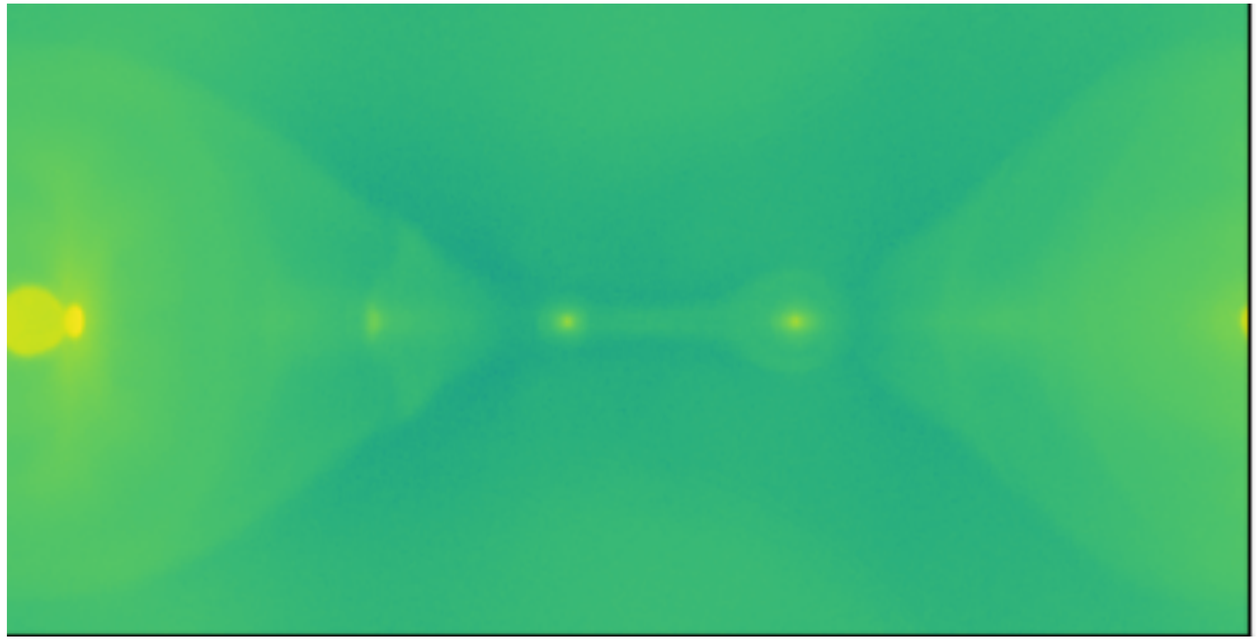
Relativistic reconnection, explicit or semi-implicit methods?

K. Schoeffler[1]

F. Bacchini[2]

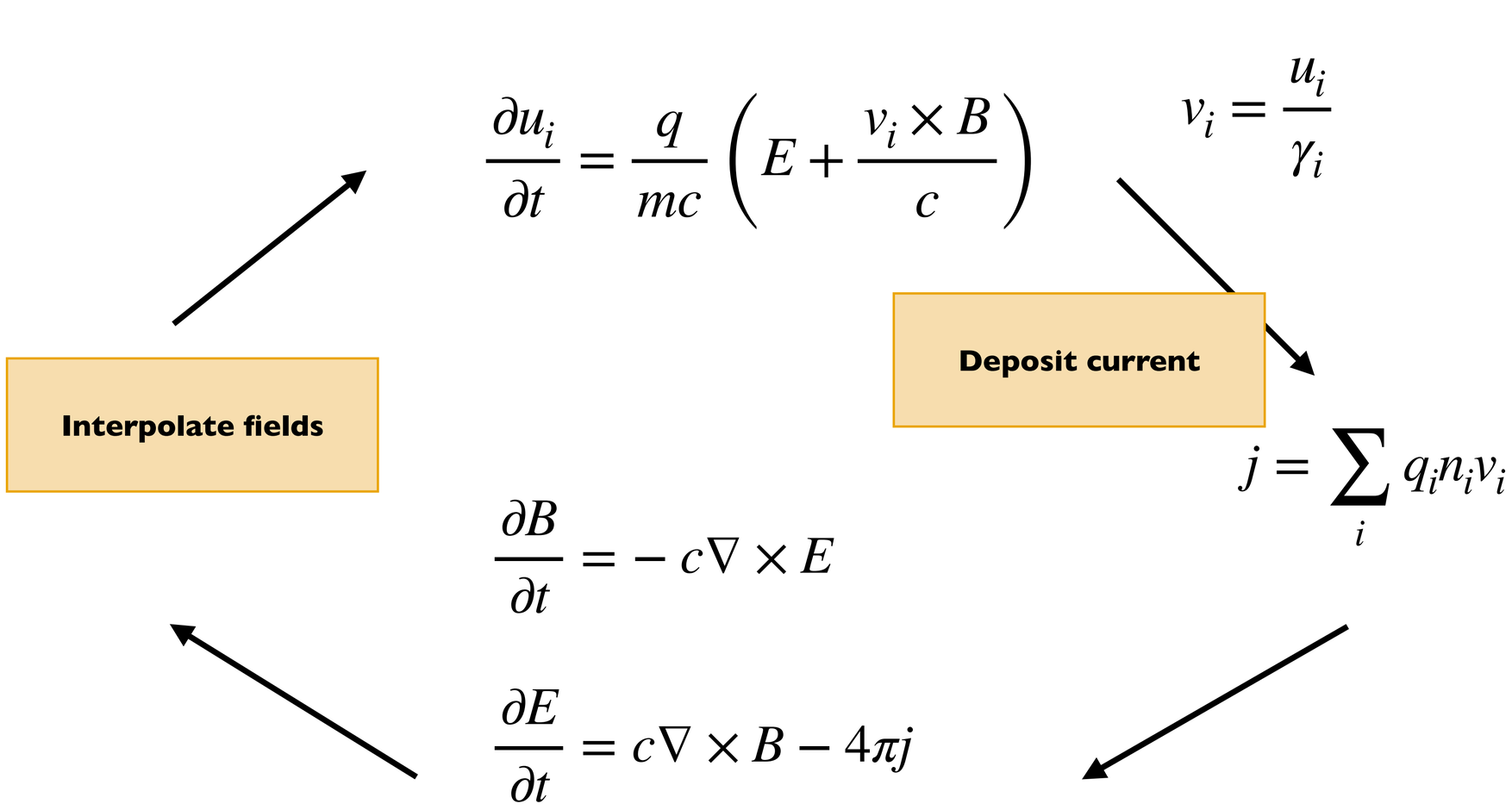
M. E. Innocenti[1]



1 Institut für Theoretische Physik, Ruhr-Universität Bochum, Germany

2 Centre for mathematical Plasma Astrophysics, Department of Mathematics, KU Leuven, Celestijnenlaan 200B, B-3001 Leuven, Belgium

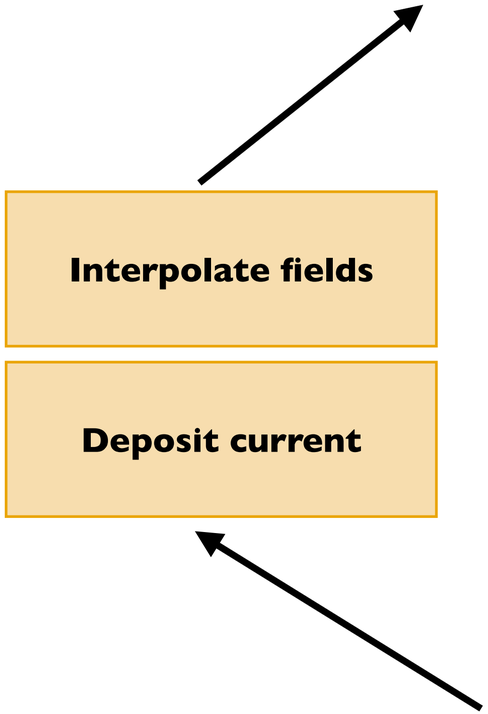
What is an explicit Particle-in-cell (PIC) code?



