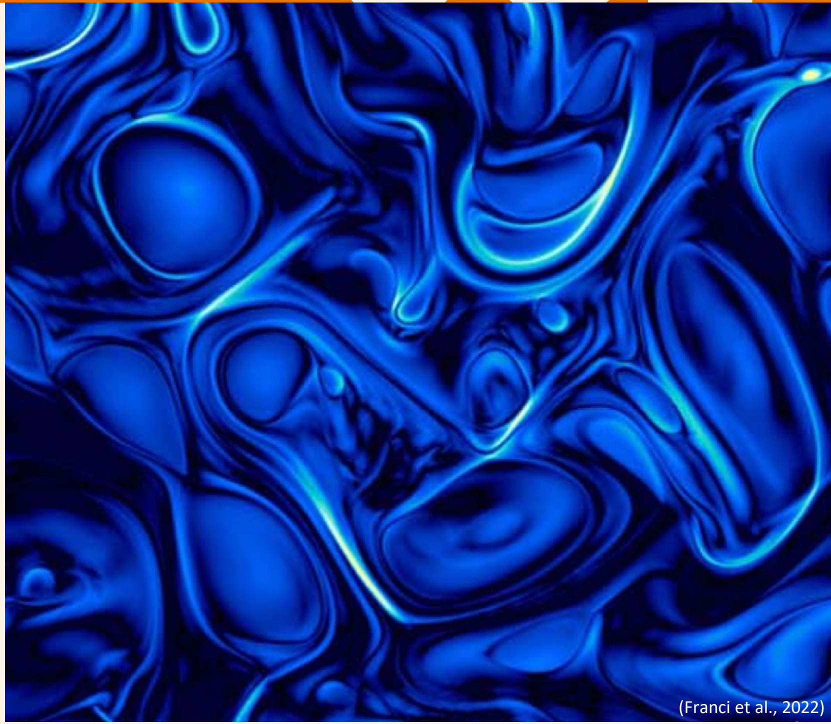

(Franci et al., 2022)

Astrophysical Plasmas and Cosmic Ray Propagation – Part II

Daniel Verscharen
University College London (@ucl),
Mullard Space Science Laboratory (@MSSLSpaceLab)

19 January 2026

@DVerscharen @dverscharen

Outline

Part I

1. A brief introduction to basic plasma physics
2. Stellar winds
3. In-situ measurement of astrophysical plasmas

Part II

1. Plasma turbulence
2. Cosmic-ray transport

Plasma turbulence

All natural systems are *dissipative* at some scale.
Extend MHD equations with viscosity and resistivity:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{U}) = 0$$

$$\rho \frac{\partial \boldsymbol{U}}{\partial t} + \rho \, (\boldsymbol{U} \cdot \nabla) \, \boldsymbol{U} = -\nabla P + \frac{1}{4\pi} \left[(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} \right] + \mu \nabla^2 \boldsymbol{U}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{U} \times \boldsymbol{B}) + \frac{c^2}{4\pi\sigma} \nabla^2 \boldsymbol{B}$$

Turbulence is *multi-scale disorder* in which non-linear interactions transfer energy across scales.

Turbulence is *multi-scale disorder* in which non-linear interactions transfer energy across scales.

Compare the non-linear term and the dissipative term in the momentum equation:

$$\rho \frac{\partial \boldsymbol{U}}{\partial t} + \rho \left(\boldsymbol{U} \cdot \nabla \right) \boldsymbol{U} = -\nabla P + \frac{1}{4\pi} \left[\left(\nabla \times \boldsymbol{B} \right) \times \boldsymbol{B} \right] + \mu \nabla^2 \boldsymbol{U}$$

Turbulence is *multi-scale disorder* in which non-linear interactions transfer energy across scales.

Compare the non-linear term and the dissipative term in the momentum equation:

$$\rho \frac{\partial \boldsymbol{U}}{\partial t} + \rho \left(\boldsymbol{U} \cdot \nabla \right) \boldsymbol{U} = -\nabla P + \frac{1}{4\pi} \left[\left(\nabla \times \boldsymbol{B} \right) \times \boldsymbol{B} \right] + \mu \nabla^2 \boldsymbol{U}$$

$$\rho \left(\boldsymbol{U} \cdot \nabla \right) \boldsymbol{U} \sim \frac{\rho U^2}{L} \qquad \qquad \mu \nabla^2 \boldsymbol{U} \sim \frac{\mu U}{L^2}$$

Turbulence is *multi-scale disorder* in which non-linear interactions transfer energy across scales.

Compare the non-linear term and the dissipative term in the momentum equation:

$$\rho \frac{\partial \boldsymbol{U}}{\partial t} + \rho \left(\boldsymbol{U} \cdot \nabla \right) \boldsymbol{U} = -\nabla P + \frac{1}{4\pi} \left[\left(\nabla \times \boldsymbol{B} \right) \times \boldsymbol{B} \right] + \mu \nabla^2 \boldsymbol{U}$$

$$\rho \left(\boldsymbol{U} \cdot \nabla \right) \boldsymbol{U} \sim \frac{\rho U^2}{L}$$

$$\mu \nabla^2 \boldsymbol{U} \sim \frac{\mu U}{L^2}$$

$$\frac{\rho \left(\boldsymbol{U} \cdot \nabla \right) \boldsymbol{U}}{\mu \nabla^2 \boldsymbol{U}} \sim \frac{\rho U L}{\mu} = Re$$

Reynolds number is usually very large in most natural plasmas –> most plasmas are turbulent!

Power spectrum of turbulent fluctuations

Assumption: At intermediate scales, energy just transfers from scale to scale (locally) without dissipation and at a constant rate (no pile-up of energy at certain scales).

Power spectrum of turbulent fluctuations

Assumption: At intermediate scales, energy just transfers from scale to scale (locally) without dissipation and at a constant rate (no pile-up of energy at certain scales).

Consider turbulent eddy of scale ℓ and velocity difference (fluctuation) δU_ℓ.
The typical eddy turn-over time is $\tau \sim \ell / \delta U_\ell$.

Power spectrum of turbulent fluctuations

<u>Assumption:</u> At intermediate scales, energy just transfers from scale to scale (locally) without dissipation and at a constant rate (no pile-up of energy at certain scales).

Consider turbulent eddy of scale ℓ and velocity difference (fluctuation) δU_ℓ.
The typical eddy turn-over time is $\tau \sim \ell / \delta U_\ell$.

Rate of energy transfer is $\epsilon \sim W/\tau$, where the energy $W \sim \delta U_\ell^2$.

Combining relations: $W \sim (\epsilon \ell)^{2/3} \sim \left(\frac{\epsilon}{k}\right)^{2/3}$

Power spectrum of turbulent fluctuations

<u>Assumption:</u> At intermediate scales, energy just transfers from scale to scale (locally) without dissipation and at a constant rate (no pile-up of energy at certain scales).

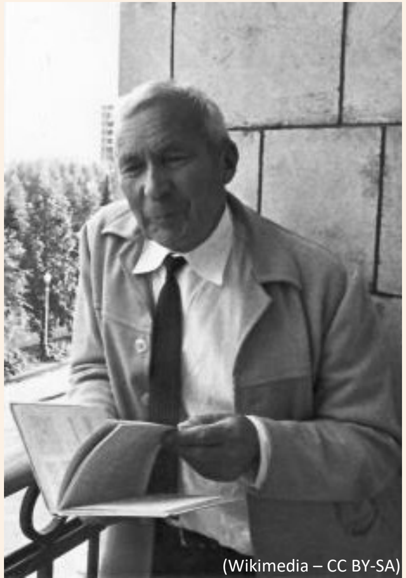
Consider turbulent eddy of scale ℓ and velocity difference (fluctuation) δU_ℓ.
The typical eddy turn-over time is $\tau \sim \ell/\delta U_\ell$.

