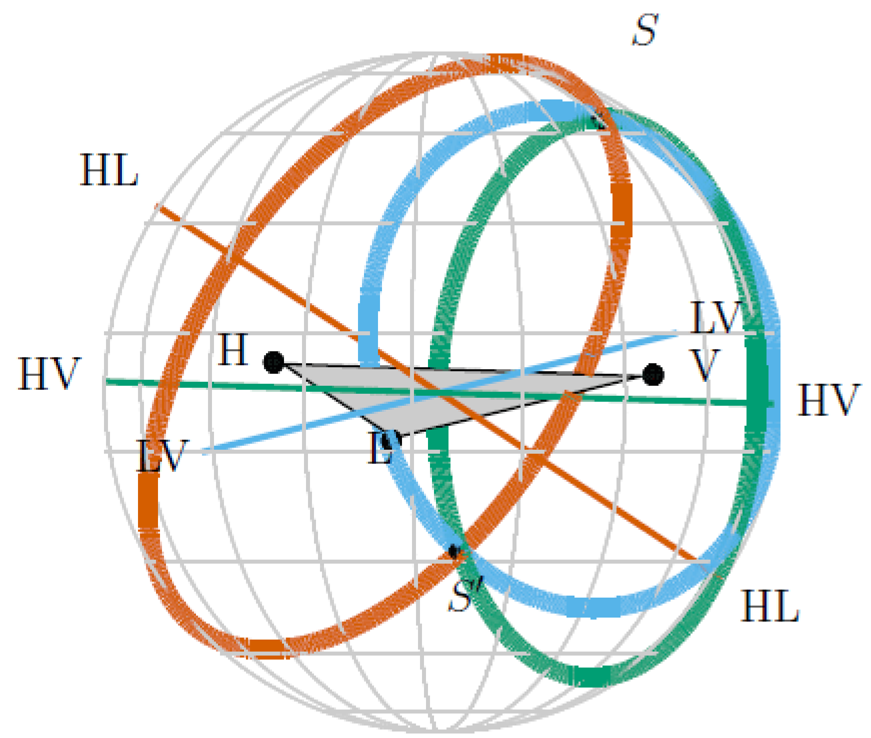
- or misaligned spins

Abbott et al. 2020, GWTC-2, 2020,
https://arxiv.org/abs/2010.14527

Sky localization (RA, DEC):

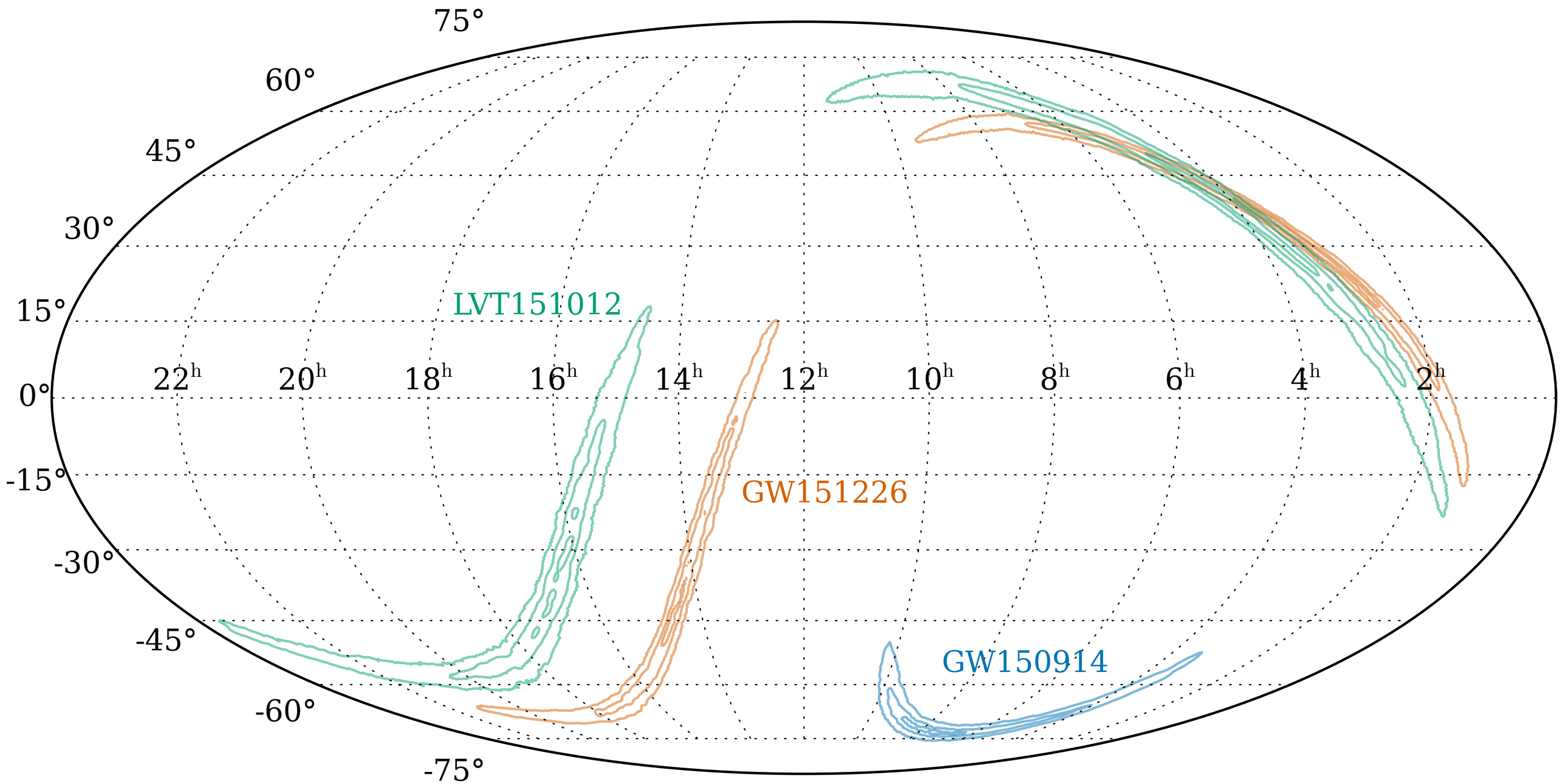
Measured by time delay between two detectors
(+ phase, + amplitude)





Sky localization (RA, DEC):

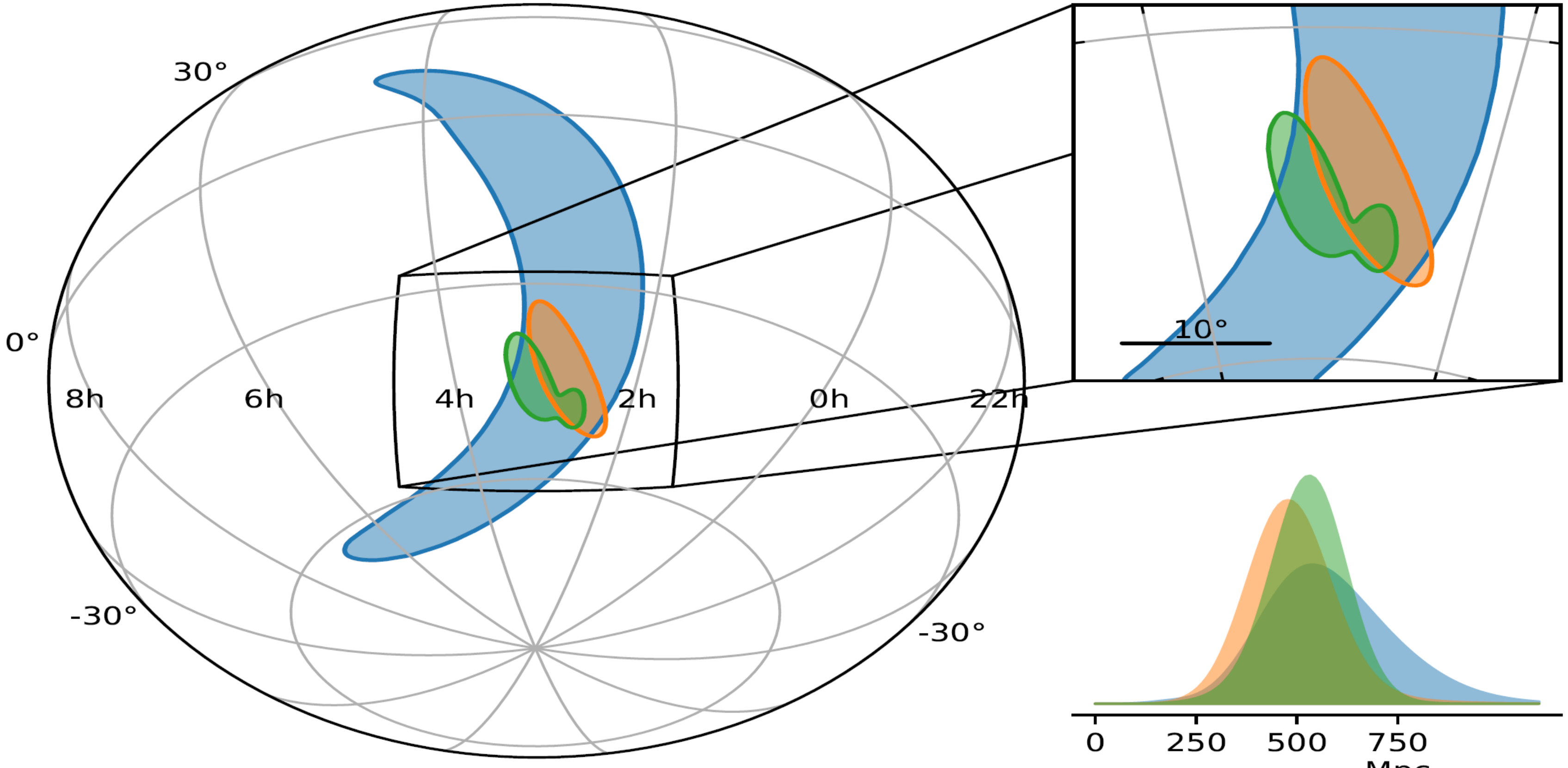
Sky map with only 2 detectors (GW150914, GW151226, LVT151012)
(hundreds of square degrees 90% credible area)



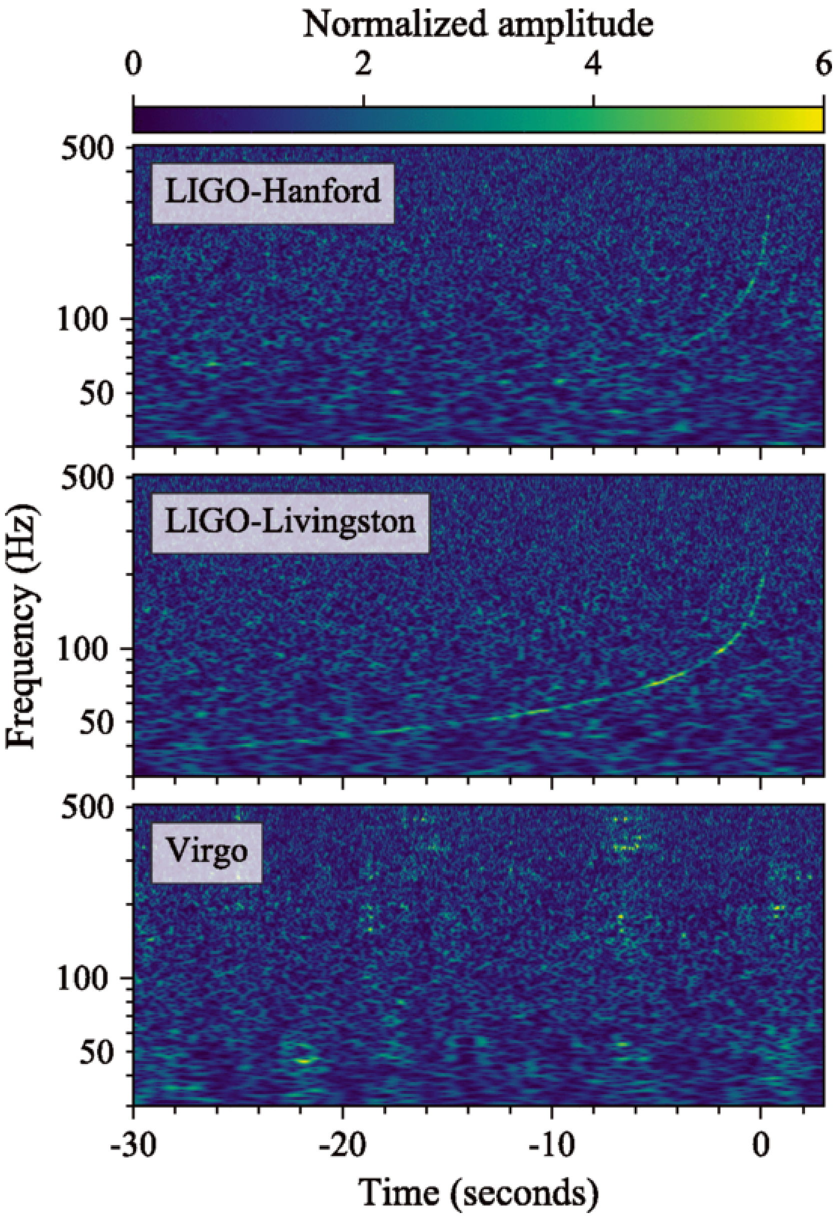
Sky localization (RA, DEC):