$u_{i,n+1}$ and E_{n+1} are calculated based on only the fields E_n and momenta $u_{i,n}$ from the previous time step

Leads to instabilities if time steps and spaces are not fully resolved

What is an implicit Particle-in-cell (PIC) code?



$$\frac{\partial u_i}{\partial t} = \frac{q}{mc}\left(E + \frac{v_i \times B}{c}\right)$$

$$j = \sum_i q_i n_i v_i \qquad v_i = \frac{u_i}{\gamma_i}$$

$$\frac{\partial B}{\partial t} = -c\nabla \times E$$

$$\frac{\partial E}{\partial t} = c\nabla \times B - 4\pi j$$

All equations are solved self-consistently in terms of both $u_{i,n}$ / E_n and $u_{i,n+1}$ / E_{n+1} usually by iteration

Avoids instabilities, allowing for under-resolution of time and space

What can we under-resolve

RUHR UNIVERSITÄT BOCHUM RUB

We don't need to resolve the Debye scales

If we are mainly interested in ion scales, we can under-resolve the electron scales

Spatial resolution

$$\lambda_D/dx > 1$$

$$d_e/dx > 1$$

$$d_i/dx > 1$$

semi-implicit schemes tend to be $\sim 10\times$ slower

Factor of $\sqrt{m_i/m_e}c/v_T$ faster for each space direction

Most benefit in 3D

$$\lambda_D^2 = \frac{T}{4\pi mne^2} \qquad d_e^2 = \frac{m_e c^2}{4\pi mne^2} \qquad d_i^2 = \frac{m_i c^2}{4\pi mne^2}$$

$$\lambda_D = \frac{v_T}{c} d_e = \frac{v_T}{c}\sqrt{\frac{m_e}{m_i}} d_i$$

What can we under-resolve

RUHR UNIVERSITÄT BOCHUM RUB

We don't need to resolve the Debye scales

If we are mainly interested in ion scales, we can under-resolve the electron scales

Spatial resolution

$$\lambda_D/dx > 1$$

$$d_e/dx > 1$$

$$d_i/dx > 1$$

semi-implicit schemes tend to be $\sim 10\times$ slower

Factor of $\sqrt{m_i/m_e}c/v_T$ faster for each space and time direction

Most benefit in 3D

$$\lambda_D^2 = \frac{T}{4\pi mne^2} \qquad d_e^2 = \frac{m_e c^2}{4\pi mne^2} \qquad d_i^2 = \frac{m_i c^2}{4\pi mne^2}$$

$$\lambda_D = \frac{v_T}{c} d_e = \frac{v_T}{c}\sqrt{\frac{m_e}{m_i}} d_i$$

Time resolution

$$dx/dt > c$$

$$dx/dt > v_T$$

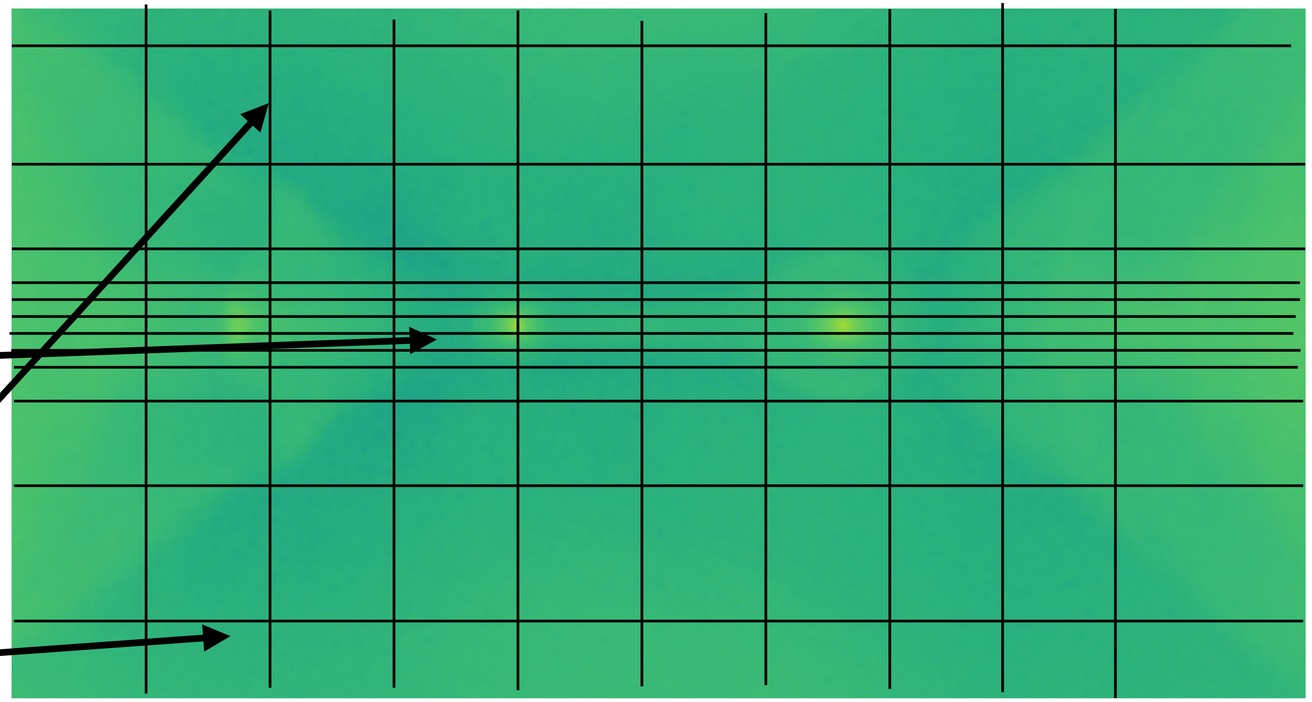
What can we under-resolve

RUHR UNIVERSITÄT BOCHUM RUB

We only need to resolve regions where kinetic effects are important: Particularly useful with non-standard grids (eg. log scale grids)

Kinetic effects important

Under-resolved fluid model suffices



Relativistic implicit schemes

Factor $\gamma_{i,n}$ makes system of equations nonlinear, and more difficult to solve

Factor $\gamma_{i,n}$ also reduces scale separation, for relativistic temperatures and velocities

No benefit in pair plasma

$$\frac{\partial u_i}{\partial t} = \frac{q}{mc}\left(E + \frac{v_i \times B}{c}\right)$$

$$j = \sum_i q_i n_i v_i$$

$$v_i = \frac{u_i}{\gamma_i}$$

$$\frac{\partial B}{\partial t} = -c\nabla \times E$$

$$\frac{\partial E}{\partial t} = c\nabla \times B - 4\pi j$$

Relativistic implicit schemes

RUHR UNIVERSITÄT BOCHUM RUB

Factor $\gamma_{i,n}$ makes system of equations nonlinear, and more difficult to solve

Factor $\gamma_{i,n}$ also reduces scale separation, for relativistic temperatures and velocities

No benefit in pair plasma

Useful in moderately relativistic cases: where electrons are relativistic but ions are not

Relativistic implicit schemes

RUHR UNIVERSITÄT BOCHUM RUB

Factor $\gamma_{i,n}$ makes system of equations nonlinear, and more difficult to solve

Factor $\gamma_{i,n}$ also reduces scale separation, for relativistic temperatures and velocities

No benefit in pair plasma

Useful in moderately relativistic cases: where electrons are relativistic but ions are not

More radiative cooling for electrons may help retain a scale separation

RelSIM

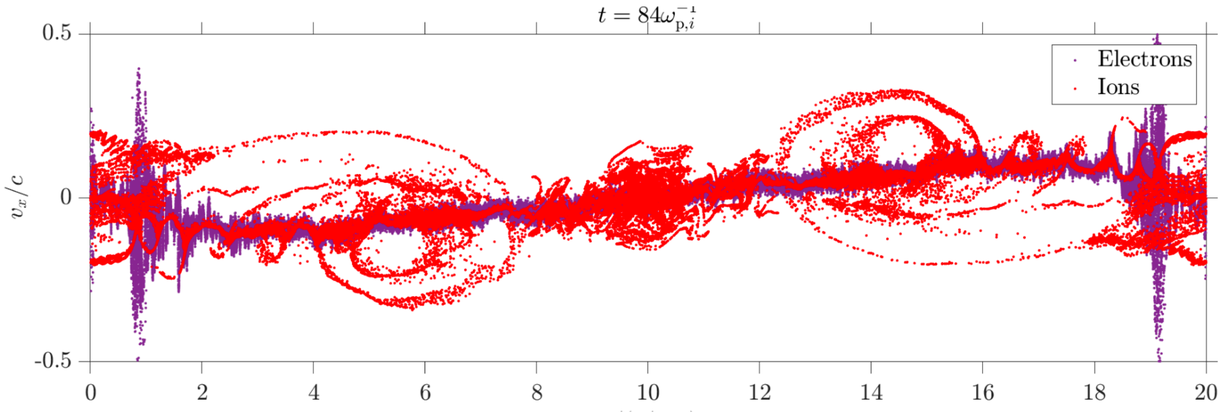
RUHR UNIVERSITÄT BOCHUM RUB

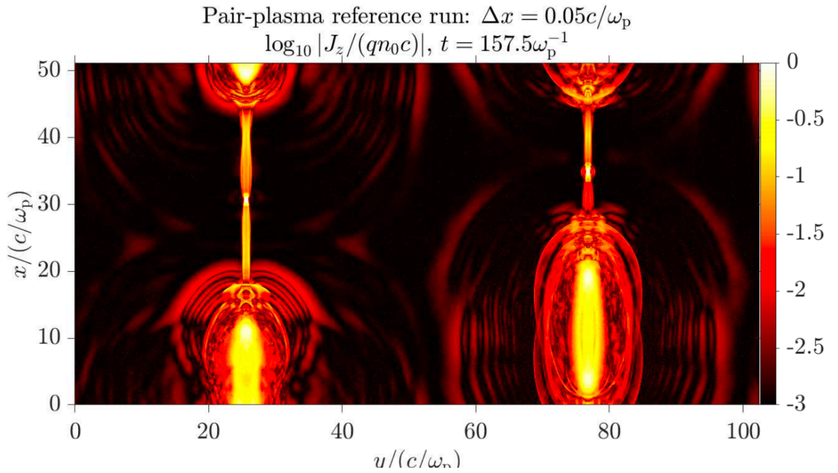
Relativistic Semi-Implicit code RelSIM

Bacchini 2023 [https://doi.org/10.48550/arXiv.2306.04685]

RelSIM is EcSIM (Lapenta 2017): modified such that it can do simulations of relativistic plasmas.