Rate of energy transfer is $\epsilon \sim W/\tau$, where the energy $W \sim \delta U_\ell^2$.

Combining relations: $W \sim (\epsilon\ell)^{2/3} \sim \left(\frac{\epsilon}{k}\right)^{2/3}$

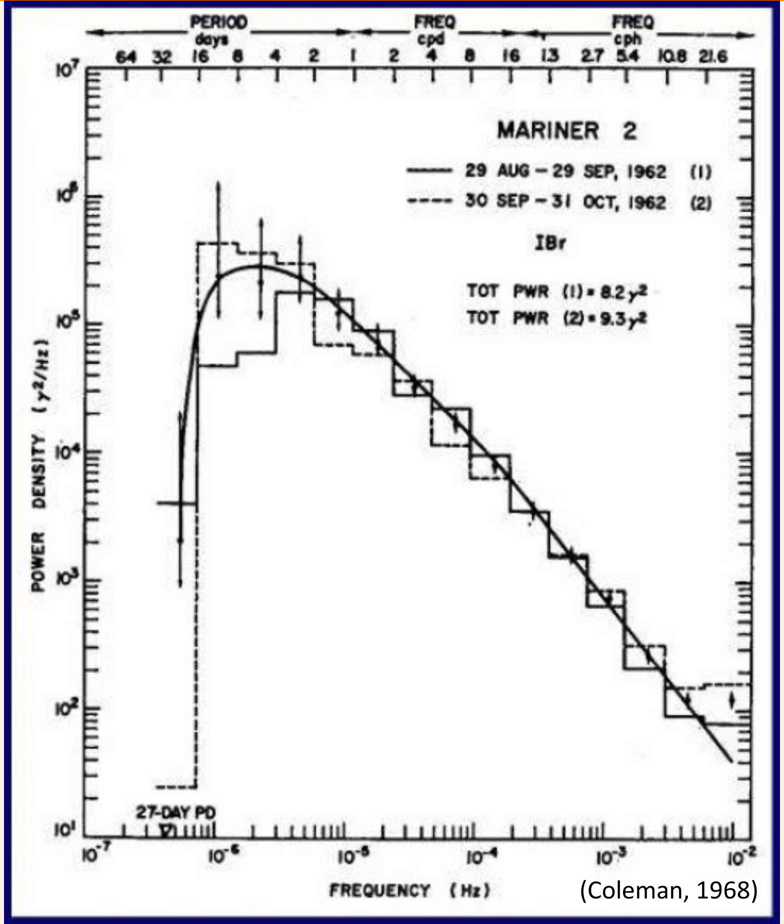
Define power spectral density as $P \sim \frac{W}{k}$ so that $P \sim \epsilon^{2/3} k^{-5/3}$

Kolmogorov spectrum



(Wikimedia – CC BY-SA)

Measurement of plasma turbulence in the solar wind



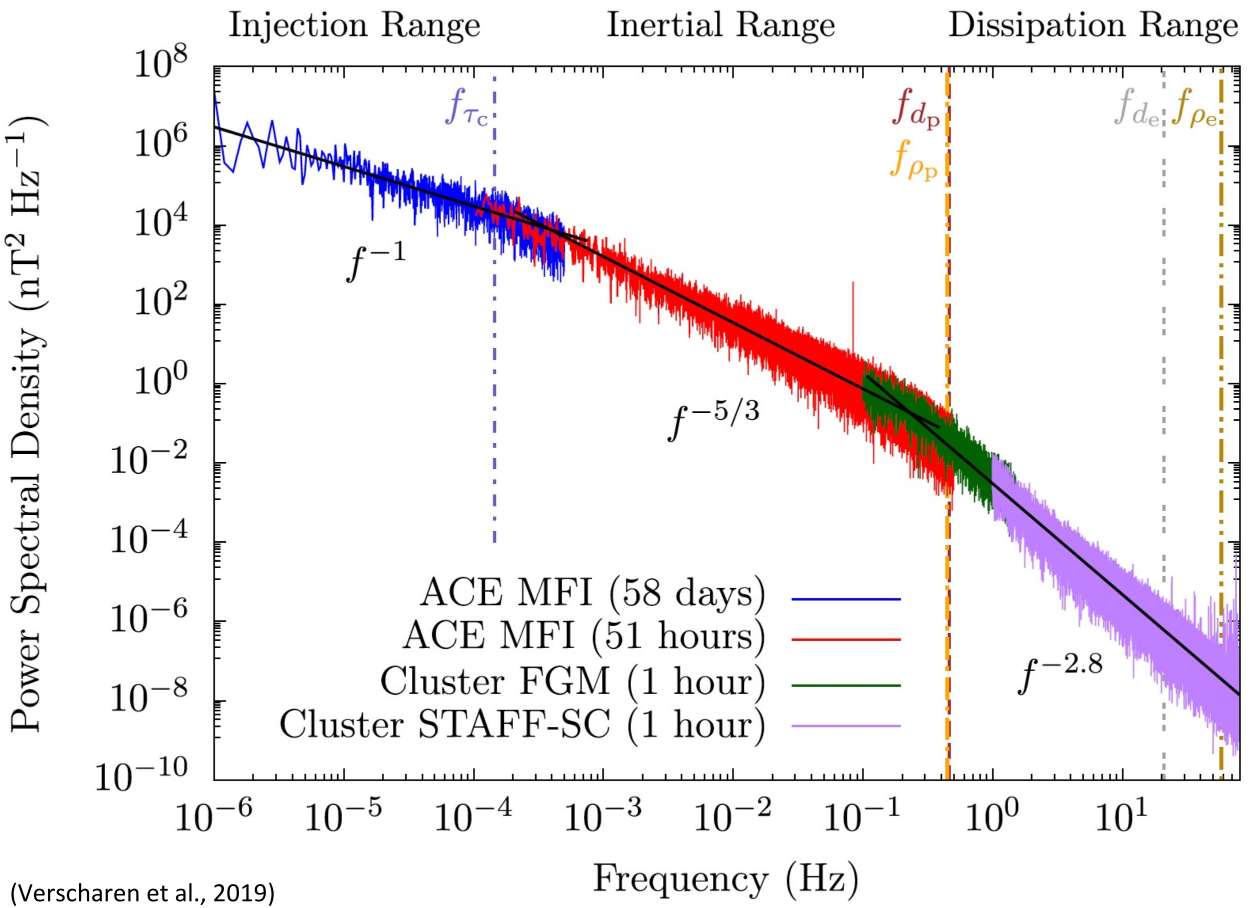
(Coleman, 1968)

Very early measurements from Mariner 2 show Kolmogorov spectrum in the magnetic field of the solar wind.

Remember: the magnetic field is frozen in the plasma flow, so similar spectra are expected for plasma velocity and magnetic field.

Frequency in the spacecraft frame translates to wavenumber in plasma frame.