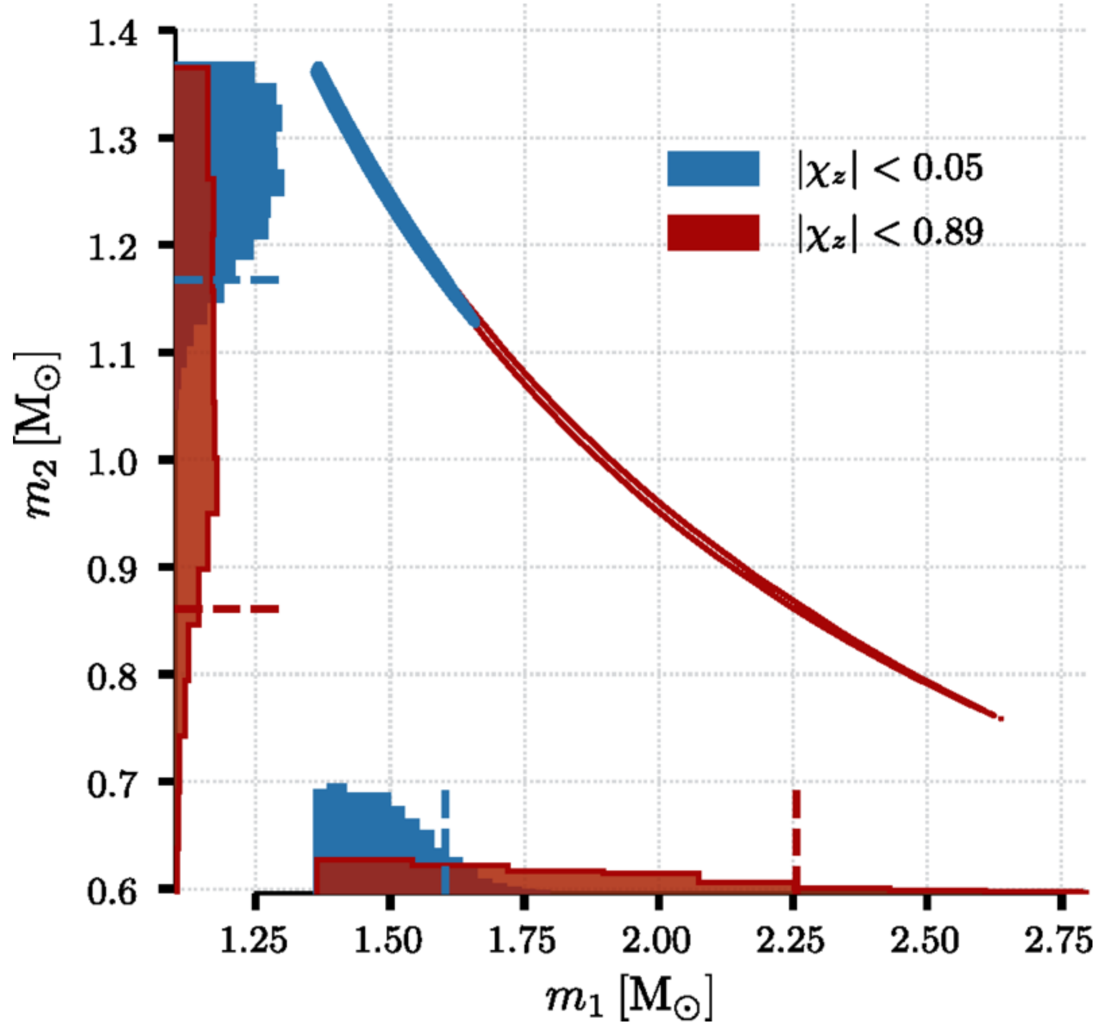
Sky map with 3 detectors (GW170814)
(TENS of square degrees 90% credible area)

Rapid LIGO only localization
Rapid LIGO+ Virgo localization
Refined LIGO+ Virgo localization



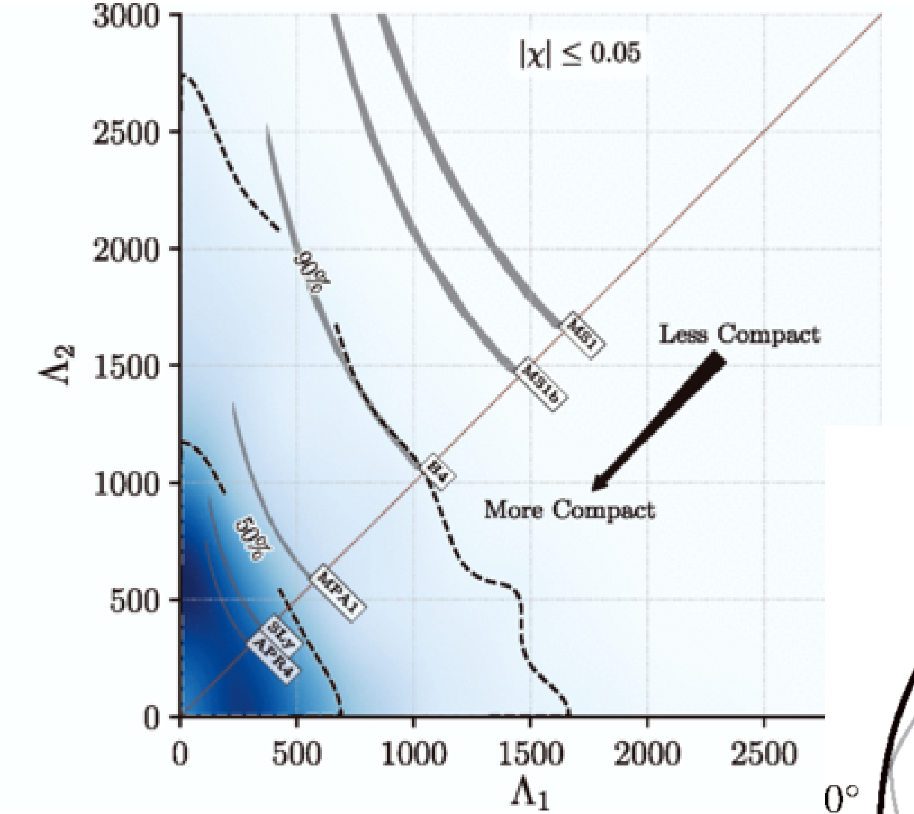
Some interesting individual events: GW170817

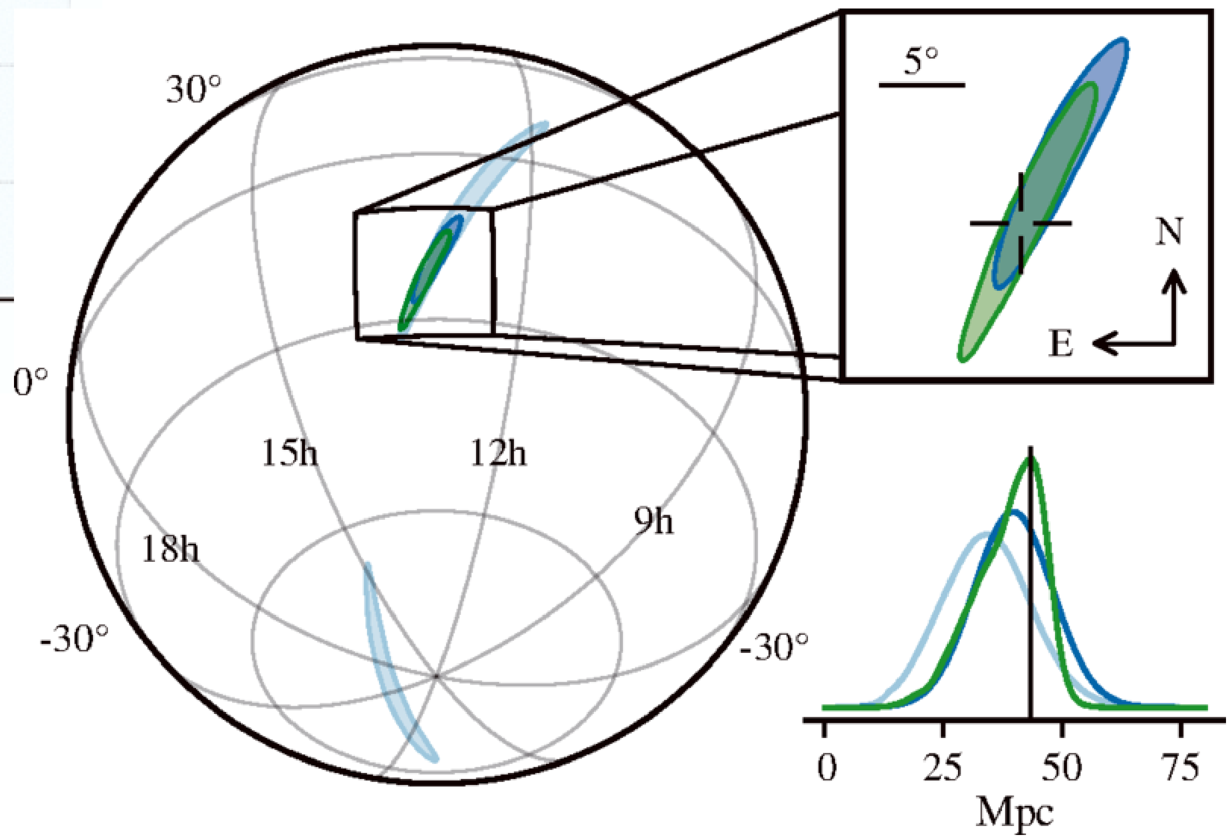




Abbott et al. 2017

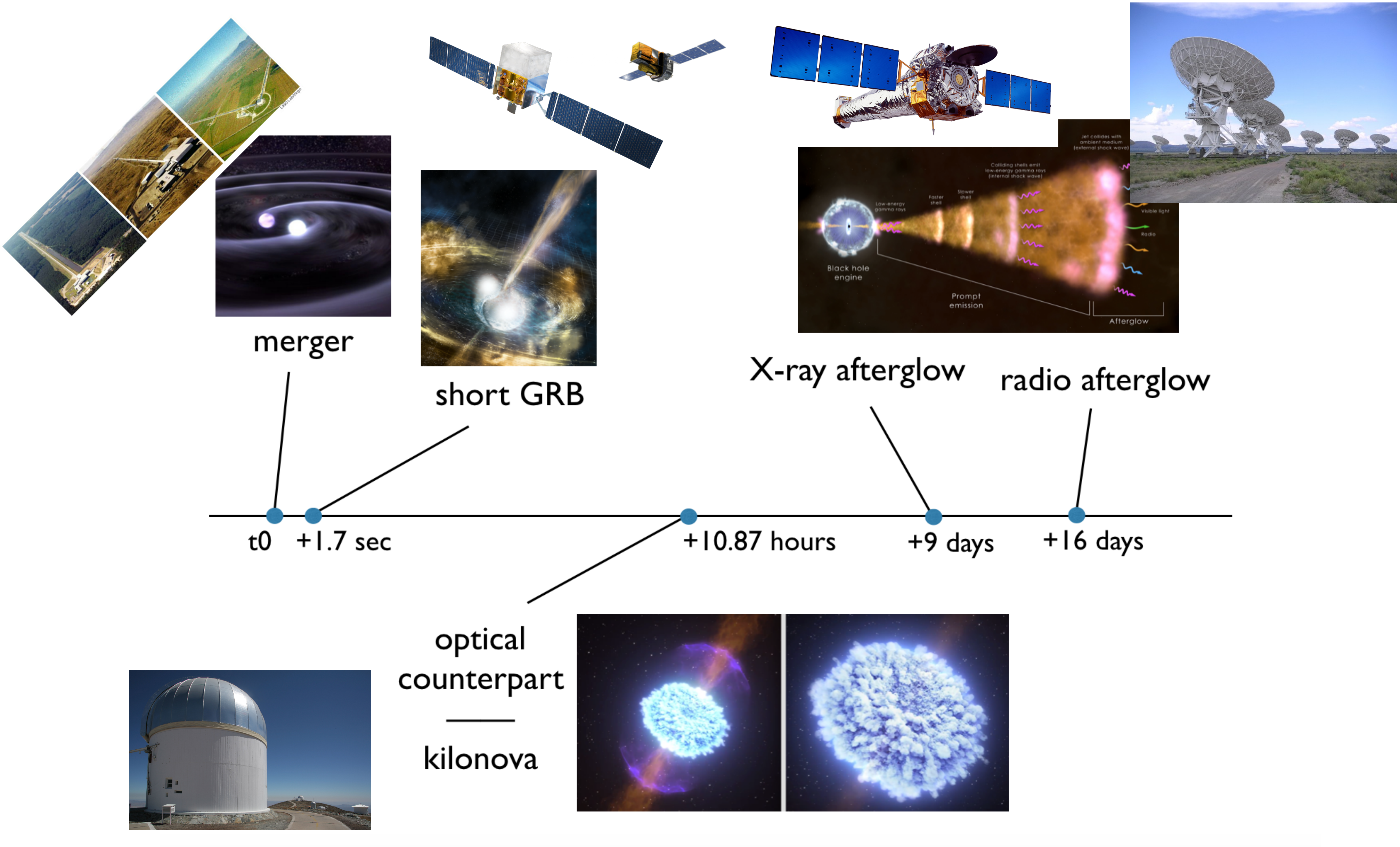
Some interesting individual events: GW170817