Tested two-stream

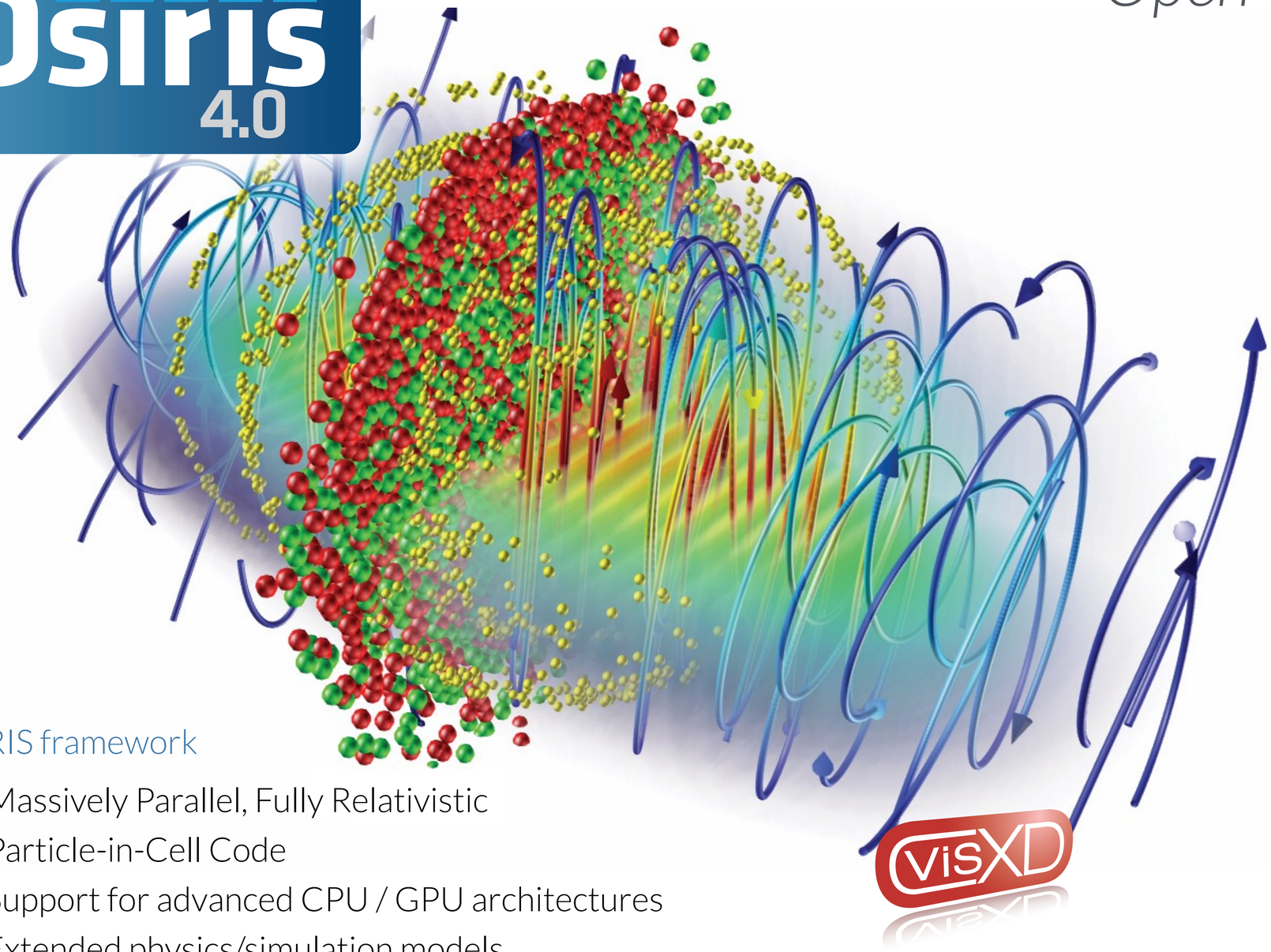




Tested relativistic pair reconnection

Osiris 4.0

Open-source version available



OSIRIS framework

- Massively Parallel, Fully Relativistic Particle-in-Cell Code
- Support for advanced CPU / GPU architectures
- Extended physics/simulation models
- AI/ML surrogate models and data-driven discovery

Open-access model

- 40+ research groups worldwide are using OSIRIS
- 400+ publications in leading scientific journals
- Large developer and user community
- Detailed documentation and sample inputs files available
- Support for education and training

Using OSIRIS 4.0

- The code can be used freely by research institutions after signing an MoU
- Open-source version at:

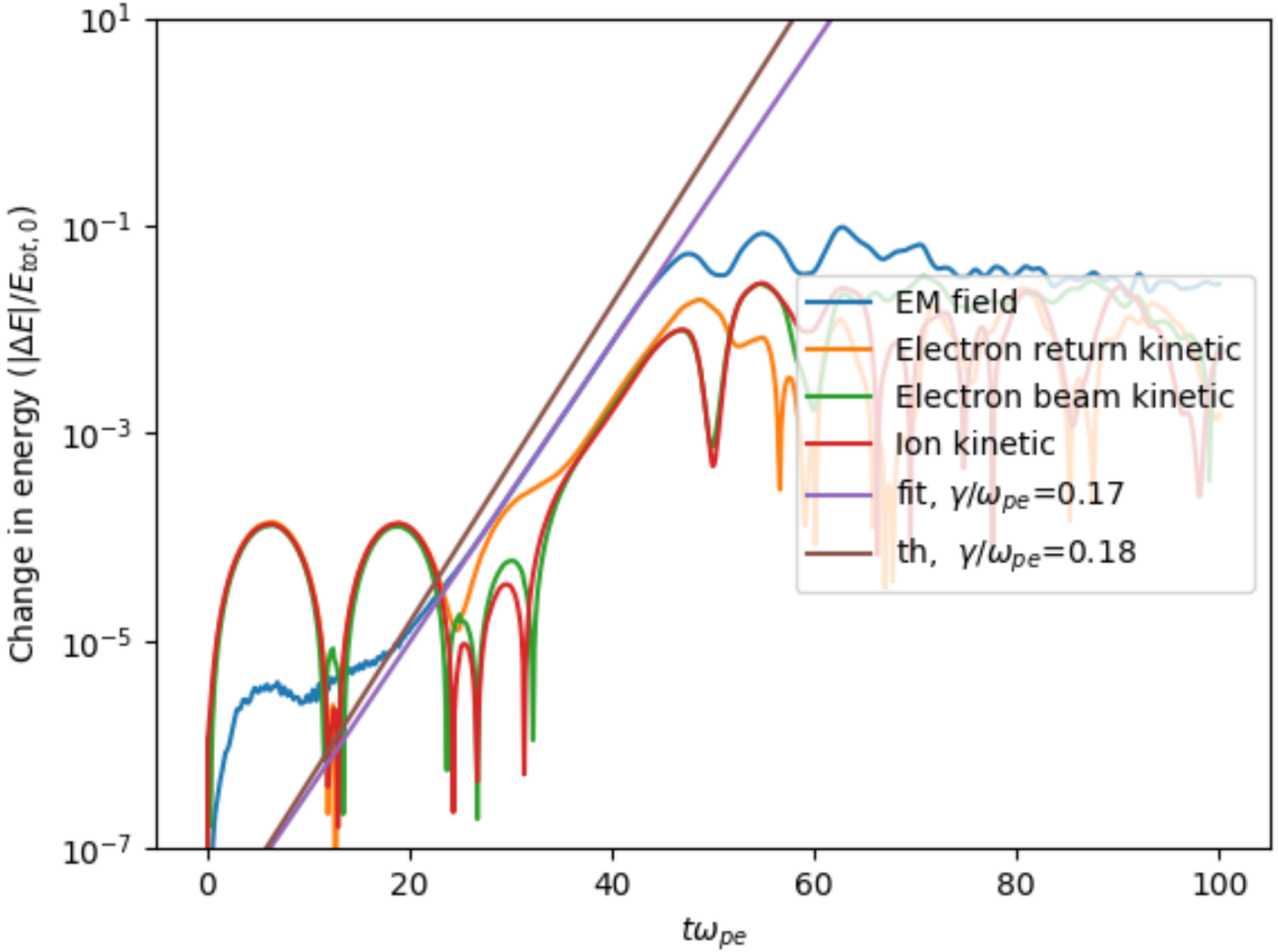
https://osiris-code.github.io/

UNIVERSITY OF MICHIGAN · UCLA · TÉCNICO LISBOA

Ricardo Fonseca: ricardo.fonseca@tecnico.ulisboa.pt

Explicit method recovers Relativistic two-stream growth rate

RUHR UNIVERSITÄT BOCHUM RUB

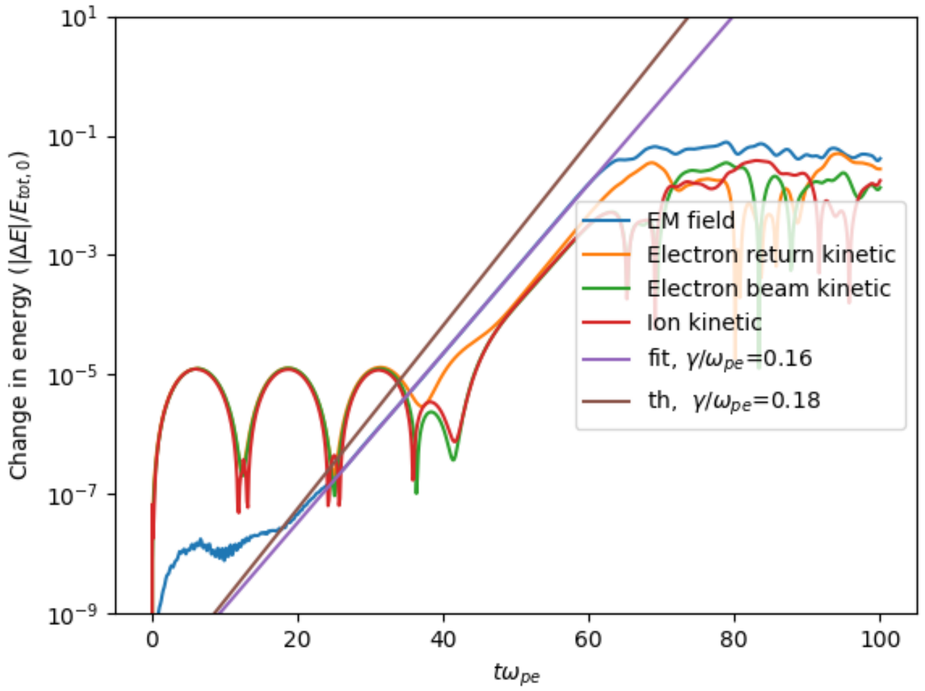


with OSIRIS

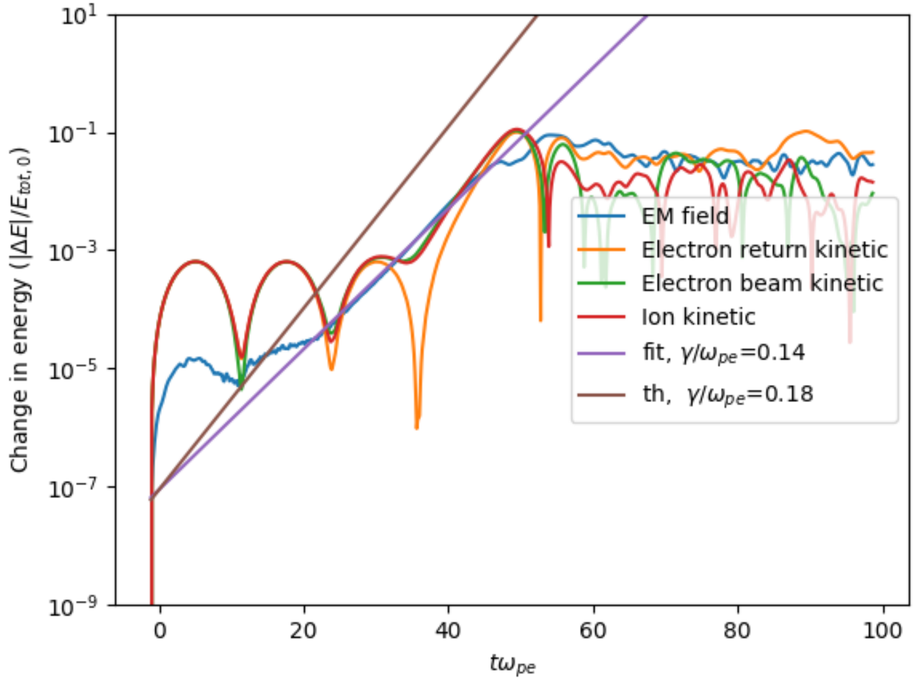
Both Explicit and implicit methods recover theoretical rate