Measurement of plasma turbulence in the solar wind



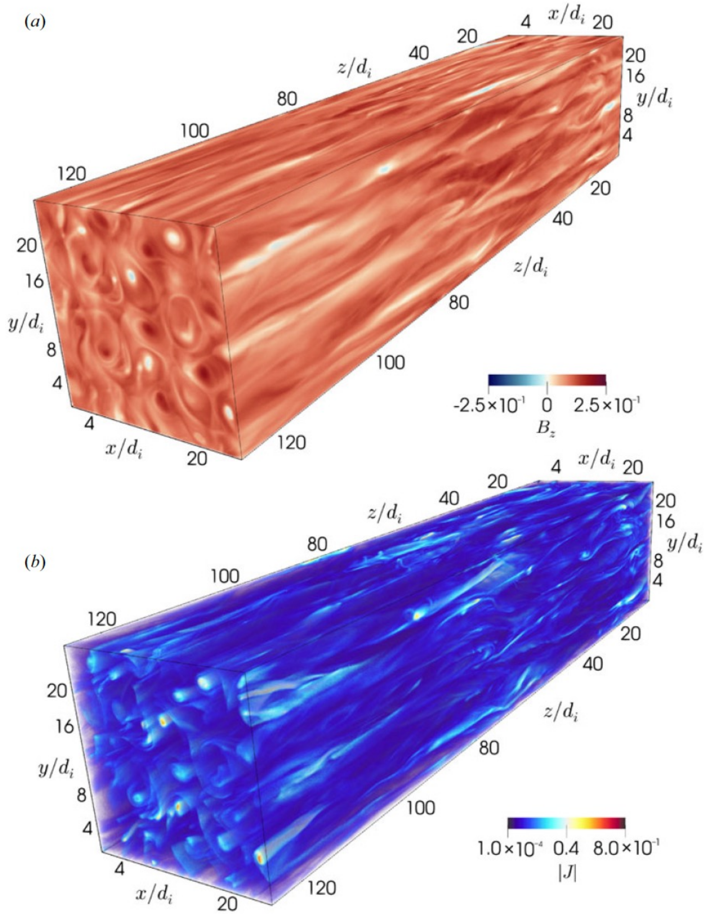
(Verscharen et al., 2019)

Today, we have much more detailed measurements.

- Injection range
- Inertial range
- Dissipation range

Breakpoints at characteristic scales.

Intermittency



Turbulence develops ***intermittency***: non-uniform spatial distribution of fluctuations at given scales.

In a plasma, this means that *current sheets* ($\nabla\times\boldsymbol{B} = 4\pi\boldsymbol{j}/c$) and *vorticity sheets* ($\nabla\times\boldsymbol{U} = \boldsymbol{\Omega}$) form.

These coherent structures are locations of strong ***dissipation***.

PIC simulation by Agudelo Rueda, DV, et al. (2021)

How do particles interact with turbulent fluctuations?

There are many channels through which turbulent electromagnetic fields can interact with charged particles in a plasma.

In a broad spectrum of wave-like fluctuations with frequencies ω, wave vectors k, and random phases, ***resonant wave-particle interactions*** can occur.

How do particles interact with turbulent fluctuations?

There are many channels through which turbulent electromagnetic fields can interact with charged particles in a plasma.

In a broad spectrum of wave-like fluctuations with frequencies ω, wave vectors k, and random phases, ***resonant wave-particle interactions*** can occur.

Landau resonance:

$$\omega = k_{\parallel} v_{\parallel}$$

Particle moves along background magnetic field with same speed as wave

How do particles interact with turbulent fluctuations?

There are many channels through which turbulent electromagnetic fields can interact with charged particles in a plasma.

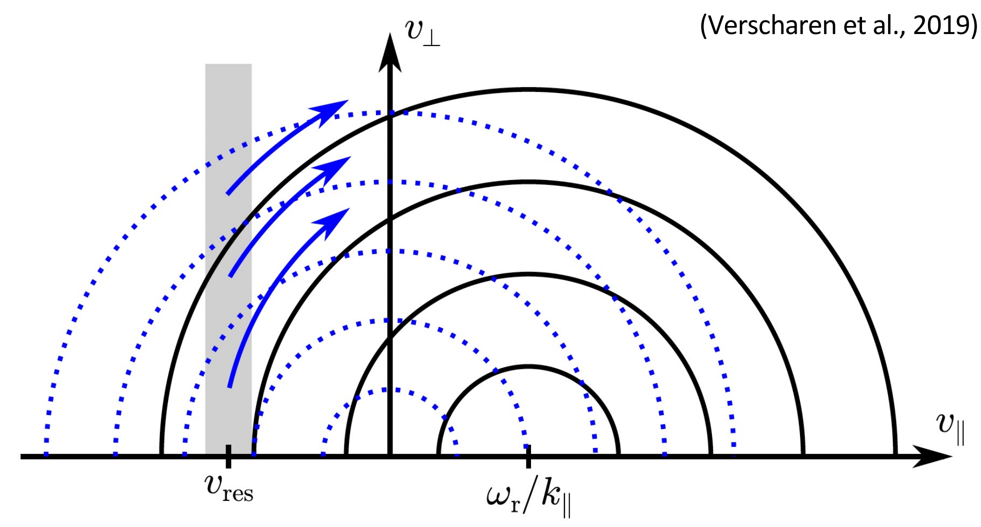
In a broad spectrum of wave-like fluctuations with frequencies ω, wave vectors k, and random phases, ***resonant wave-particle interactions*** can occur.

Landau resonance: $\omega = k_{\|} v_{\|}$ Particle moves along background magnetic field with same speed as wave

Cyclotron resonance: $\omega = k_{\|} v_{\|} \pm \Omega_j$ Particle experiences during its gyration a constant perpendicular wave field

Resonant interactions lead to diffusion in velocity space (quasi-linear theory):

$$\frac{\partial f_{0j}}{\partial t} = \lim_{V\to\infty} \sum_{n=-\infty}^{+\infty} \frac{q_j^2}{8\pi^2 m_j^2} \int \frac{1}{v_\perp V} \hat{G} v_\perp \delta\left(\omega_{k\mathrm{r}} - k_\parallel v_\parallel - n\Omega_j\right) \left|\psi_k^{j,n}\right|^2 \hat{G} f_{0j}\, \mathrm{d}^3 k$$

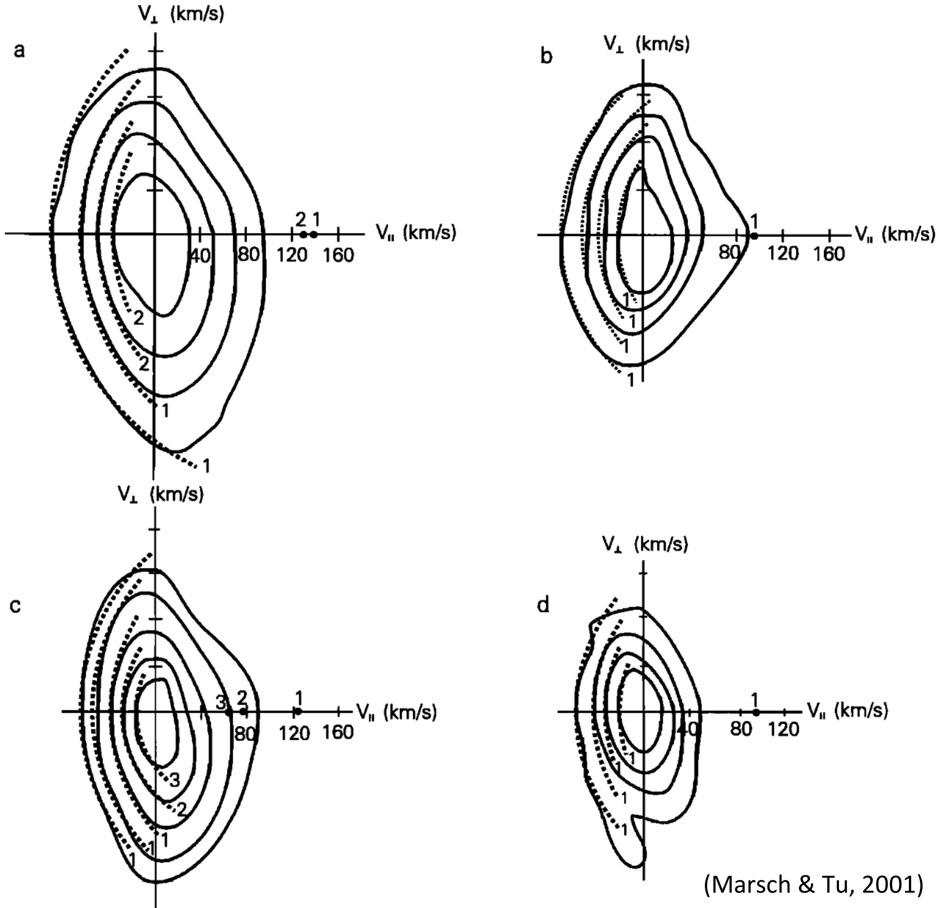


$$\hat{G} \equiv \left(1 - \frac{k_\parallel v_\parallel}{\omega_{k\mathrm{r}}}\right) \frac{\partial}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega_{k\mathrm{r}}} \frac{\partial}{\partial v_\parallel}$$