Abbott et al. 2017

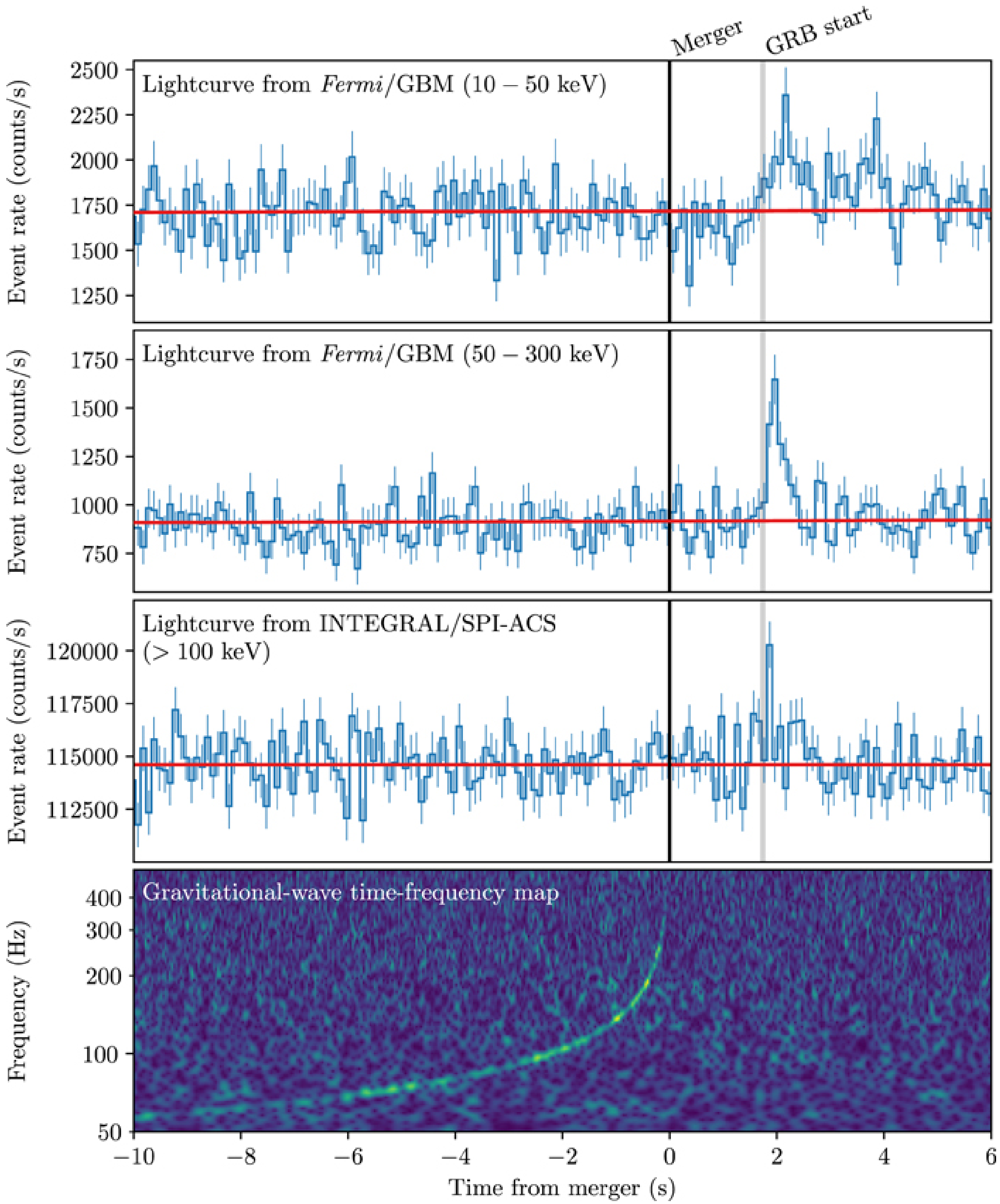
Some interesting individual events: GW170817



e.g. Abbott+ 2017, ApJ, 848, L13

Credit: slide from Riccardo Ciolfi + LVC

Some interesting individual events: GW170817



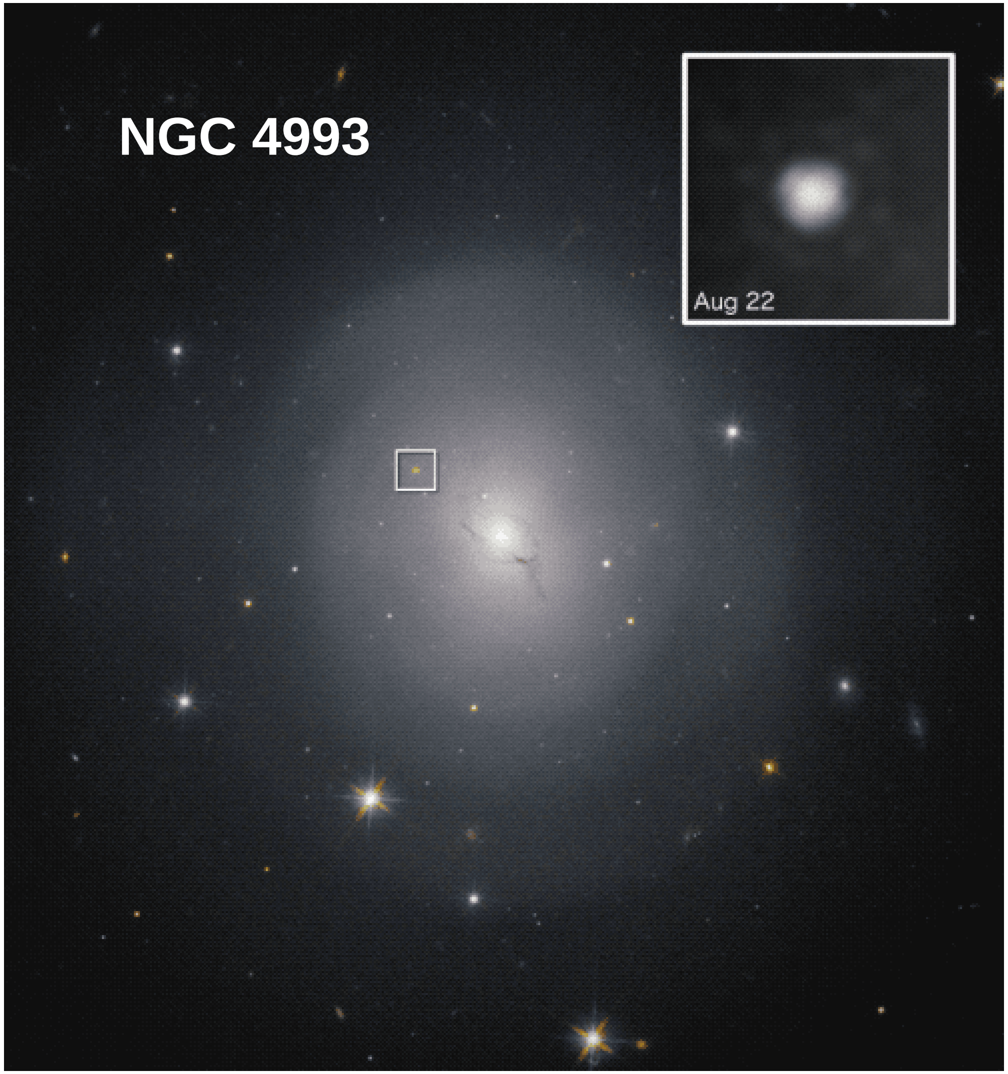
FERMI satellite:

Faint short GRB
~ 1.7 s after merger

→ First direct evidence that BNS mergers are associated with short GRBs

Abbott+ 2017, ApJ, 848, L13
Goldstein+ 2017, ApJ, 848, L14

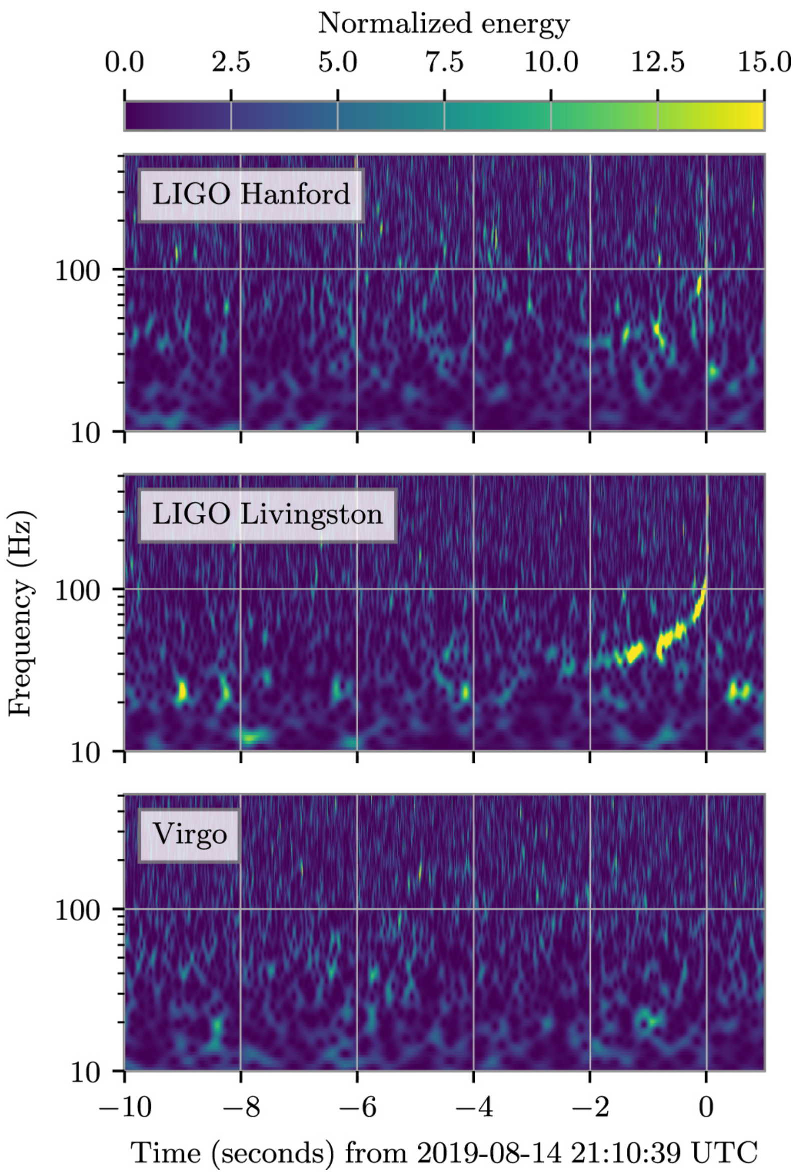
Some interesting individual events: GW170817