RUHR UNIVERSITÄT BOCHUM RUB

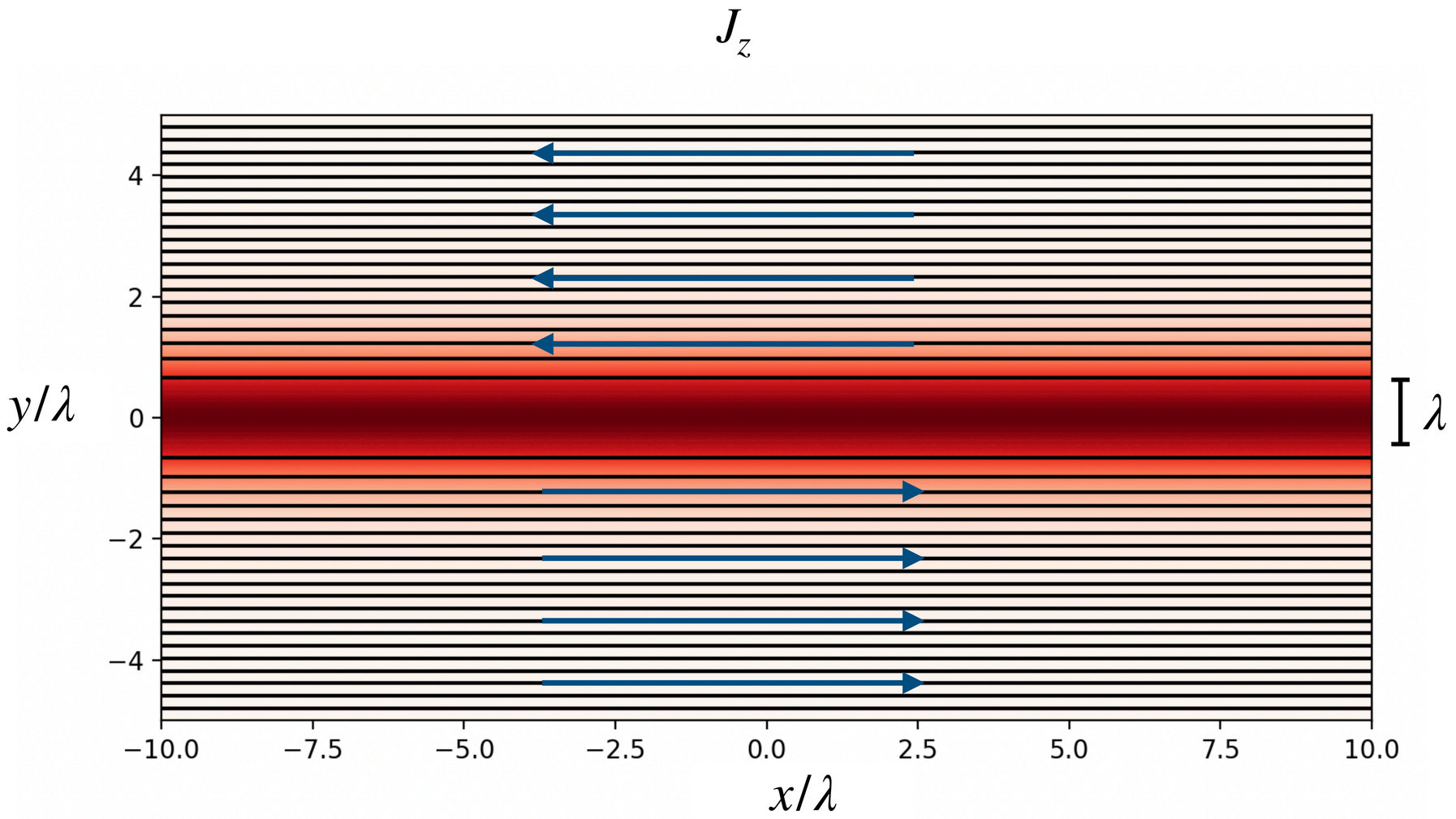
OSIRIS



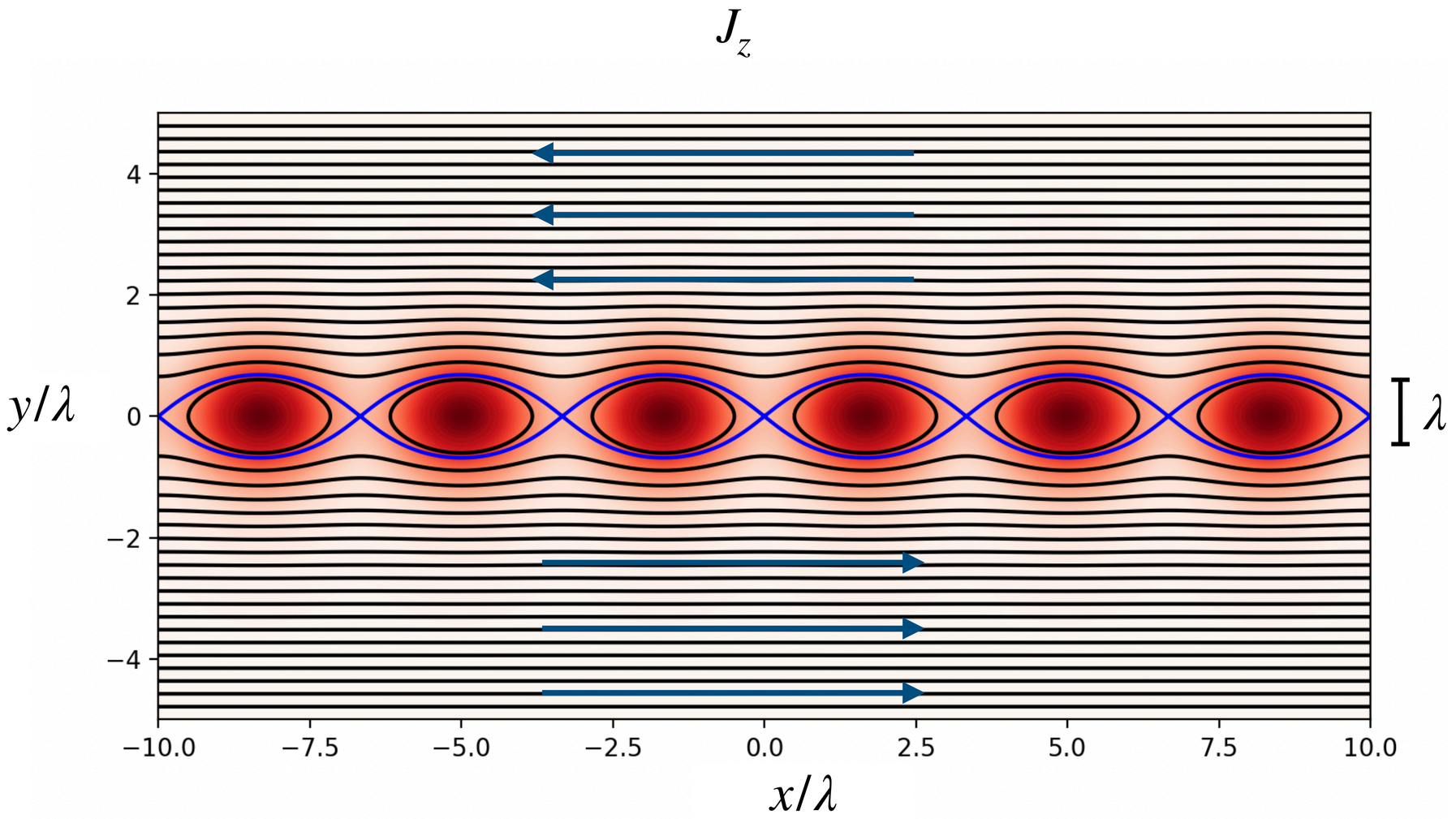
RelSIM



Tearing mode



Tearing mode

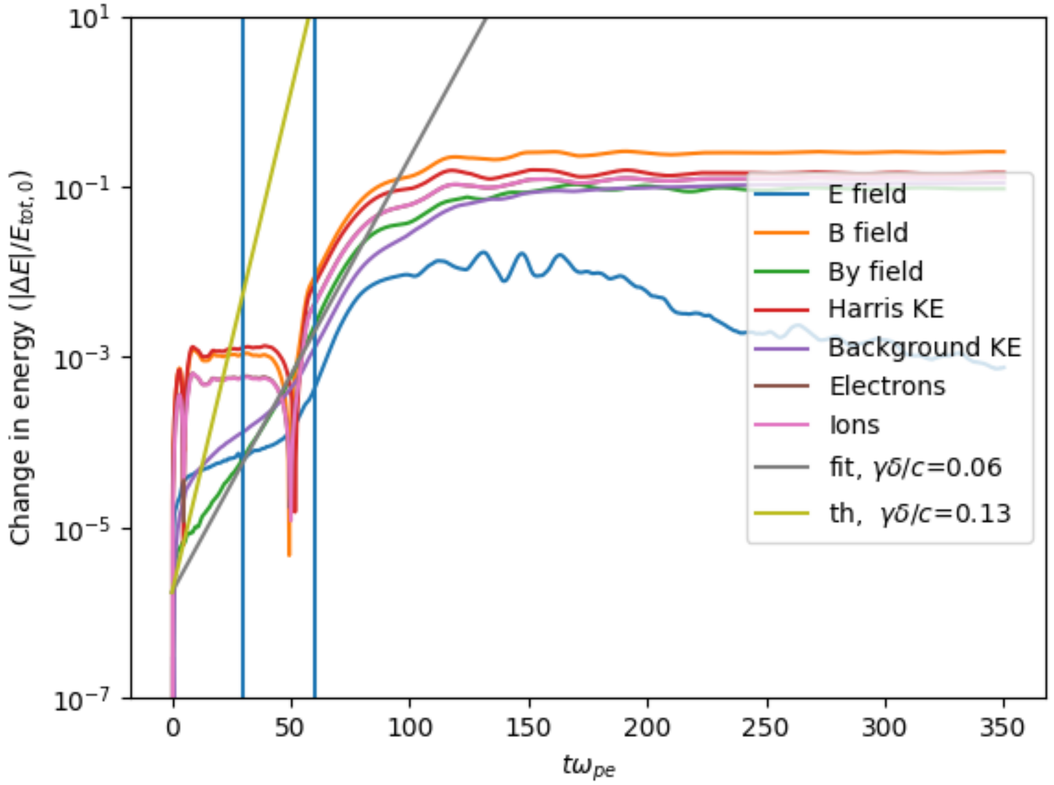


Relativistic tearing electron-positron (explicit simulation)

RUHR UNIVERSITÄT BOCHUM RUB

OSIRIS

$$\frac{L_x}{\lambda} = 10 \qquad \frac{L_y}{\lambda} = 20$$



Theory From Zelenyi 1979

$$\frac{T_{e,H}}{m_e c^2} = 1 \qquad \frac{u_d}{c} = 0.816$$

$$\frac{\lambda}{d_e} = 1.0$$

$$\frac{T_{e,b}}{m_e c^2} = 0.01 \qquad \frac{n_0}{n_b} = 6$$

$$\sigma_c = 10$$

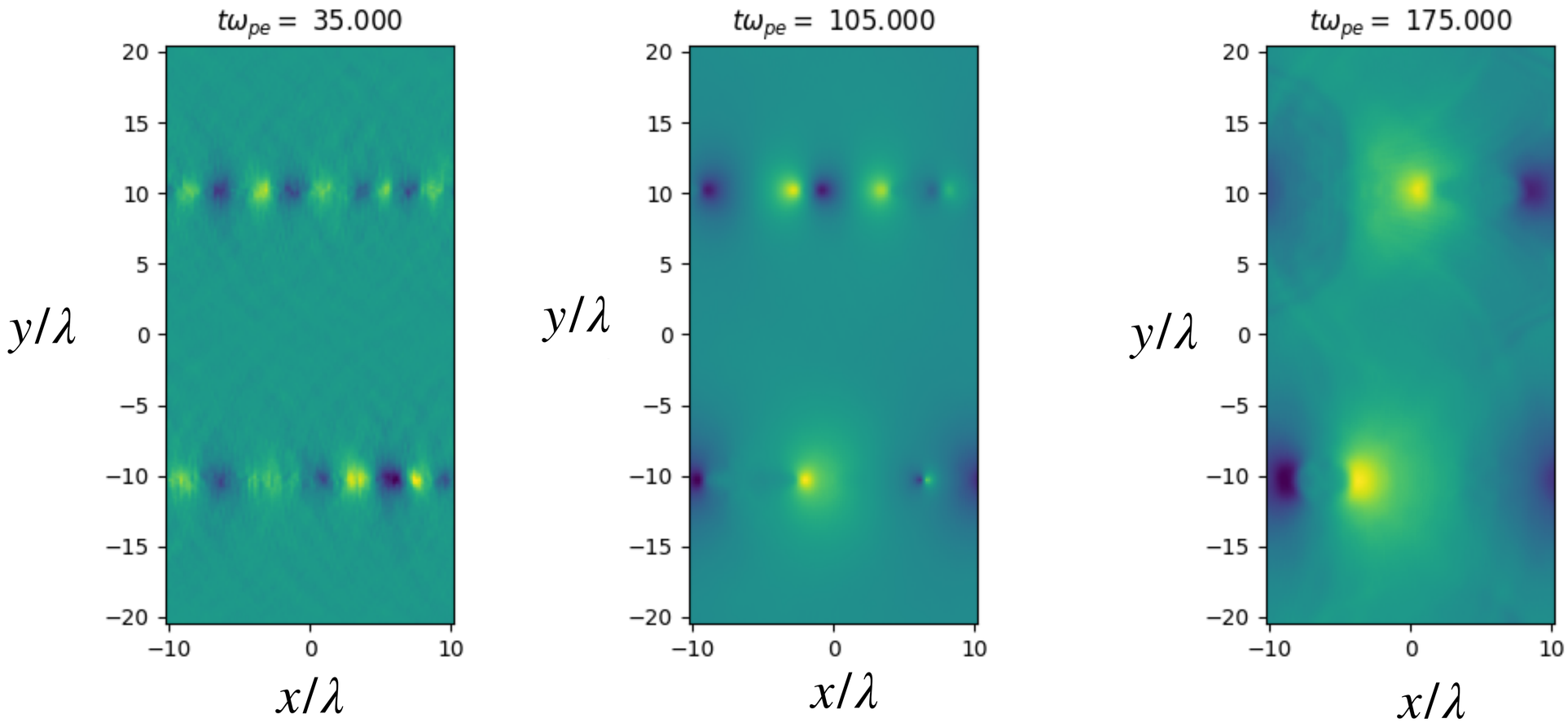
We can essentially ignore background

Relativistic tearing electron-positron (explicit simulation)