$$\psi_k^{j,n} \equiv \frac{1}{\sqrt{2}} \left[E_{k,\mathrm{r}} e^{i\phi_k} J_{n+1}(\xi_j) + E_{k,\mathrm{l}} e^{-i\phi_k} J_{n-1}(\xi_j)\right] + \frac{v_\parallel}{v_\perp} E_{kz} J_n(\xi_j)$$

q_j: charge, m_j: mass, $(v_\perp, v_\parallel)$: velocity components, k: wavevector, ϕ_k azimuthal angle of wavevector, Ω_j: gyro-frequency, ω_{kr}: wave frequency, E_k: polarised electric-field components, J_n: Bessel function, $\xi_j = k_\perp v_\perp / \Omega_j$

How do particles interact with turbulent fluctuations?



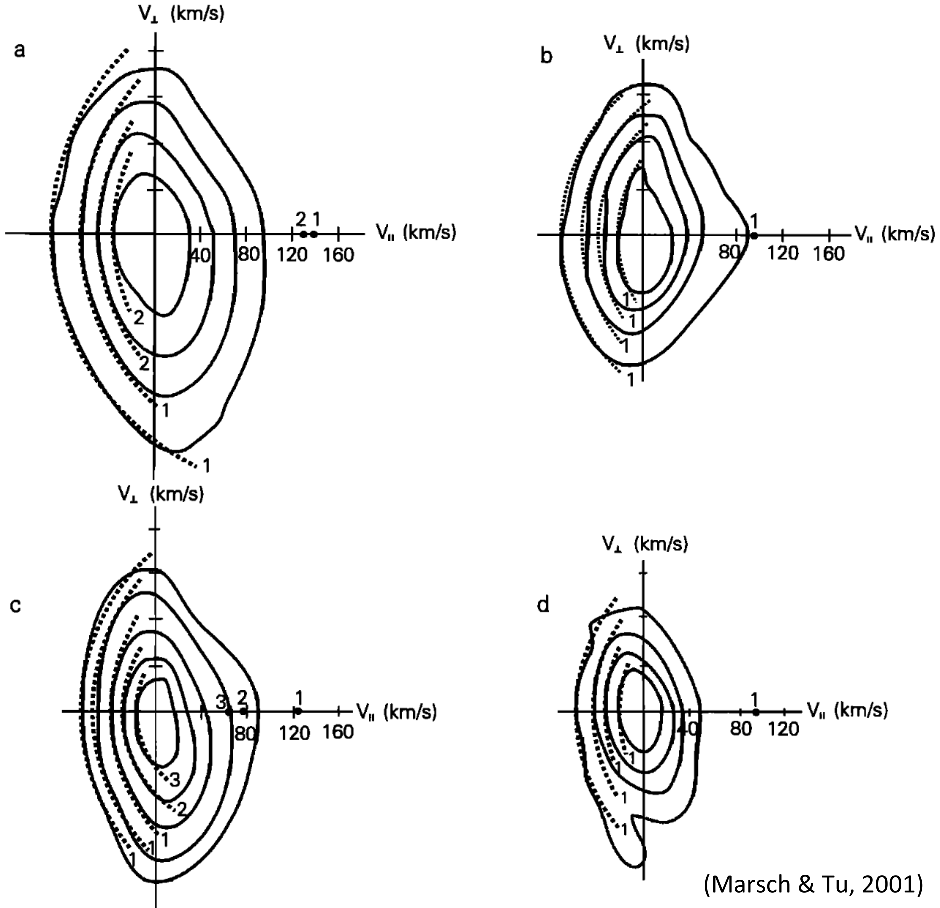
(Marsch & Tu, 2001)

In-situ measurements show evidence for quasi-linear diffusion of thermal protons in the solar wind.

Resonant wave-particle interactions with ion-cyclotron waves.

One possible channel for turbulent dissipation and plasma heating.

How do particles interact with turbulent fluctuations?



(Marsch & Tu, 2001)

In-situ measurements show evidence for quasi-linear diffusion of thermal protons in the solar wind.

Resonant wave-particle interactions with ion-cyclotron waves.

One possible channel for turbulent dissipation and plasma heating.

What happens to energetic particles?

Cosmic-ray transport

Our fundamental set of equations describes energetic particles too. Just include relativistic effects for $f_j(\boldsymbol{r}, \boldsymbol{p}, t)$:

$$\frac{\partial f_j}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_j}{\partial \boldsymbol{r}} + q_j \left(\boldsymbol{E} + \frac{1}{c} \boldsymbol{v} \times \boldsymbol{B} \right) \cdot \frac{\partial f_j}{\partial \boldsymbol{p}} = 0$$

$$\boldsymbol{v} = \frac{\boldsymbol{p}}{\gamma m_j}$$

$$\nabla \cdot \boldsymbol{E} = 4\pi \sum_j q_j \int f_j \, \mathrm{d}^3 p$$

$$\gamma = \sqrt{1 + \frac{p^2}{m_j^2 c^2}}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c} \sum_j q_j \int \boldsymbol{v} f_j \, \mathrm{d}^3 p + \frac{1}{c} \frac{\partial \boldsymbol{E}}{\partial t}$$

Let’s make the following **assumptions**:

- The cosmic rays propagate through a thermal plasma that moves at velocity $\boldsymbol{U}$.
- The plasma is filled with turbulent fluctuations and waves that scatter particles due to the Lorentz force.
- Between scatterings, the particles follow their guiding-centre motion.
- Scattering on magnetic-field fluctuations is frequent enough to keep the cosmic-ray distribution isotropic in the frame that moves with the plasma.
- Therefore, there is no tendency for the particles to move systematically in either direction.