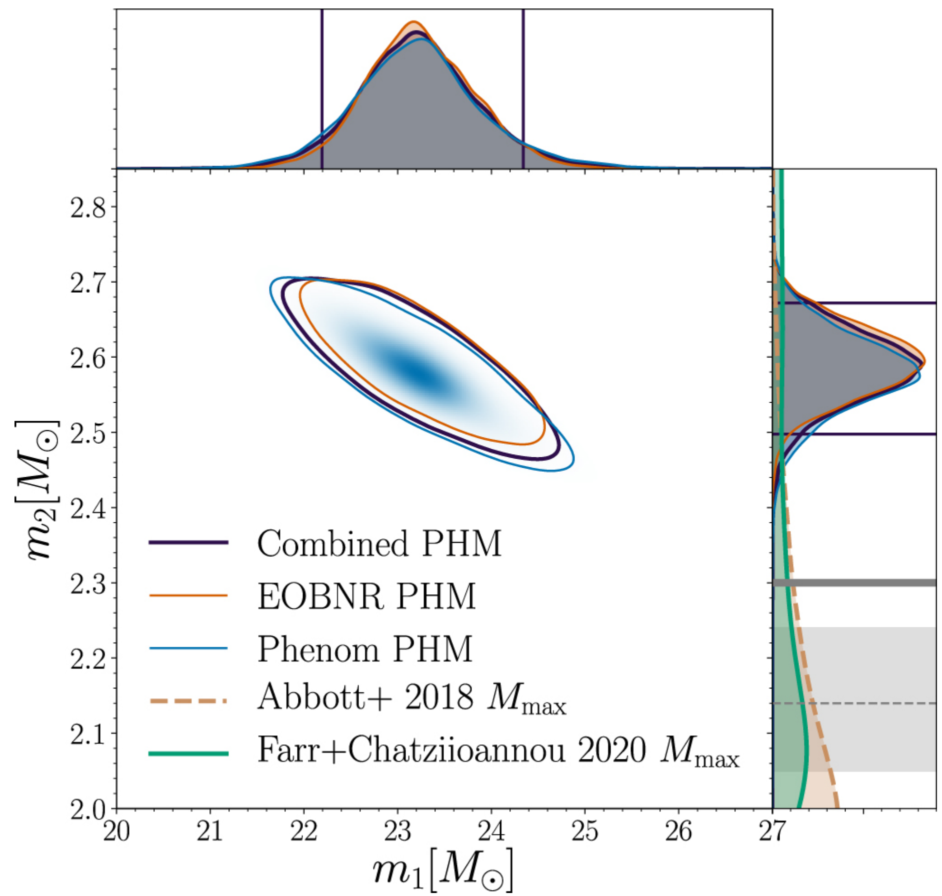


- **Early-type (S0) galaxy**
- **Mostly old stars (~ 10 Gyr)**
- **z ~ 0.0098 (Levan et al. 2017)**
- **stellar mass ~ 10^{10-11} Msun (Im et al. 2017)**

Coulter+ 2017, Sci, 358, 1556
Abbott+ 2017, ApJ, 848, L12
Abbott+ 2017, ApJ, 848, L13
Blanchard+ 2017, ApJ, 848, L22

Some interesting individual events: GW190814



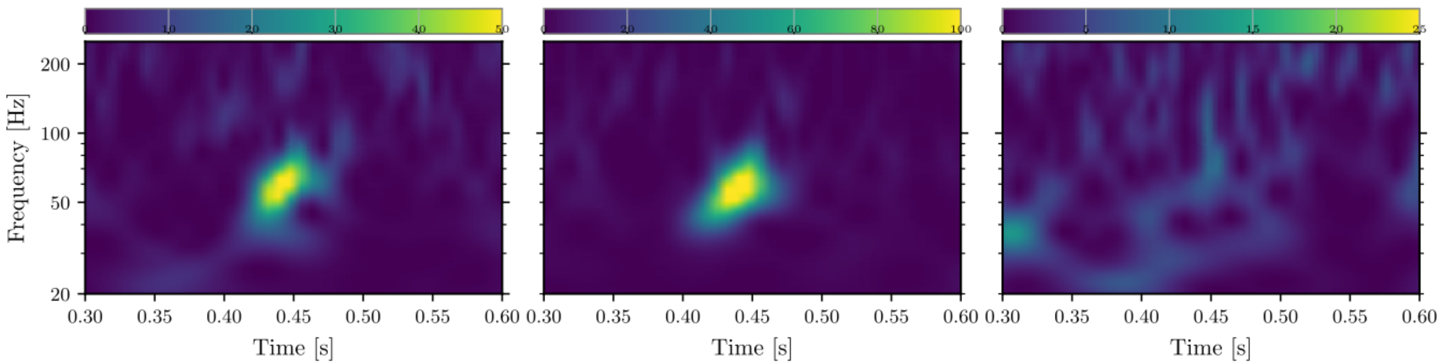


Secondary mass in the lower mass gap
Neutron star or black hole?
Mass ratio ~0.1

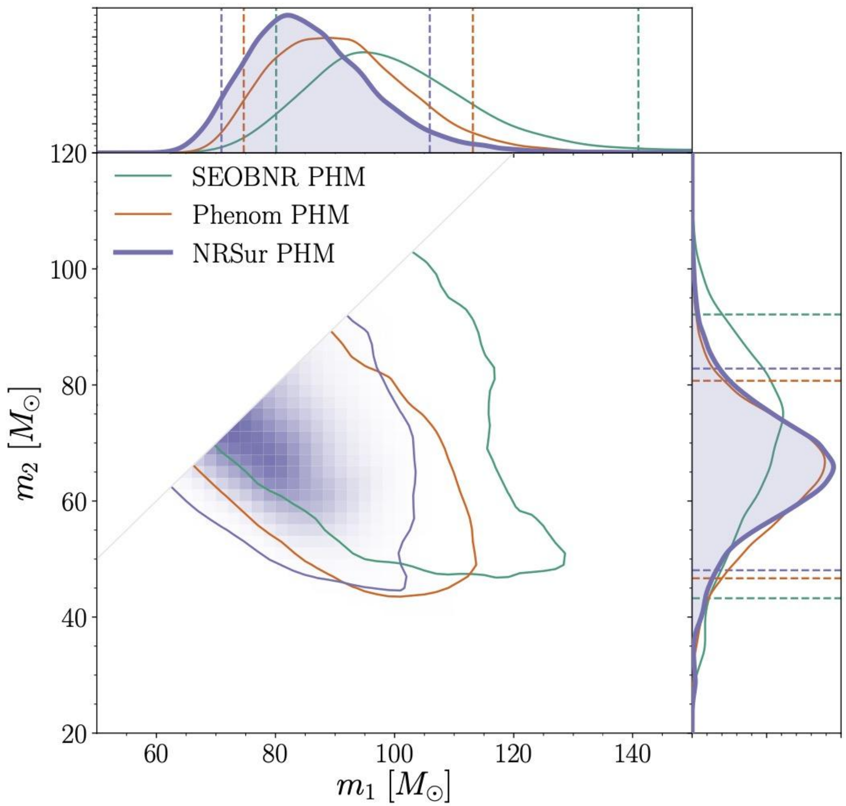
Abbott et al. 2020, GW190814

Some interesting individual events: GW190521

Normalized energy



LIGO Livingstone LIGO Hanford Virgo



Remnant mass ~ 140 Msun
First intermediate-mass black hole with gravitational waves

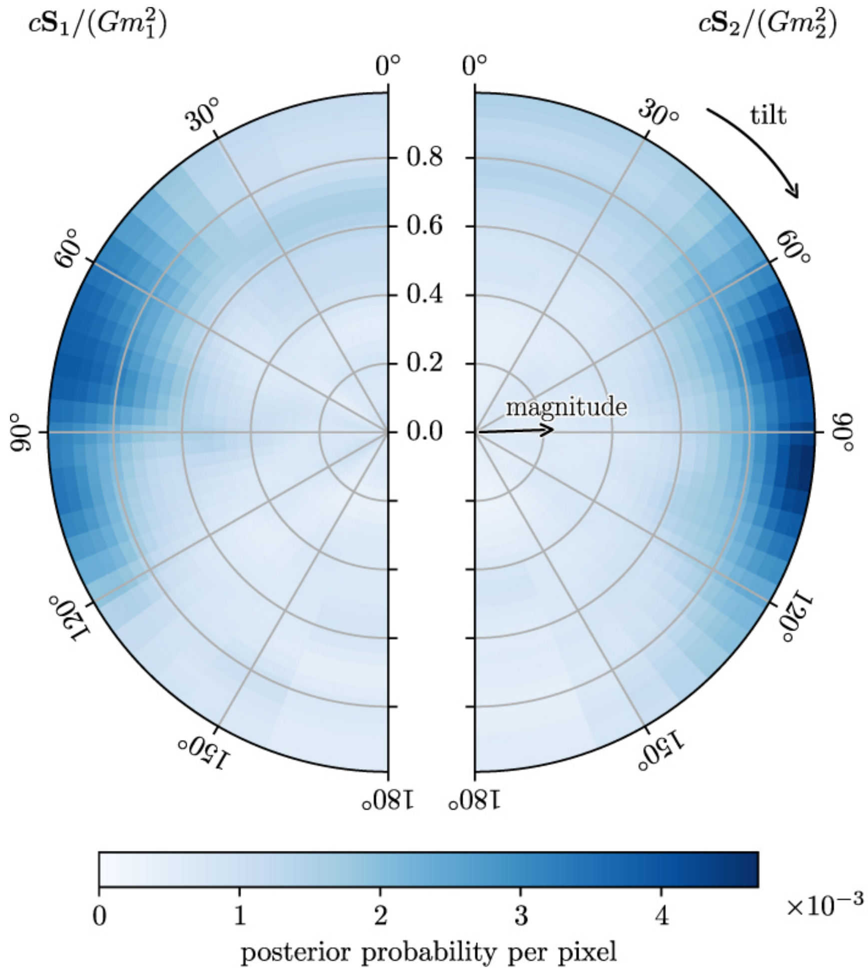
Primary mass ~ 85 Msun
First black hole in the pair-instability mass gap