RUHR UNIVERSITÄT BOCHUM RUB

OSIRIS

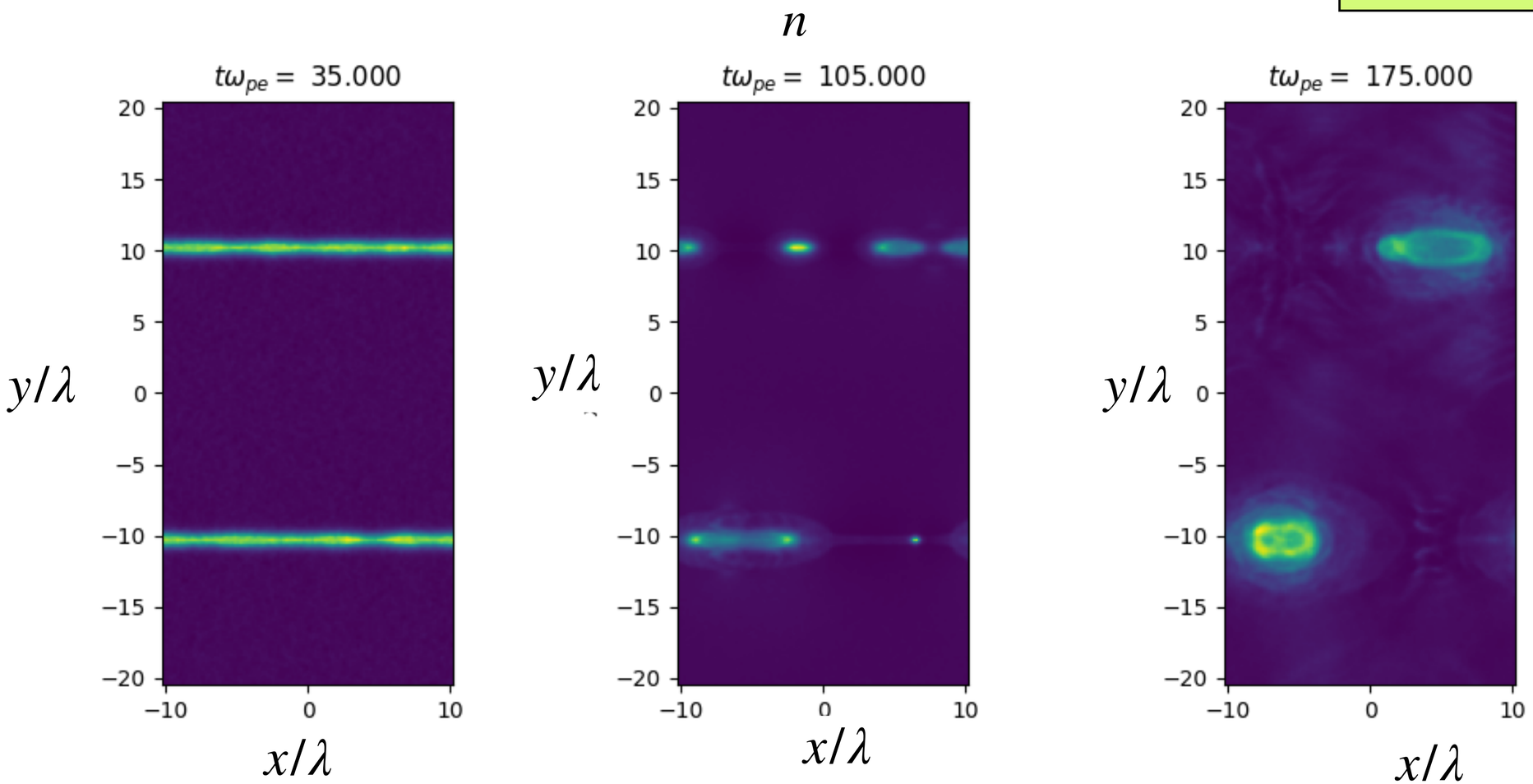
B_y



Relativistic tearing electron-positron (explicit simulation)

RUHR UNIVERSITÄT BOCHUM RUB

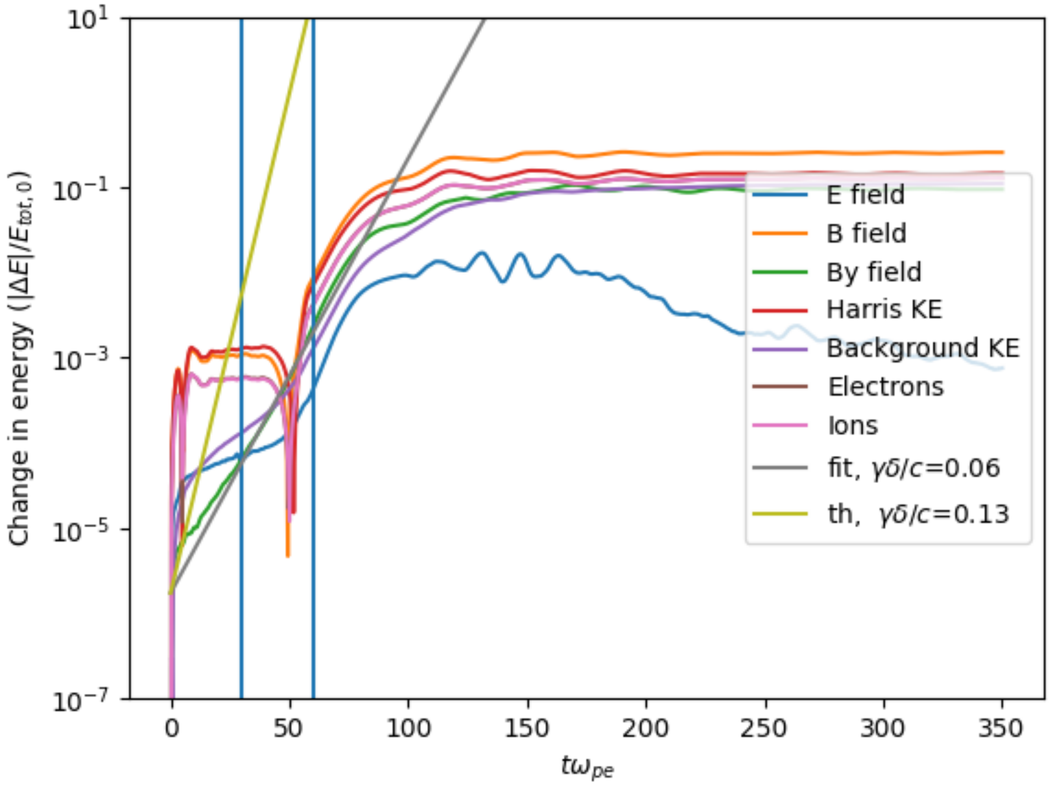
OSIRIS



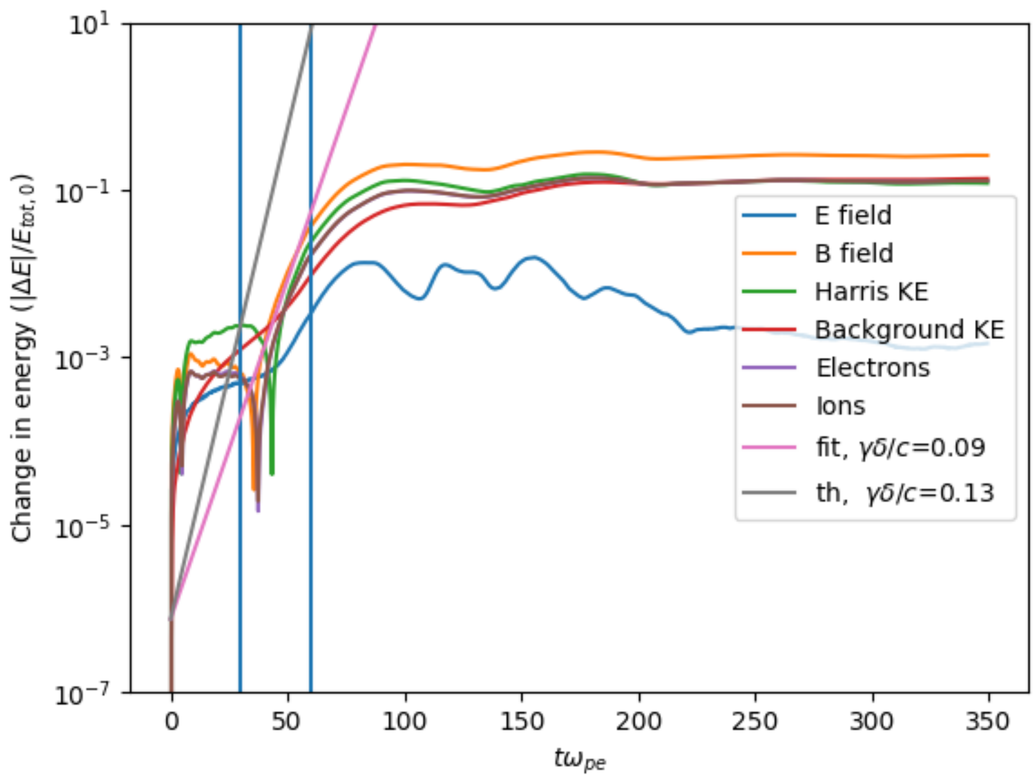
Implicit method recovers explicit result