Lorentz-transform the momentum in the Vlasov equation into the frame that co-moves with the thermal plasma (and thus with the turbulence/waves) at steady-state velocity $\boldsymbol{U}(\boldsymbol{r})$ with $v_A \ll U \ll c$:

$$\boldsymbol{p}' = \frac{\boldsymbol{p} - \boldsymbol{U}E/c^2}{\sqrt{1 - \frac{U^2}{c^2}}} \simeq \boldsymbol{p} - \gamma m_j \boldsymbol{U} + O\left(\frac{U^2}{v^2} p\right)$$

Lorentz-transform the momentum in the Vlasov equation into the frame that co-moves with the thermal plasma (and thus with the turbulence/waves) at steady-state velocity $\boldsymbol{U}(\boldsymbol{r})$ with $v_A \ll U \ll c$:

$$\boldsymbol{p}' = \frac{\boldsymbol{p} - \boldsymbol{U}E/c^2}{\sqrt{1 - \frac{U^2}{c^2}}} \simeq \boldsymbol{p} - \gamma m_j \boldsymbol{U} + O\left(\frac{U^2}{v^2} p\right)$$

$$\frac{\mathrm{d}\boldsymbol{p}'}{\mathrm{d}t} = \boldsymbol{v} \cdot \frac{\partial \boldsymbol{p}'}{\partial \boldsymbol{r}} + \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \cdot \frac{\partial \boldsymbol{p}'}{\partial \boldsymbol{p}}$$

Lorentz-transform the momentum in the Vlasov equation into the frame that co-moves with the thermal plasma (and thus with the turbulence/waves) at steady-state velocity $\boldsymbol{U}(\boldsymbol{r})$ with $v_A \ll U \ll c$:

$$\boldsymbol{p}' = \frac{\boldsymbol{p} - \boldsymbol{U}E/c^2}{\sqrt{1 - \frac{U^2}{c^2}}} \simeq \boldsymbol{p} - \gamma m_j \boldsymbol{U} + O\left(\frac{U^2}{v^2} p\right)$$

$$\frac{\mathrm{d}\boldsymbol{p}'}{\mathrm{d}t} = \boldsymbol{v} \cdot \frac{\partial \boldsymbol{p}'}{\partial \boldsymbol{r}} + \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \cdot \frac{\partial \boldsymbol{p}'}{\partial \boldsymbol{p}}$$

$$= -\gamma m_j \boldsymbol{v} \cdot \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{r}} + \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -\boldsymbol{p} \cdot \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{r}} + \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}$$

If the distribution is isotropic, we only care about the magnitude of $\boldsymbol{p}'$:

$$\frac{\mathrm{d}p'^2}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{p}' \cdot \boldsymbol{p}'\right) \qquad \Rightarrow \qquad \frac{\mathrm{d}p'}{\mathrm{d}t} = \frac{\boldsymbol{p}'}{p'} \cdot \frac{\mathrm{d}\boldsymbol{p}'}{\mathrm{d}t}$$

If the distribution is isotropic, we only care about the magnitude of $\boldsymbol{p}'$:

$$\frac{\mathrm{d}p'^2}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{p}' \cdot \boldsymbol{p}'\right) \qquad \Rightarrow \qquad \frac{\mathrm{d}p'}{\mathrm{d}t} = \frac{\boldsymbol{p}'}{p'} \cdot \frac{\mathrm{d}\boldsymbol{p}'}{\mathrm{d}t}$$

Use the result from previous slide and neglect electric fields (B-fields do not change magnitude of momentum):

$$\frac{\mathrm{d}p'}{\mathrm{d}t} = -\frac{\boldsymbol{p}'}{p'} \cdot \left[\boldsymbol{p} \cdot \frac{\partial \boldsymbol{U}}{\partial r} - \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}\right] = -\frac{\boldsymbol{p}'}{p'} \cdot \left[\left(\boldsymbol{p}' + \gamma m_j \boldsymbol{U}\right) \cdot \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{r}}\right]$$

If the distribution is isotropic, we only care about the magnitude of $\boldsymbol{p}'$:

$$\frac{\mathrm{d}p'^2}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{p}' \cdot \boldsymbol{p}') \qquad \Rightarrow \qquad \frac{\mathrm{d}p'}{\mathrm{d}t} = \frac{\boldsymbol{p}'}{p'} \cdot \frac{\mathrm{d}\boldsymbol{p}'}{\mathrm{d}t}$$

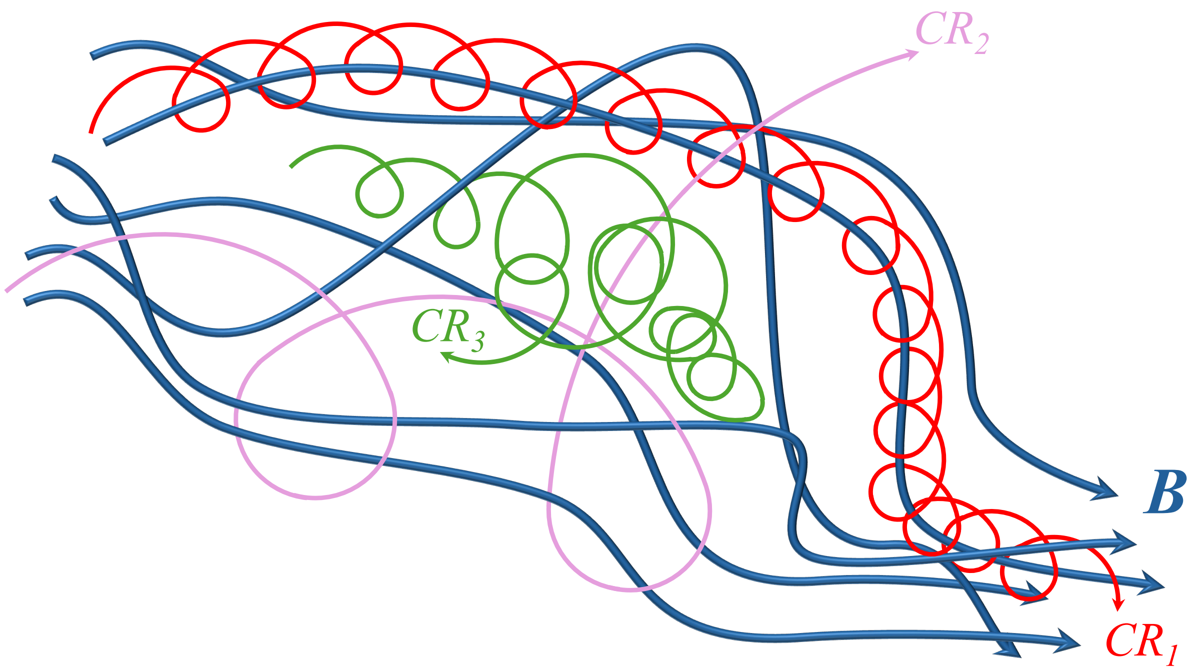
Use the result from previous slide and neglect electric fields (B-fields do not change magnitude of momentum):

$$\frac{\mathrm{d}p'}{\mathrm{d}t} = -\frac{\boldsymbol{p}'}{p'} \cdot \left[\boldsymbol{p} \cdot \frac{\partial \boldsymbol{U}}{\partial r} - \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}\right] = -\frac{\boldsymbol{p}'}{p'} \cdot \left[(\boldsymbol{p}' + \gamma m_j \boldsymbol{U}) \cdot \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{r}}\right]$$

If the distribution is isotropic in the primed frame, we can average over all directions:

$$\left\langle \frac{\mathrm{d}p'}{\mathrm{d}t} \right\rangle = \frac{1}{4\pi} \int \frac{\mathrm{d}p'}{\mathrm{d}t} \mathrm{d}\Omega = -\frac{p'}{3} \nabla \cdot \boldsymbol{U}$$

Scattering on fluctuations in the magnetic field



Scattering is most efficient for particles with $k\rho_{CR} \sim 1$.

Viewed from large scales, the particles random-walk due to this scattering.

Also cyclotron-resonant interactions are possible:

$$\omega = k_{||} v_{||} \pm \Omega_{CR}$$

The random walk of particles is a Markov process, which can be described by spatial diffusion in the form of a *Fokker-Planck equation*:

$$\left.\frac{\partial f_j}{\partial t}\right|_{\rm scat} = \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla f_j)$$

with a diffusion tensor that depends on the properties of the fluctuations.