$$m_1 = 85^{+21}_{-14} M_\odot \qquad m_2 = 66^{+17}_{-18} M_\odot$$

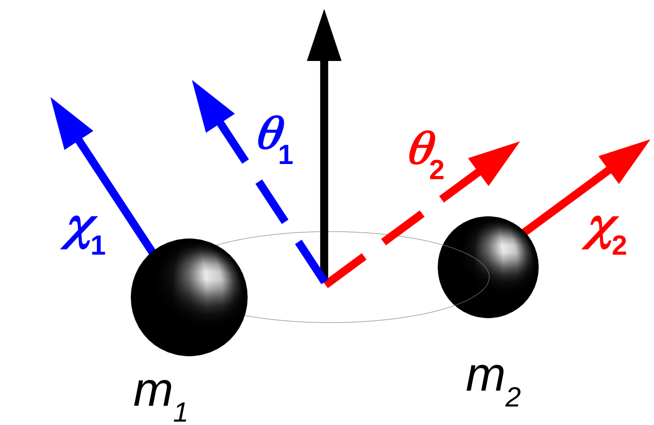
Abbott et al. 2020, detection paper

Abbott et al. 2020, astrophysical implications

Some interesting individual events: GW190521



Mild evidence for
large spins
nearly in the orbital plane



$\chi = c\, \mathbf{S}_1 / (G\, m_1^2)$

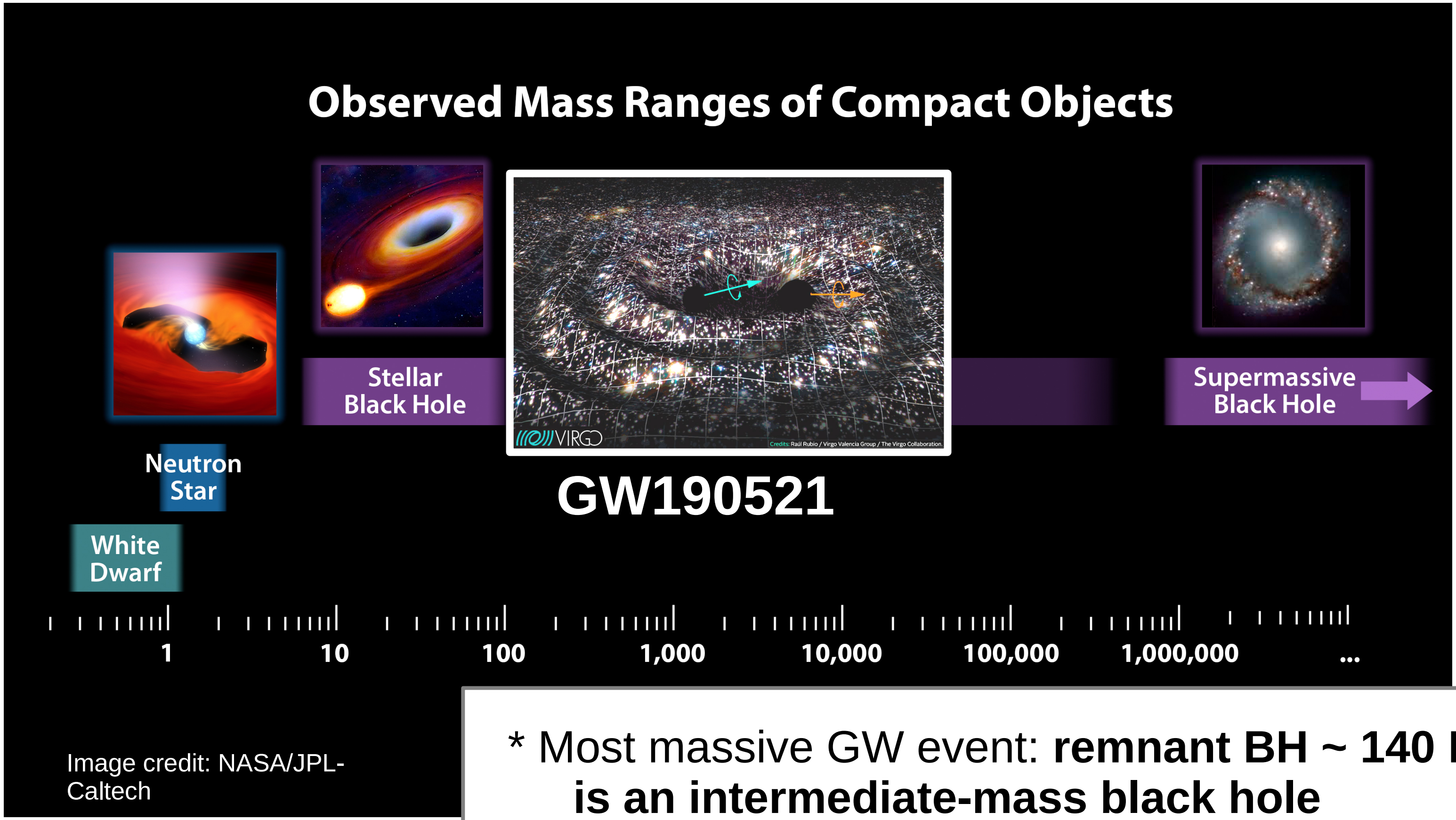
dimensionless spin

θ := tilt angle

Abbott et al. 2020, detection paper

Abbott et al. 2020, astrophysical implications

Some interesting individual events: GW190521



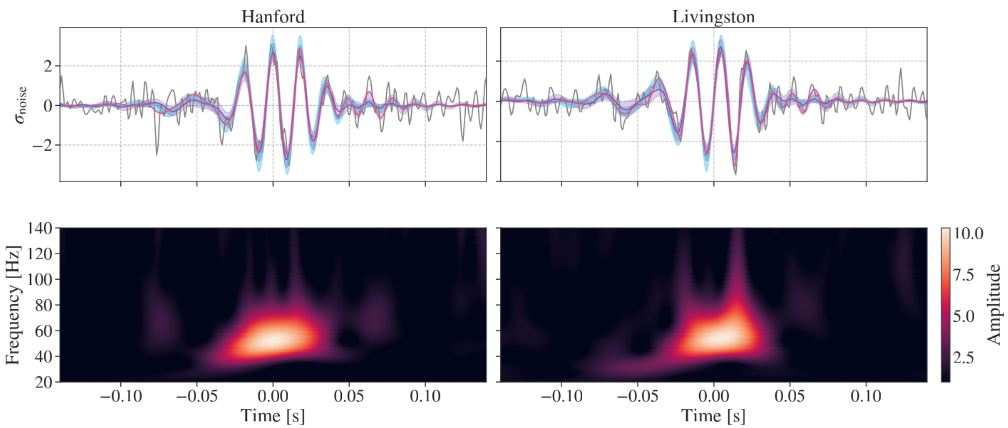
* Most massive GW event: **remnant BH ~ 140 M⊙ is an intermediate-mass black hole**

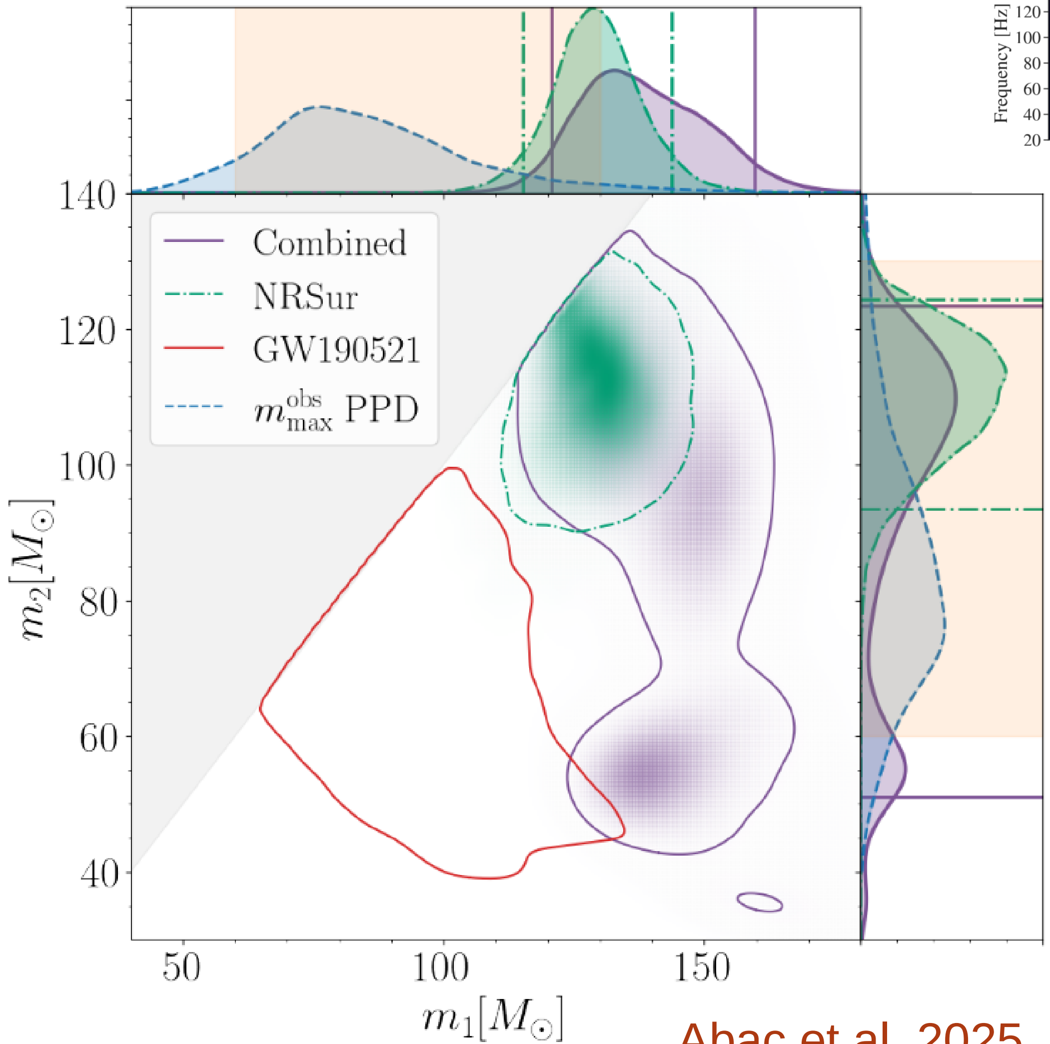
* Evidence for large spin components in the orbital plane → dynamical origin?

* Primary mass ~ 85 M⊙ in the "pair instability mass gap"

Abbott et al. 2020, GW190521 discovery, https://arxiv.org/abs/2009.01075
Abbott et al. 2020, GW190521 implications, https://arxiv.org/abs/2009.01190

Some interesting individual events: GW231123





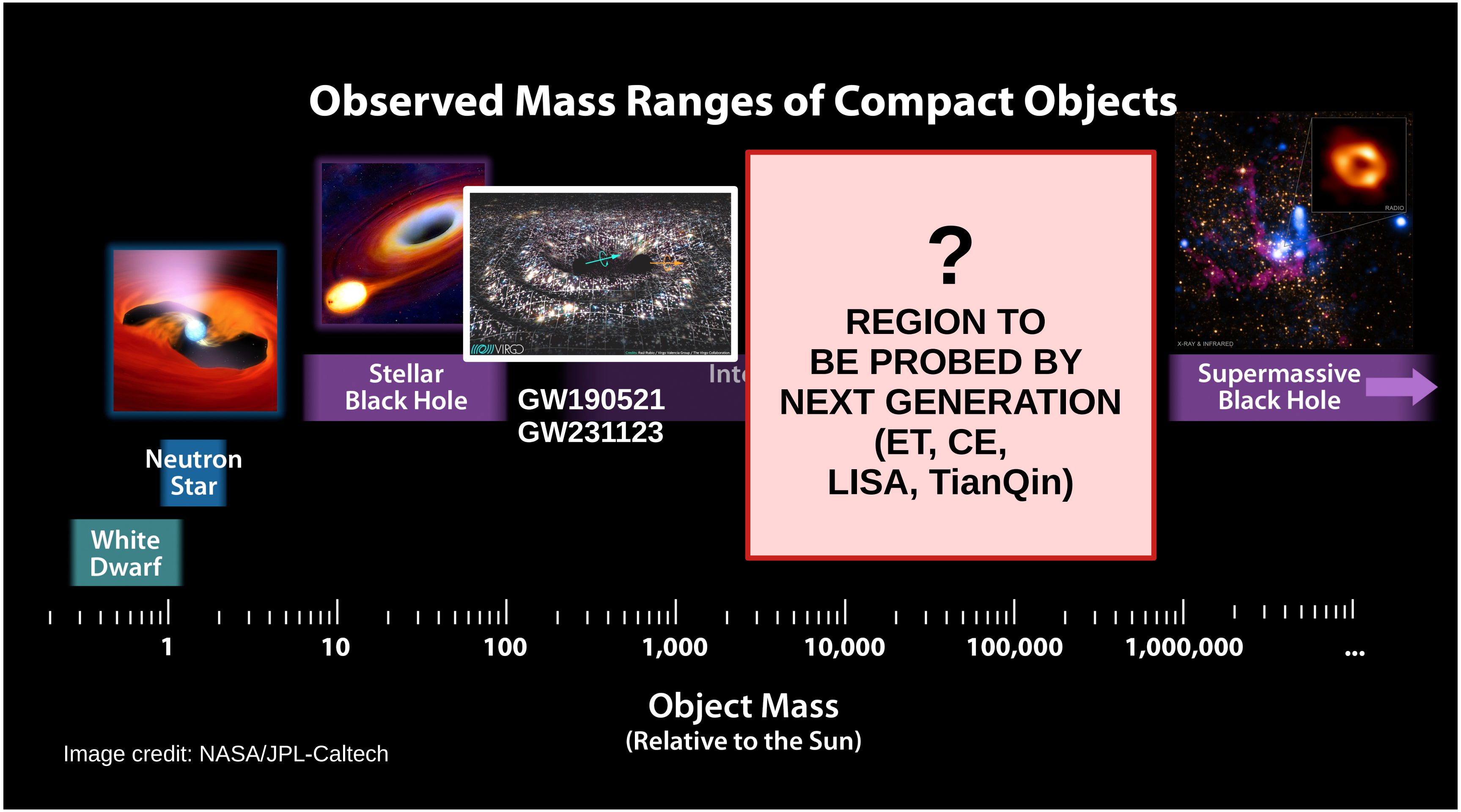
Abac et al. 2025

Final mass ~ 240 Msun
Most massive black hole with gravitational waves

Primary mass ~ 137 Msun

High spins (0.8-0.9): hierarchical black hole mergers?