RUHR UNIVERSITÄT BOCHUM RUB

OSIRIS



RelSIM

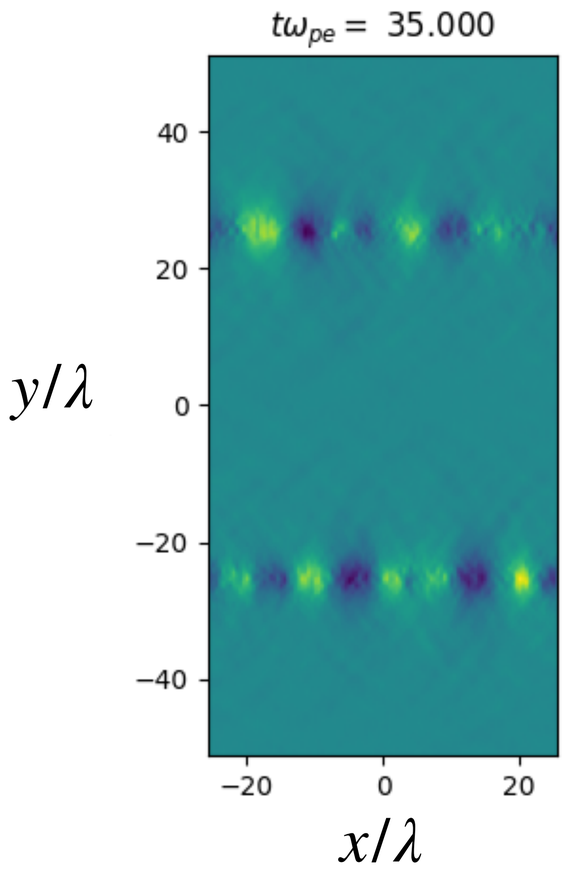


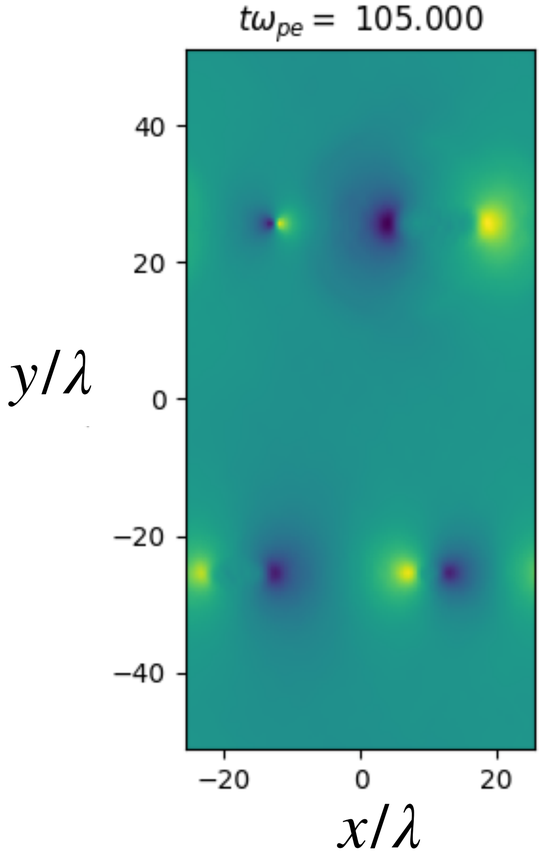
Implicit method recovers explicit result