With these considerations, our transport equation for *isotropic cosmic-ray transport* is given by the Parker equation:

$$\frac{\partial f_j}{\partial t} + \boldsymbol{U} \cdot \frac{\partial f_j}{\partial \boldsymbol{r}} - \frac{p}{3} (\nabla \cdot \boldsymbol{U}) \frac{\partial f_j}{\partial p} = \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla f_j) + Q$$

Notes:

- We have removed the primes from the momentum.
- The distribution function depends on $\boldsymbol{r}$, p, and t.
- We can add a source/sink term Q on the right-hand side.

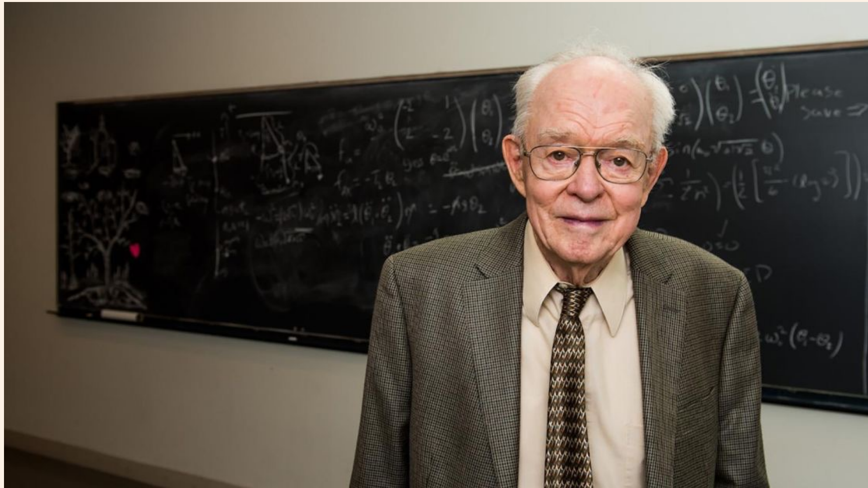
Transport equation for cosmic rays

With these considerations, our transport equation for *isotropic cosmic-ray transport* is given by the Parker equation:

$$\frac{\partial f_j}{\partial t} + \boldsymbol{U} \cdot \frac{\partial f_j}{\partial \boldsymbol{r}} - \frac{p}{3} (\nabla \cdot \boldsymbol{U}) \frac{\partial f_j}{\partial p} = \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla f_j) + Q$$

The terms represent:

1. Temporal change in f_j
2. Convection with $\boldsymbol{U}$
3. Adiabatic change of energy
4. Spatial diffusion and drift
5. Sources and sinks



With these considerations, our transport equation for *isotropic cosmic-ray transport* is given by the Parker equation:

$$\frac{\partial f_j}{\partial t} + \boldsymbol{U} \cdot \frac{\partial f_j}{\partial \boldsymbol{r}} - \frac{p}{3}\left(\nabla \cdot \boldsymbol{U}\right)\frac{\partial f_j}{\partial p} = \nabla \cdot \left(\boldsymbol{\kappa} \cdot \nabla f_j\right) + Q$$

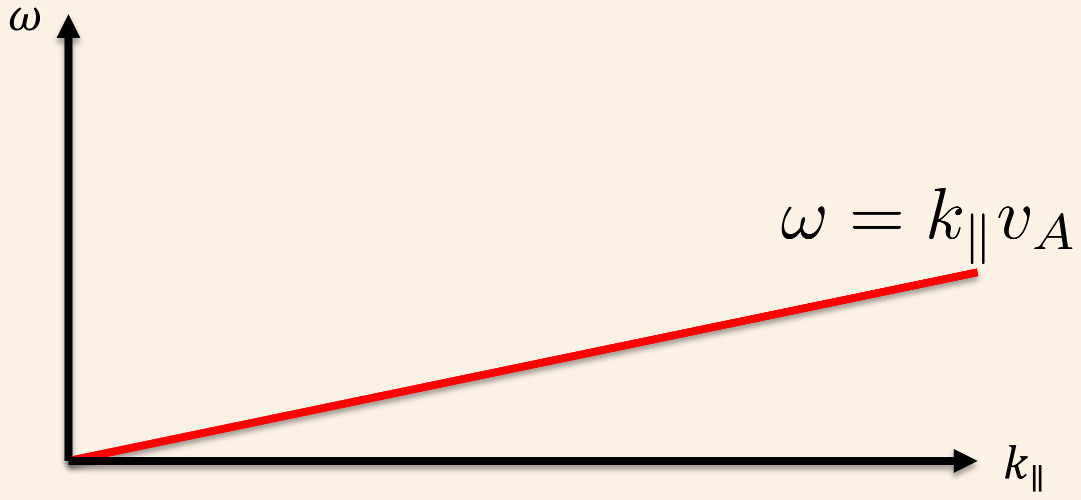
The terms represent:

1. Temporal change in f_j
2. Convection with $\boldsymbol{U}$
3. Adiabatic change of energy
4. Spatial diffusion and drift
5. Sources and sinks

Depending on assumptions, different versions of the transport equation can be found.

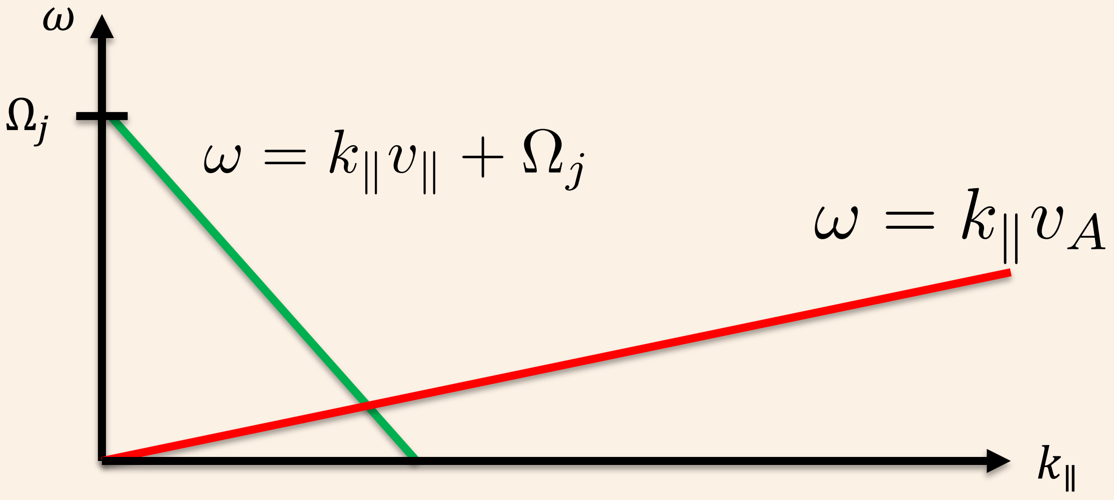
Self-consistency also applies to cosmic rays, so they can also modify the electromagnetic field.