Some interesting individual events: GW231123



Populations:

Now it is time to remove observational biases: a population perspective

1) The data { *d* }

Bayesian framework (LALInference, Veitch et al. 2015)
to extract posterior samples of **parameters of each GW event (e.g., masses, spins)**

2) Simple astrophysical models $\pi(\theta|\Lambda)$

e.g. for primary mass a **power law**
with parameters θ (e.g., mass)
and hyper-parameters Λ (e.g., slope of power law, min and max mass)
to be fitted to the data

3) Hierarchical Bayesian approach

to measure the hyper-parameters Λ of the population models
marginalizing over the properties (AKA parameters) of individual GW events

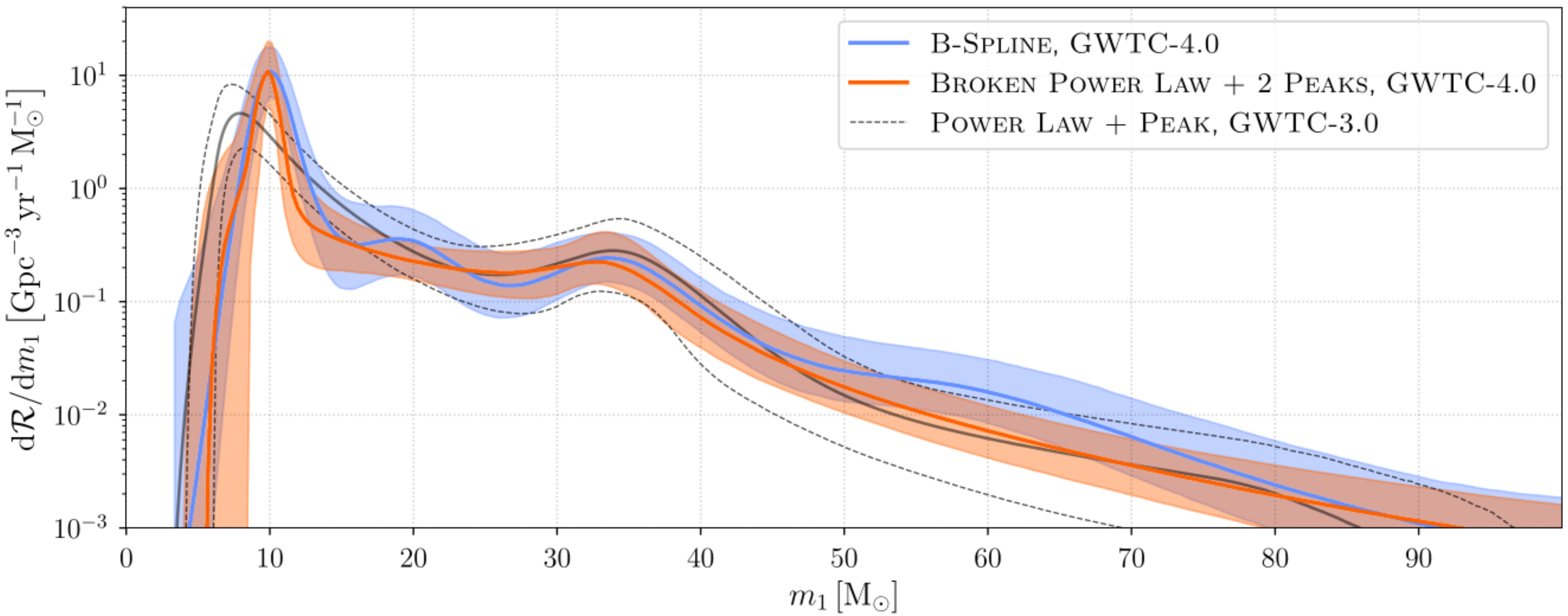
$$\mathcal{L}(\{d\}|\Lambda, N) \propto \underbrace{N^{N_{\rm det}} e^{-N\xi(\Lambda)}}_{\text{expected number of detections: encodes observational bias}} \prod_{i=1}^{N_{\rm det}} \int \underbrace{\mathcal{L}(d_i|\theta)}_{\text{single-event likelihood}} \underbrace{\pi(\theta|\Lambda)}_{\text{PRIOR: astrophysical model}} d\theta$$

In practice, we use a log-uniform prior on *N*
and estimate the single-event likelihood from LVK posterior samples obtained with prior $\pi_{\rm def}(\theta)$
then integrals over likelihood can be replaced with weighted average over discrete samples

$$\mathcal{L}(\{d\}|\Lambda) \propto \Pi_{i=1}^{N_{det}} \frac{\int \mathcal{L}(d_i|\theta)\,\pi(\theta|\Lambda)\,d\theta}{\xi(\Lambda)} \propto \Pi_{i=1}^{N_{det}} \frac{1}{\xi(\Lambda)} \langle \frac{\pi(\theta|\Lambda)}{\pi_{\rm def}(\theta)} \rangle$$

Populations:

GWTC-4 population paper

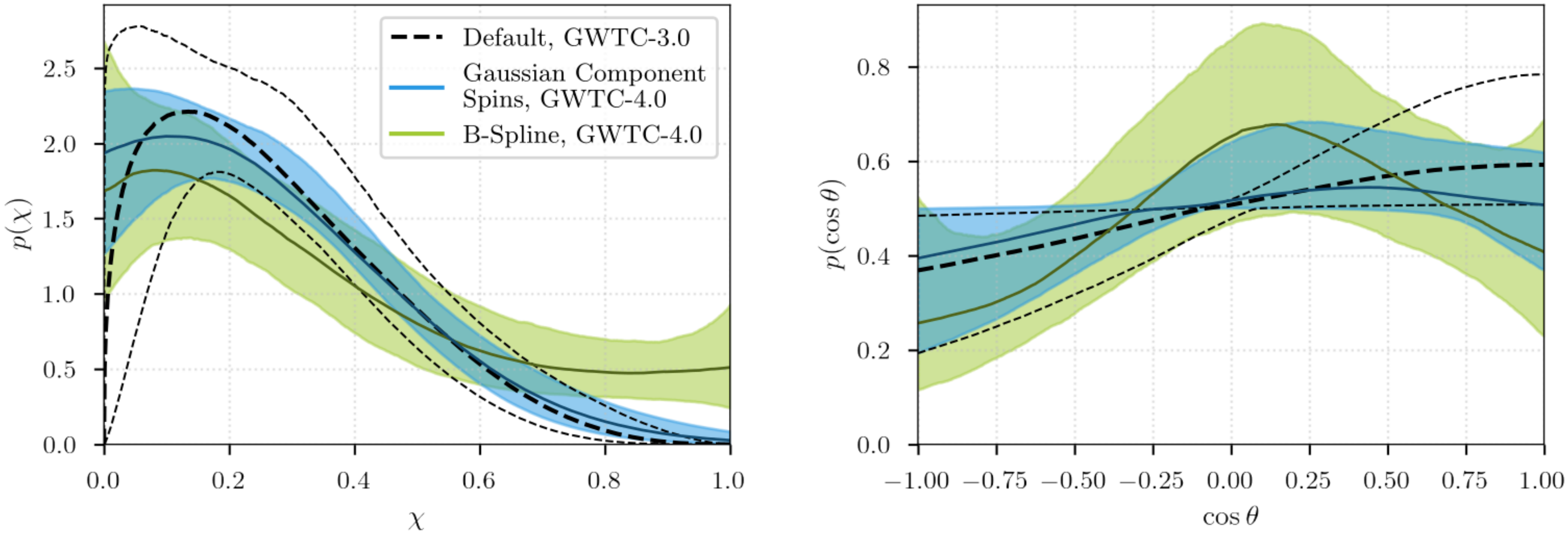


* Primary black hole mass 10 Msun peak
* Secondary feature: 35 Msun peak
* Long tail extending up to ~100 Msun → inside mass gap

Encode key information about the origin of black holes from their progenitor stars

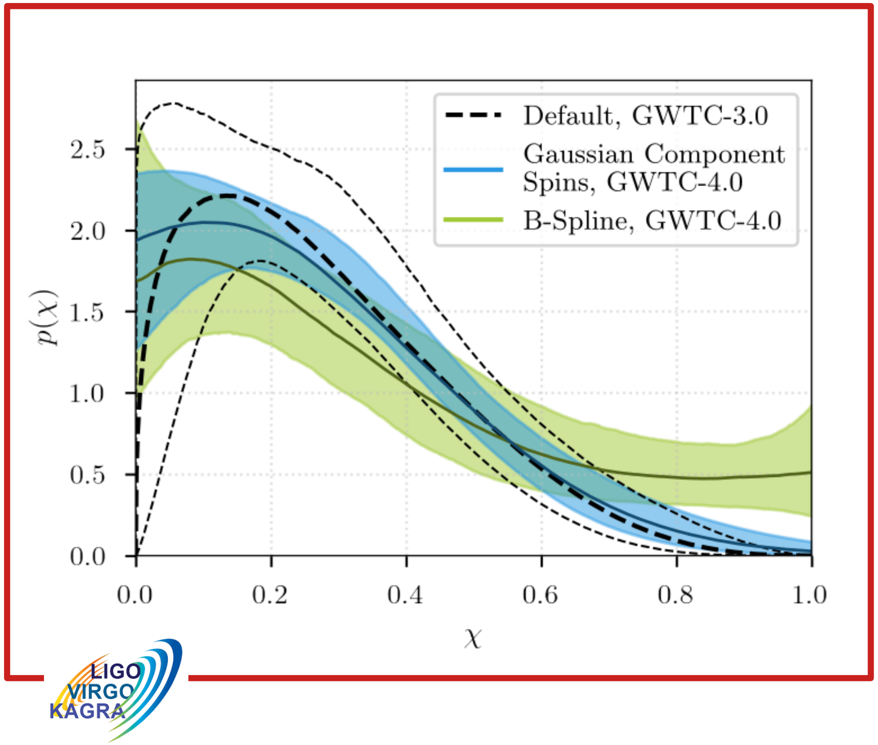
Populations:

GWTC-4 population paper



Populations:

GWTC-4 population paper

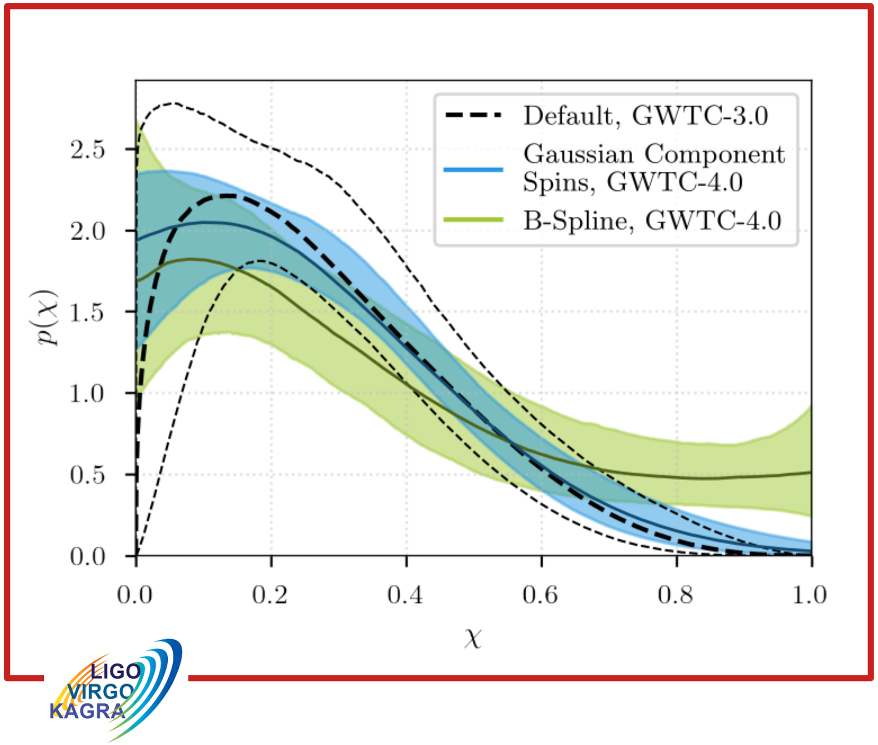


Black holes in binary black holes spin “slowly”

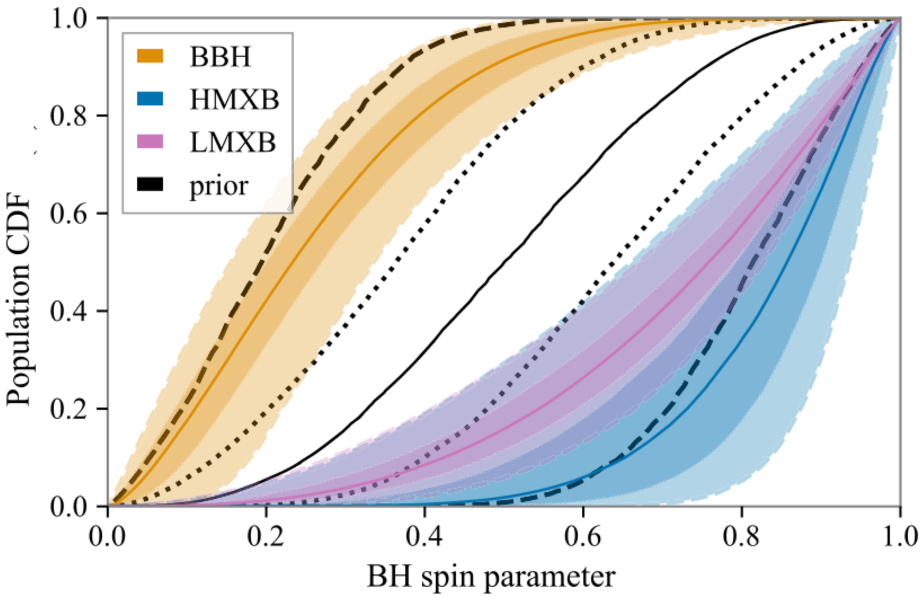
$$\chi \equiv \frac{J\,c}{G\,m_{\rm BH}^2}$$

Populations:

GWTC-4 population paper



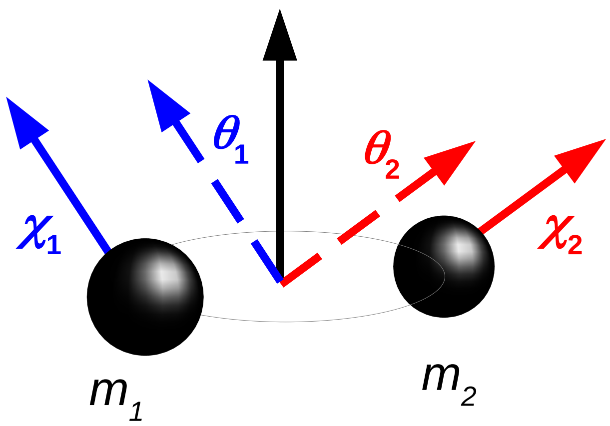
Fishbach & Kalogera 2022, X-ray binaries



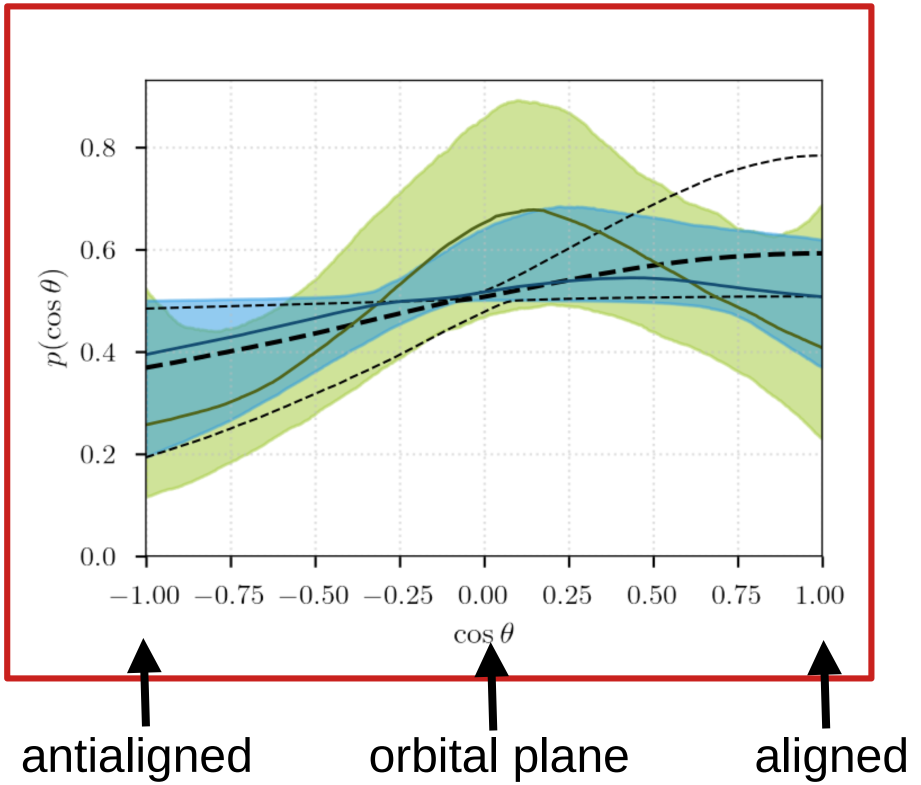
Black holes in binary black holes spin “slowly”

$$\chi \equiv \frac{J\,c}{G\,m_{\rm BH}^2}$$

Populations:



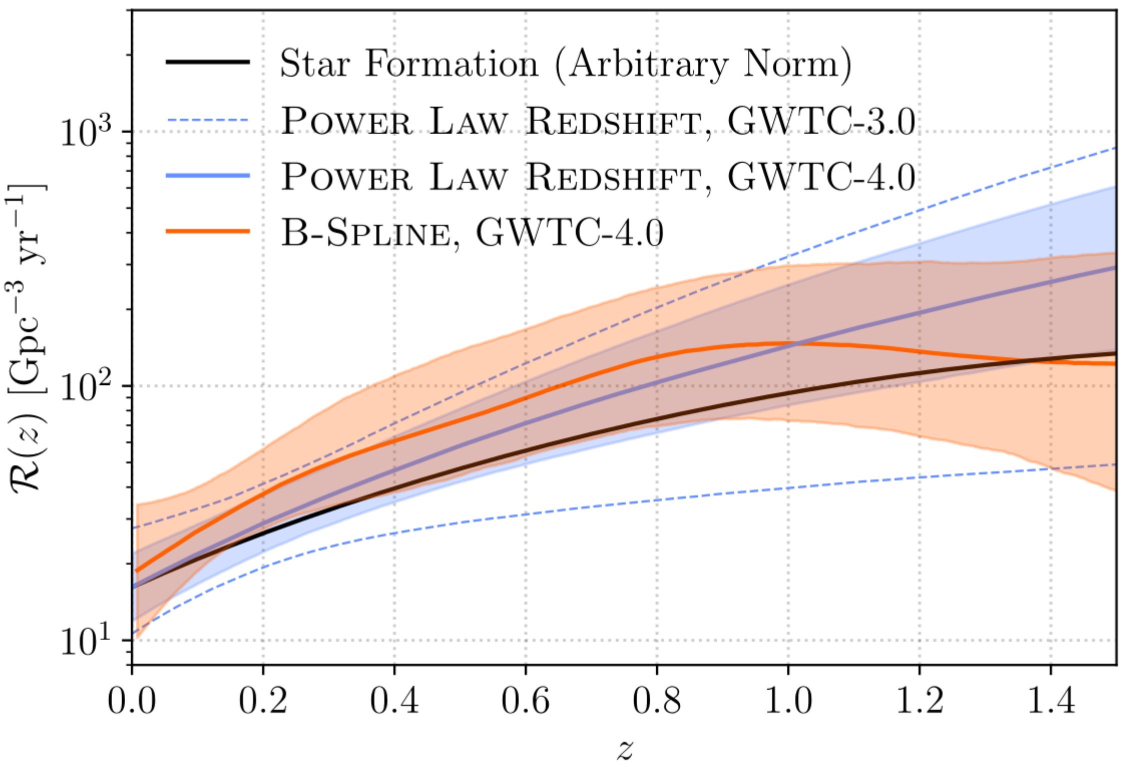
GWTC-4 population paper



Weak preference for alignment with orbital angular momentum

Populations:

GWTC-4 population paper



MERGER RATE DENSITY
at redshift $z = 0$

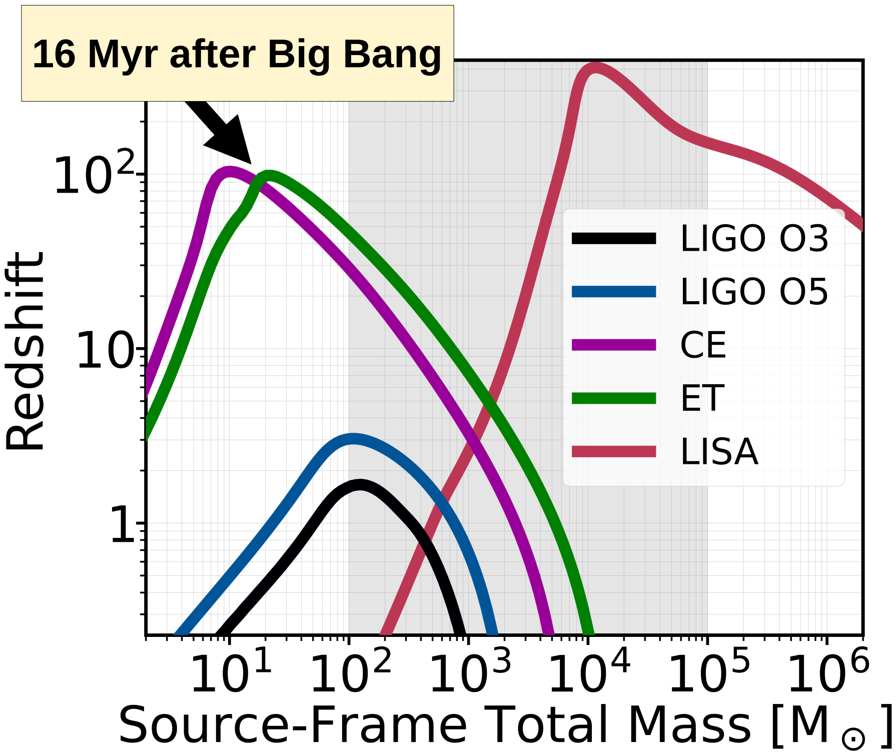
7.6–250 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$
for binary neutron stars,

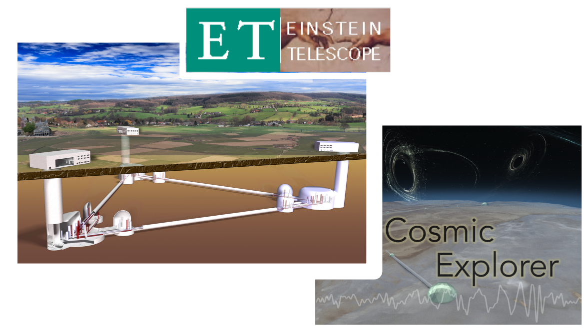
9.1–84 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$
for neutron star–black hole binaries,

14–26 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$
for binary black holes

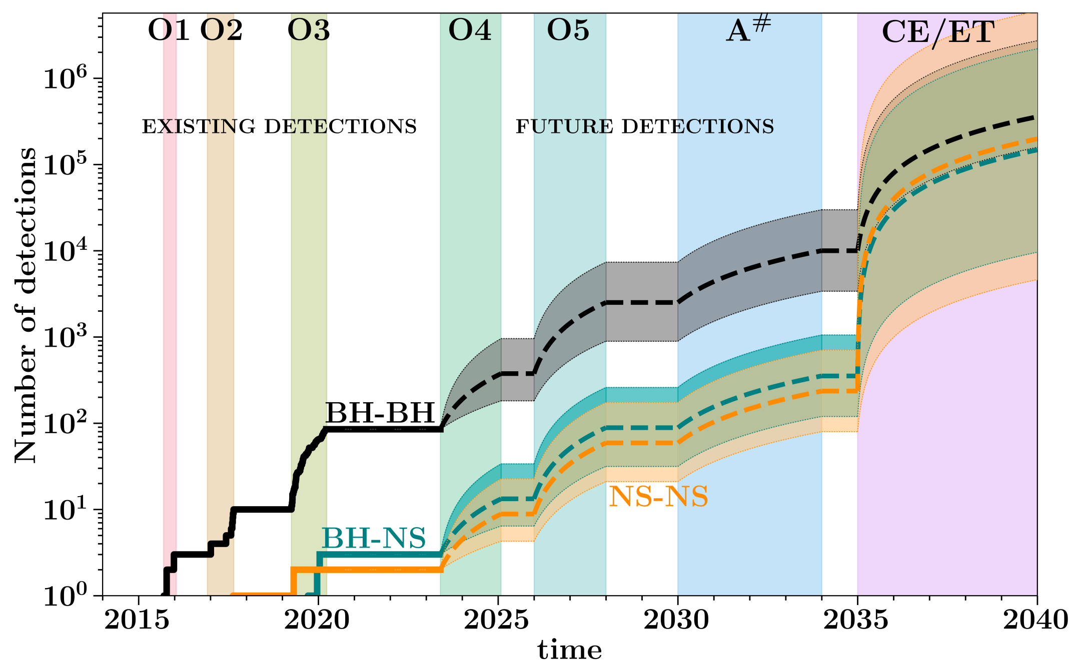
Black hole rate is problematic for the models