Dispersion relation for Alfvén waves: $\omega = k_{\|} v_A$



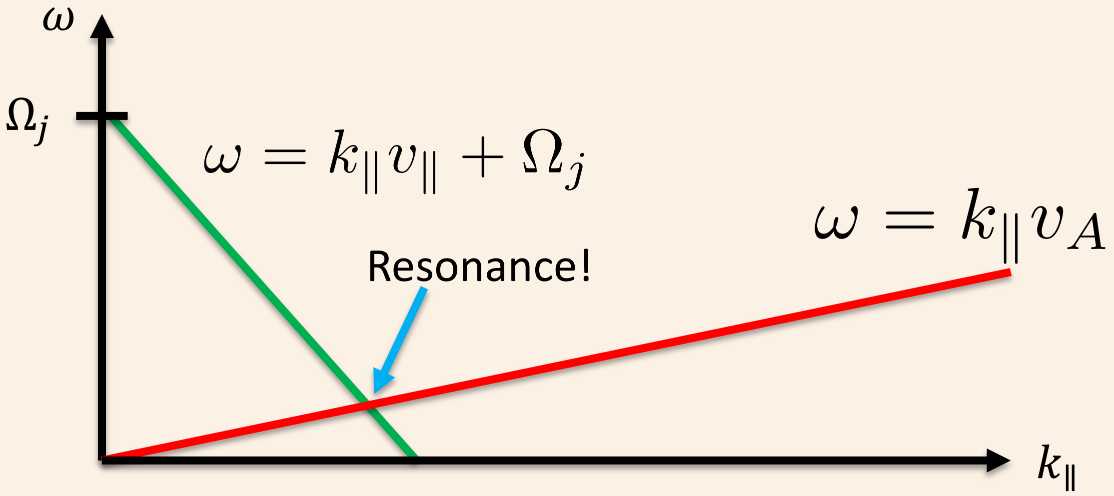
Self-consistency also applies to cosmic rays, so they can also modify the electromagnetic field.

Dispersion relation for Alfvén waves: $\omega = k_{\|} v_A$

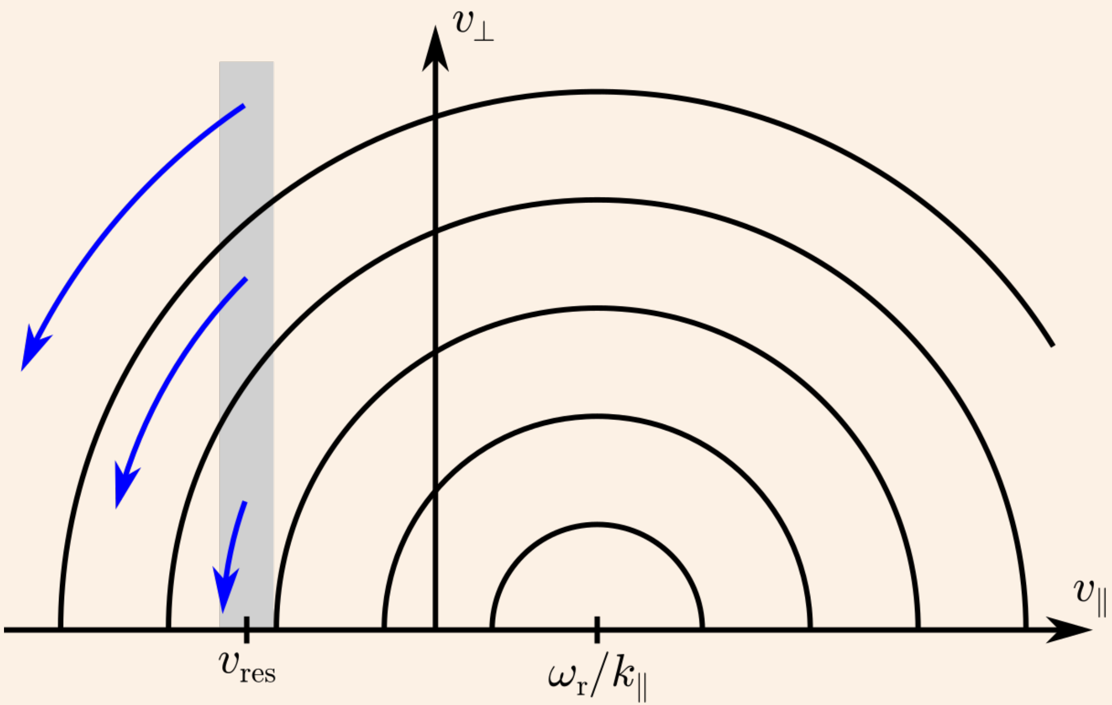


Self-consistency also applies to cosmic rays, so they can also modify the electromagnetic field.

Dispersion relation for Alfvén waves: $\omega = k_{\parallel} v_A$



At the resonance: $$k_\| v_\mathrm{A} = k_\| v_\mathrm{res} + \Omega_j$$



Resonant particles undergo quasi-linear diffusion.

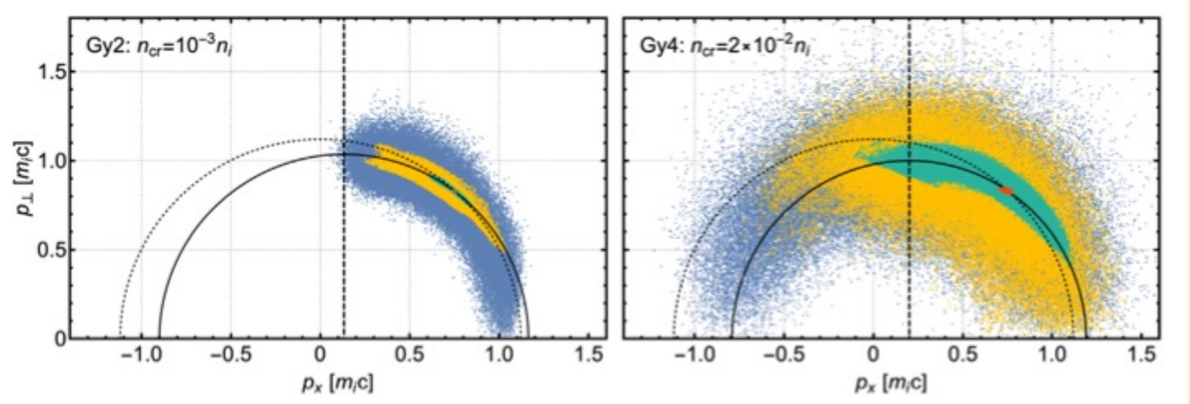
If these particles lose kinetic energy, this can drive the resonant waves unstable.

Energy gain/loss during the quasi-linear diffusion is defined by the drift speed of the CR population compared to the wave phase speed (free energy).

Waves are driven unstable at a scale comparable to CR gyro-radius.

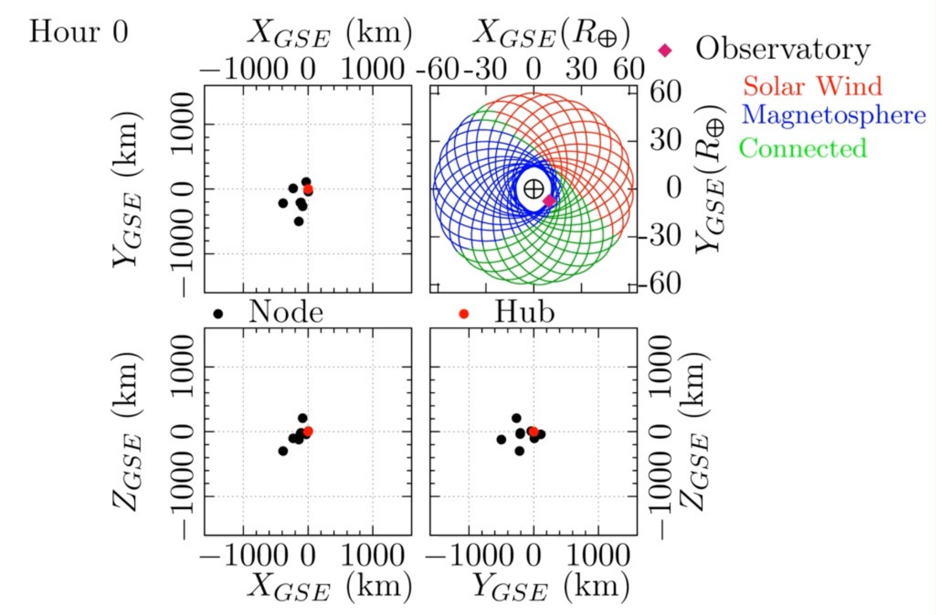
Through such instabilities, CRs can generate the scattering fluctuations themselves (self-regulation of transport).

Particle-in-cell simulations show consistent resonant scattering behaviour (plus nonlinear trapping etc.).

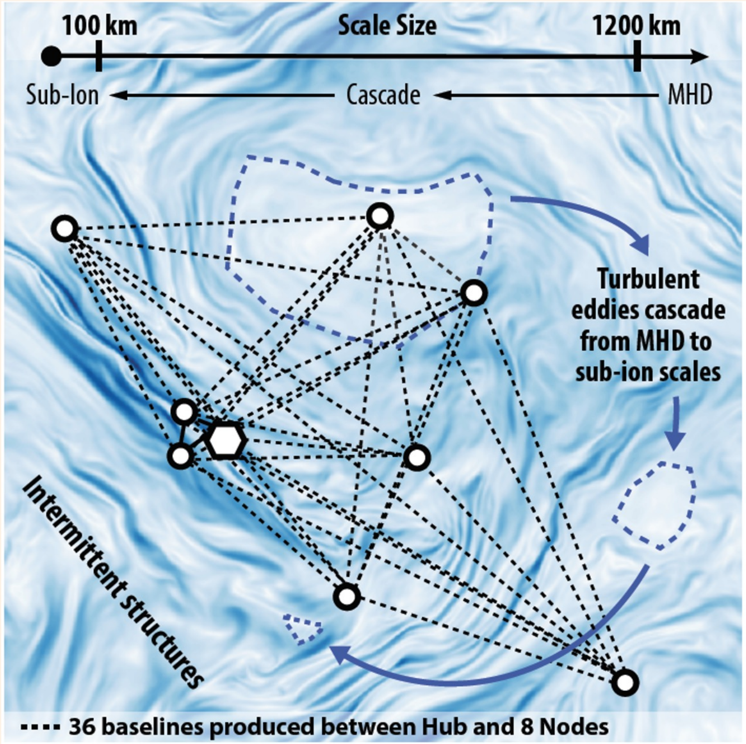


(Different times: red, green, yellow, blue; Holcomb & Spitkovsky, 2019)

HelioSwarm: multi-point study of plasma turbulence



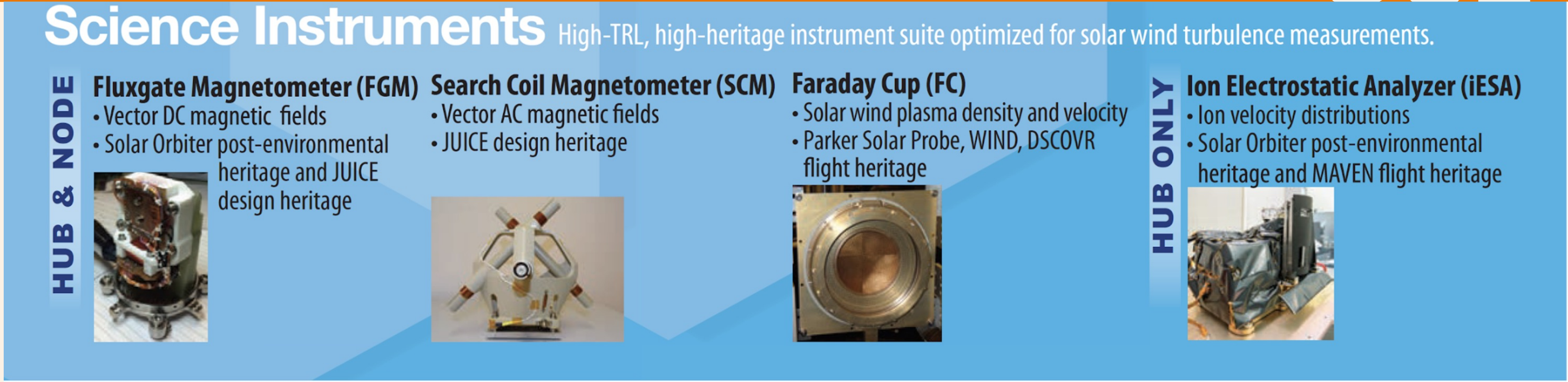
<u>Challenges</u>: AIT, swarm operations, telemetry



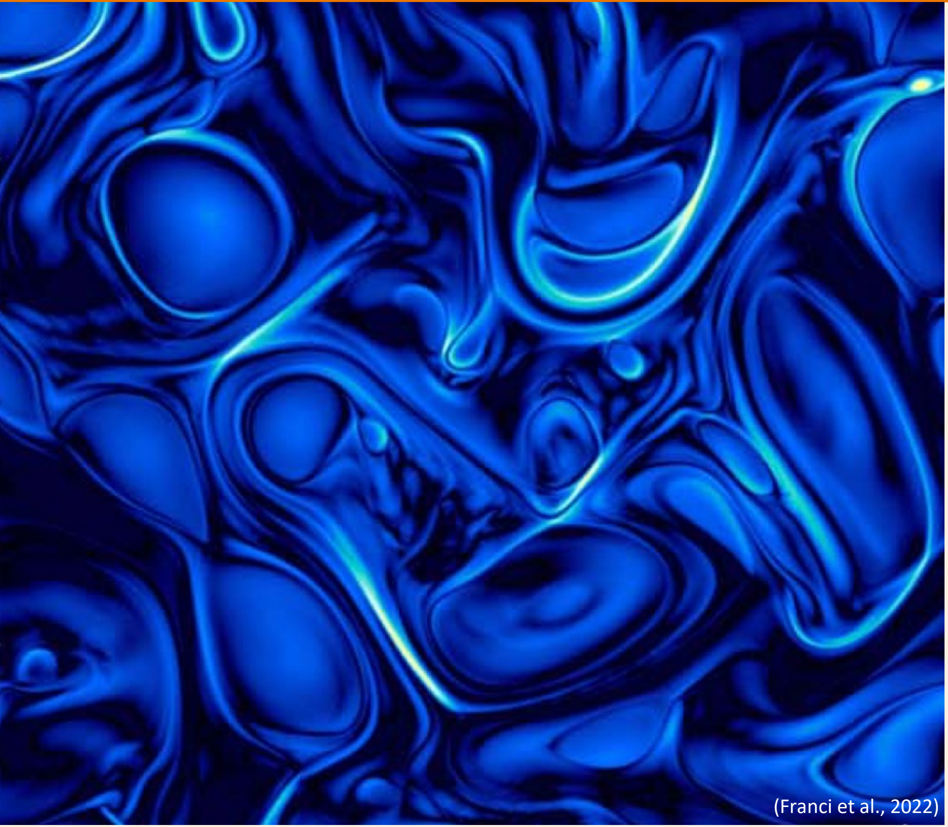
- Launch of 9 spacecraft in 2029.
- Goals: reveal the 3D spatial structure and dynamics of *turbulence*; ascertain impact of turbulence near boundaries and large-scale structures.

(Arzamasskiy et al., 2019)

HelioSwarm: multi-point study of plasma turbulence



An electron electrostatic analyser, led by Phyllis Whittlesey (UC Berkeley), is included as a Student Collaboration Option for installation on the Hub.


(Franci et al., 2022)

- Almost all astrophysical plasmas are turbulent: multi-scale disorder with non-linear energy transfer across scales.
- Exchange of energy and momentum between fluctuations and particles.
- Cosmic-ray transport is strongly determined by turbulence.
- New space missions are on the horizon to understand plasma turbulence better.