The next generation





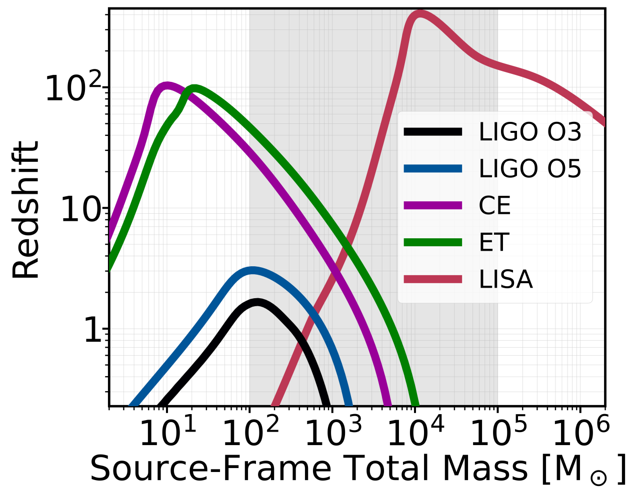
Broekgaarden et al. 2024



Ground-based: Einstein Telescope & Cosmic Explorer (> 2035)

Space-based: LISA (2035), TianQin (2034)

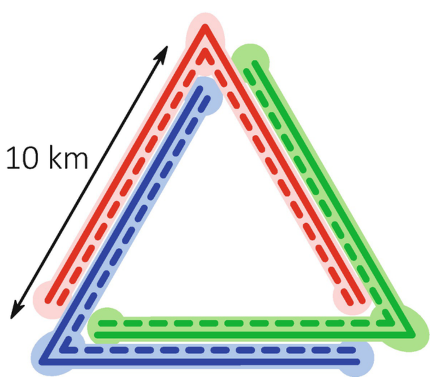
The next generation

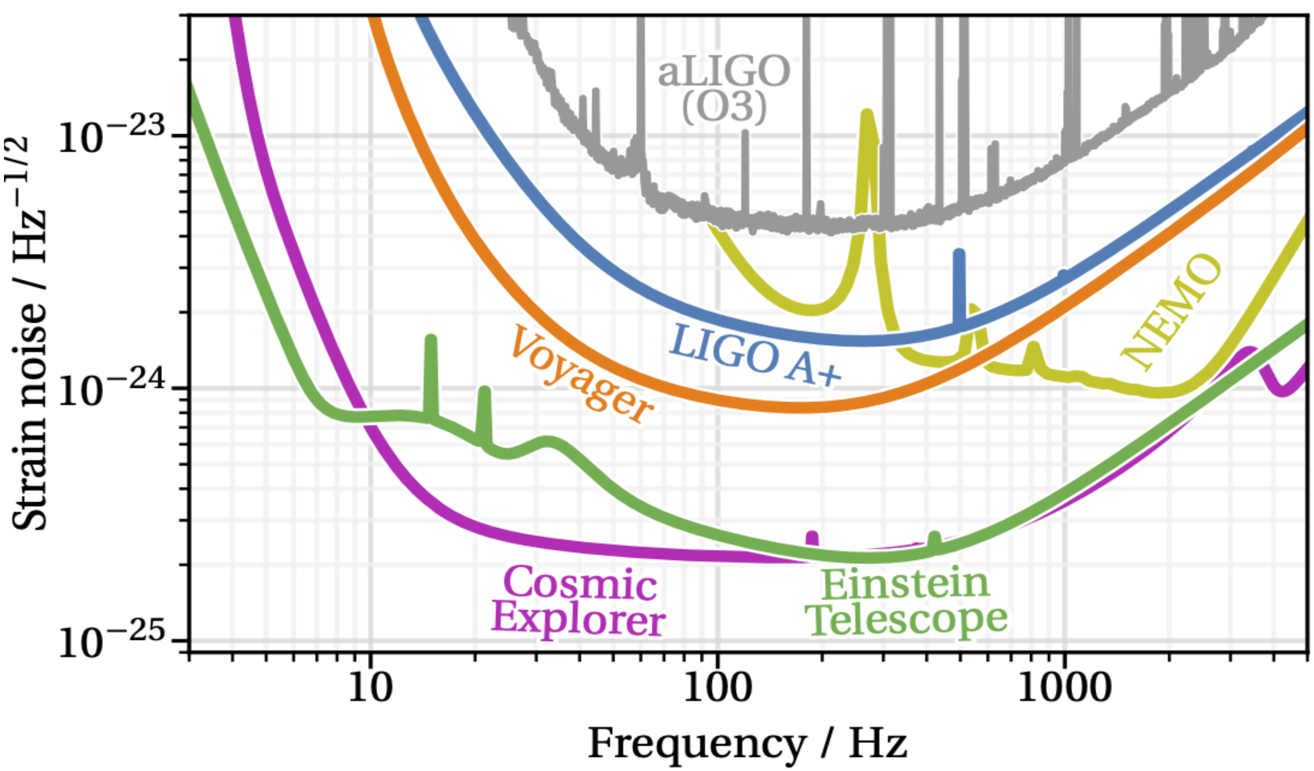


Cosmic Explorer:
* like LIGO but bigger (20km or 40 km)
* timeframe **?????**

ESFRI Strategy Report on Research Infrastructures ROADMAP 2021

Einstein Telescope:
* like LIGO but bigger (10km or 15 km)
* UNDERGROUND
(minimizes seismic and Newtonian noise)
* maybe TRIANGLE (null stream)
* timeframe > 2035

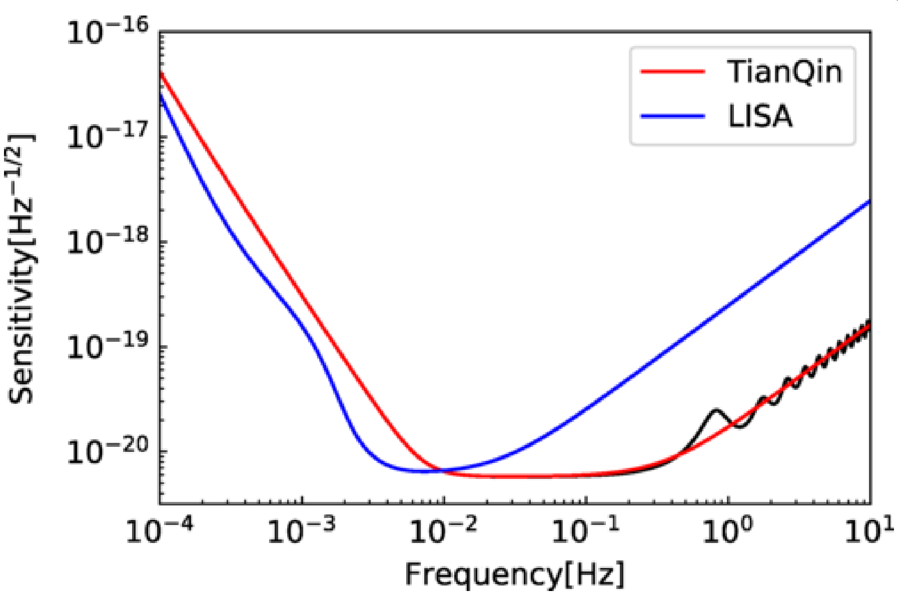




The next generation

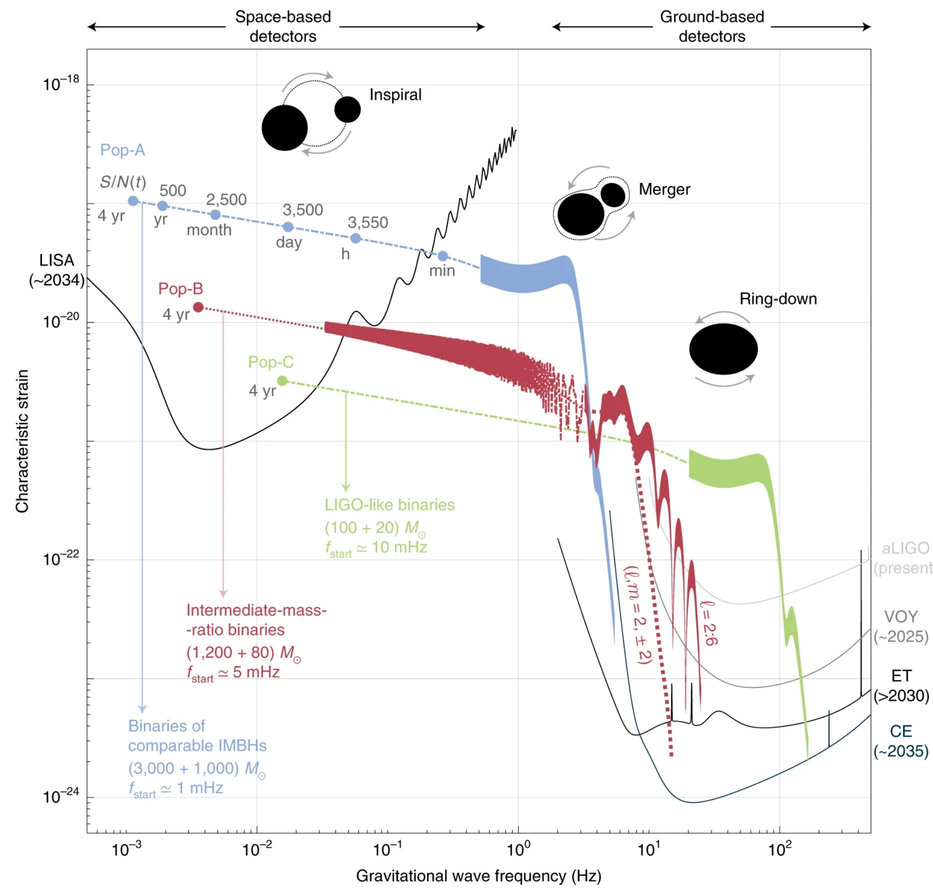
LISA (ESA / NASA)
* 3-spacecraft laser interferometer
*2.5M km arms
* timeframe: 2035 + 4 yr (10 yr?)

TIANQIN (China, Sun Yat Sen University)
* 3-spacecraft laser interferometer
* 0.1M km arms
* timeframe: 2034 + 5 yr (10 yr?)



Huang, Hu, Korol et al. 2019

MULTIBAND GW ASTRONOMY



Jani et al. 2020

Open questions from GWs

1. What determines binary compact objects' mass and spin?

2. What are the formation channels of binary compact objects?

3. What is the evolution of binary compact objects with redshift?

References:

- First detection: https://ui.adsabs.harvard.edu/abs/2016PhRvL.116f1102A/abstract
- Astrophysical interpretation of first detection: https://ui.adsabs.harvard.edu/abs/2016ApJ...818L..22A/abstract
- First binary neutron star GW170817: https://ui.adsabs.harvard.edu/abs/2017PhRvL.119p1101A/abstract
- GWTC-1 (O1+O2): https://ui.adsabs.harvard.edu/abs/2019PhRvX...9c1040A/
- Population from GWTC-1: https://ui.adsabs.harvard.edu/abs/2019ApJ...882L..24A/
- Observing scenarios paper: https://dcc.ligo.org/LIGO-P1200087-V57/public
- GWTC-2 (O1+O2+O3a): https://arxiv.org/abs/2010.14527
- Population from GWTC-2: https://ui.adsabs.harvard.edu/abs/2020arXiv201014533T/abstract
- GWTC-2.1 (O1+O2+O3a): https://ui.adsabs.harvard.edu/abs/2021arXiv210801045T/abstract
- GWTC-3 (O1+O2+O3a+O3b): https://ui.adsabs.harvard.edu/abs/2021arXiv211103606T/abstract
- Populations from GWTC-3: https://ui.adsabs.harvard.edu/abs/2021arXiv211103634T/abstract

* Public alert webpage (GRACEDB): https://gracedb.ligo.org/superevents/public/

- GWTC-4: https://ui.adsabs.harvard.edu/abs/2025arXiv250818082T/abstract
- GWTC-4 populations: https://ui.adsabs.harvard.edu/abs/2025arXiv250818083T/abstract

References:

Special event papers

- GW190412: https://ui.adsabs.harvard.edu/abs/2020PhRvD.102d3015A/abstract
- GW190425: https://ui.adsabs.harvard.edu/abs/2020ApJ...892L...3A/abstract
- GW190521 detection and science:
  https://ui.adsabs.harvard.edu/abs/2020PhRvL.125j1102A/abstract
  https://ui.adsabs.harvard.edu/abs/2020ApJ...900L..13A/abstract
- GW190814: https://ui.adsabs.harvard.edu/abs/2020ApJ...896L..44A/abstract
- GW231123: https://ui.adsabs.harvard.edu/abs/2025arXiv250708219T/abstract