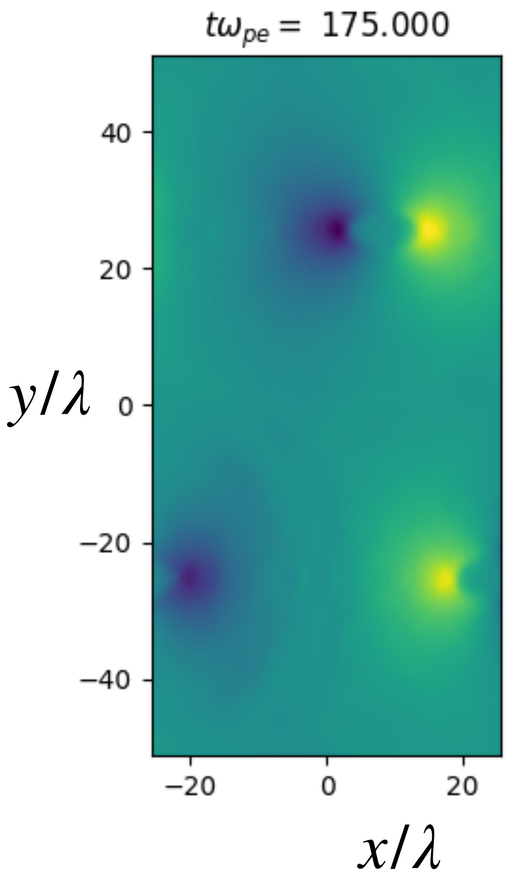
RUHR UNIVERSITÄT BOCHUM RUB

RelSIM

B_y



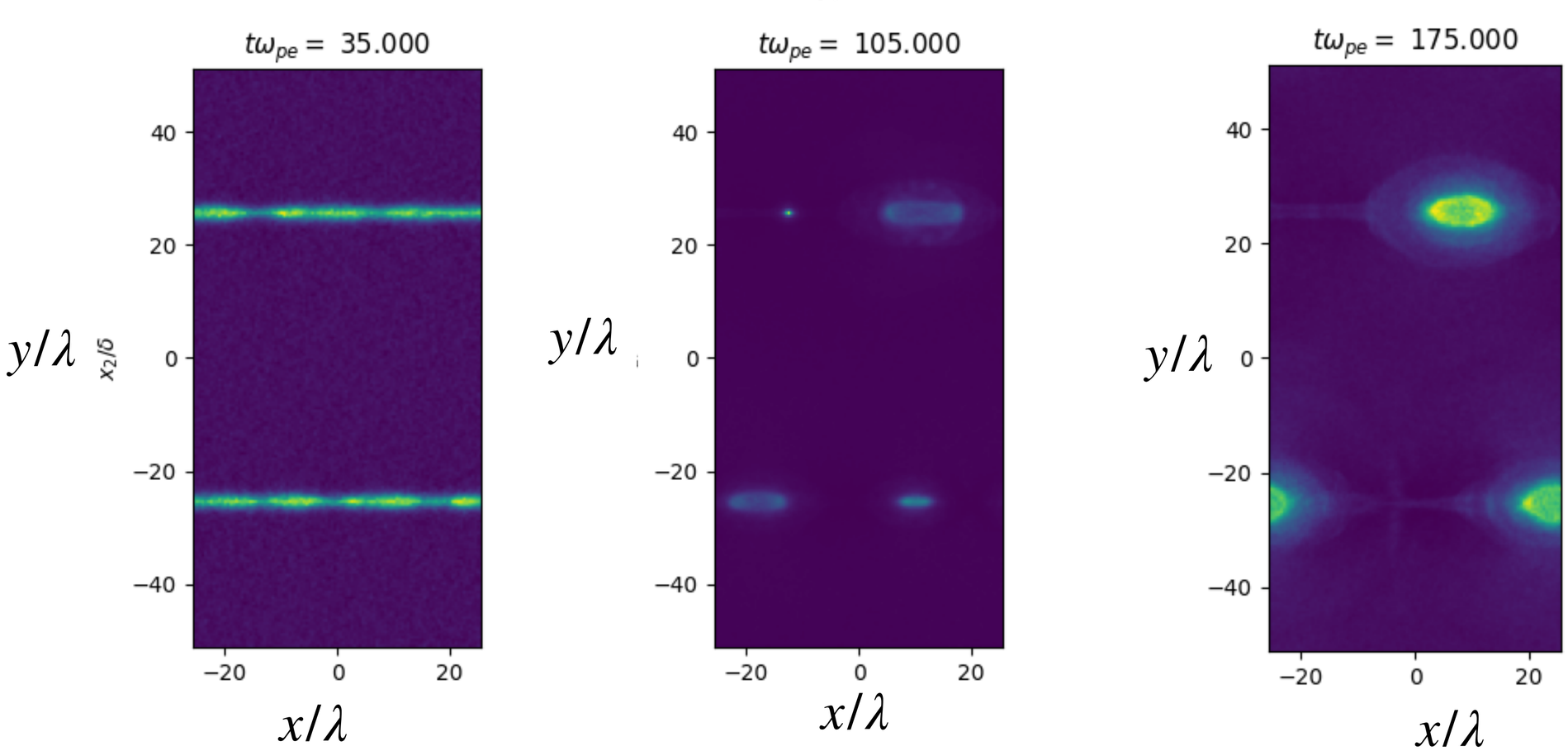




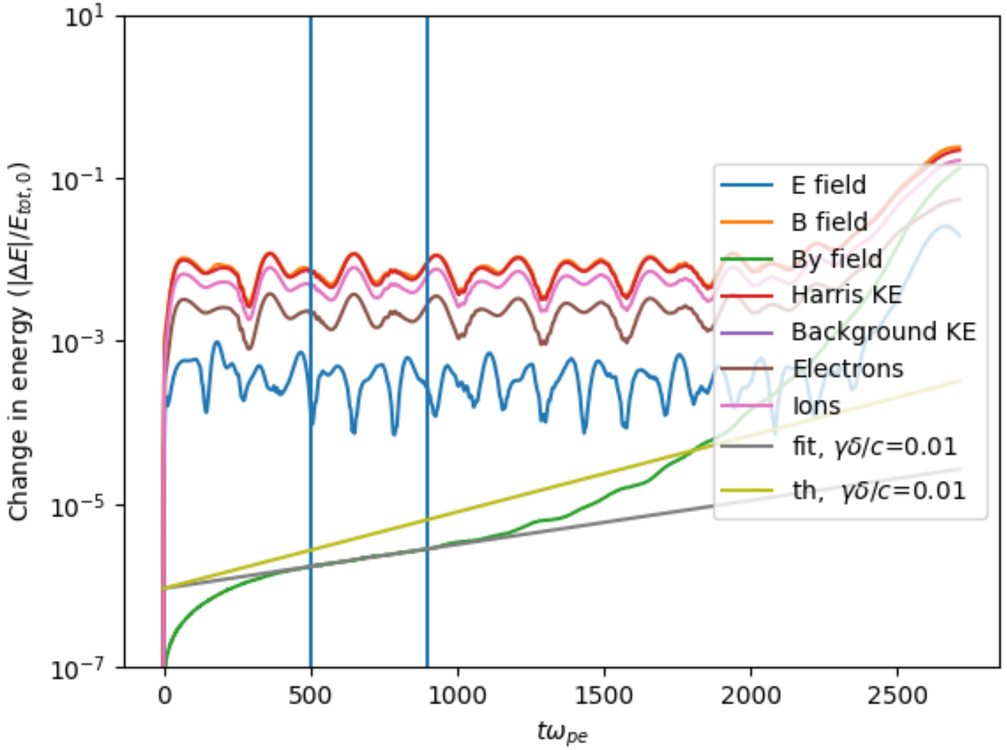
Implicit method recovers explicit result

RUHR UNIVERSITÄT BOCHUM RUB

RelSIM



Relativistic tearing electron-ion (explicit simulation)



Theory From Zelenyi 1979

$$\frac{L_x}{\lambda} = 25$$

$$\frac{T_{e,H}}{m_e c^2} = 1 \qquad \frac{u_d}{c} = 0.1 \qquad \frac{m_i}{m_e} = 25$$

$$\frac{\lambda}{d_e} = 10 \qquad \frac{\lambda}{d_i} = 1 \qquad \frac{T_i}{T_e} = 1$$

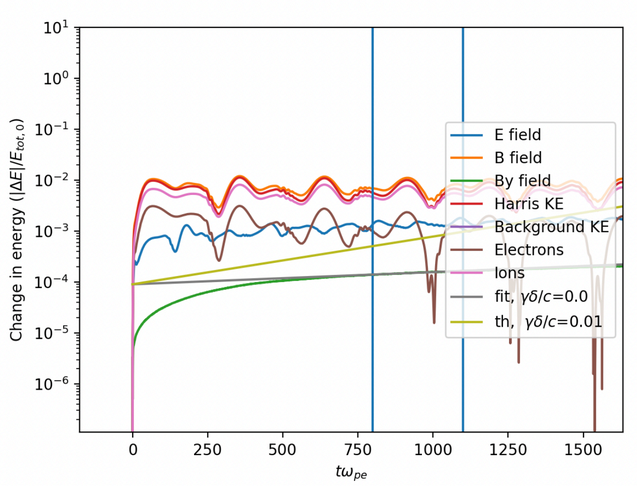
No background

$ppg = 4096$ $\lambda/dx = 16$ $\lambda_D/dx = 1.6$

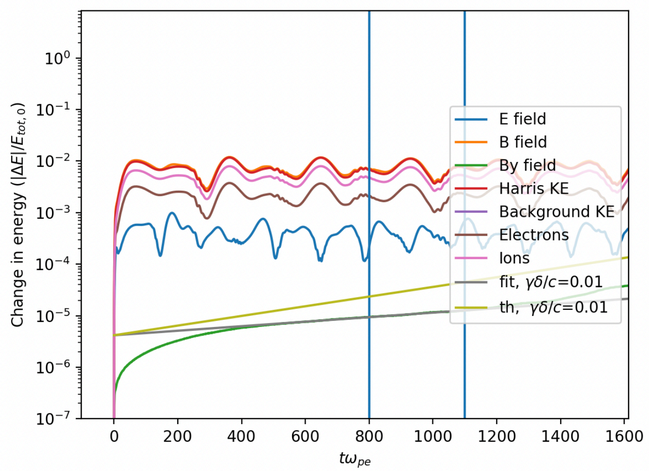
Relativistic tearing electron-ion (ppg dependence)

RUHR UNIVERSITÄT BOCHUM RUB

$ppg = 64$

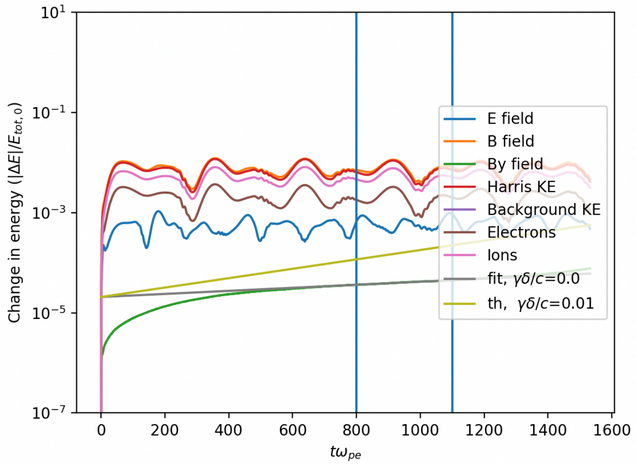


$ppg = 1024$

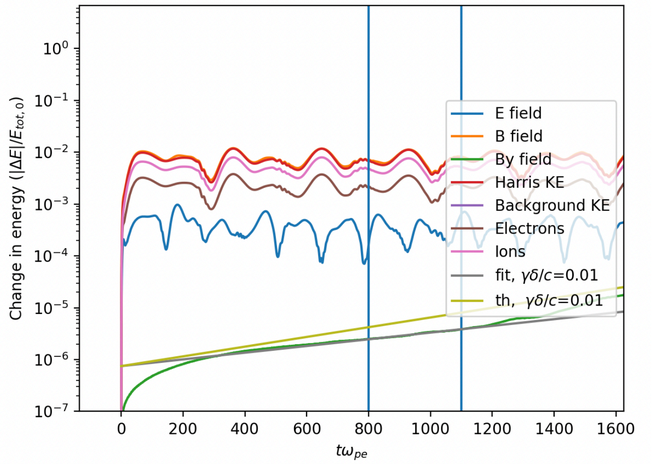


Growthrate depends on the number of particles per grid cell

$ppg = 256$



$ppg = 4096$



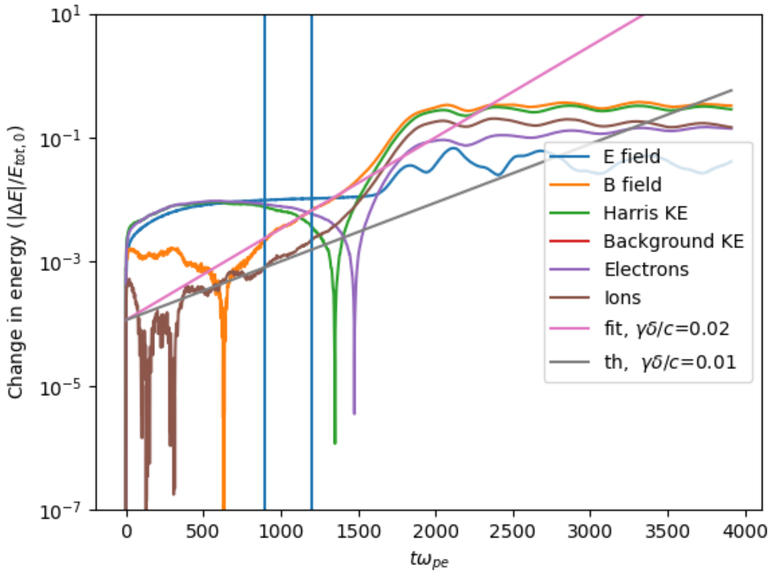
Non-linear stage becomes earlier

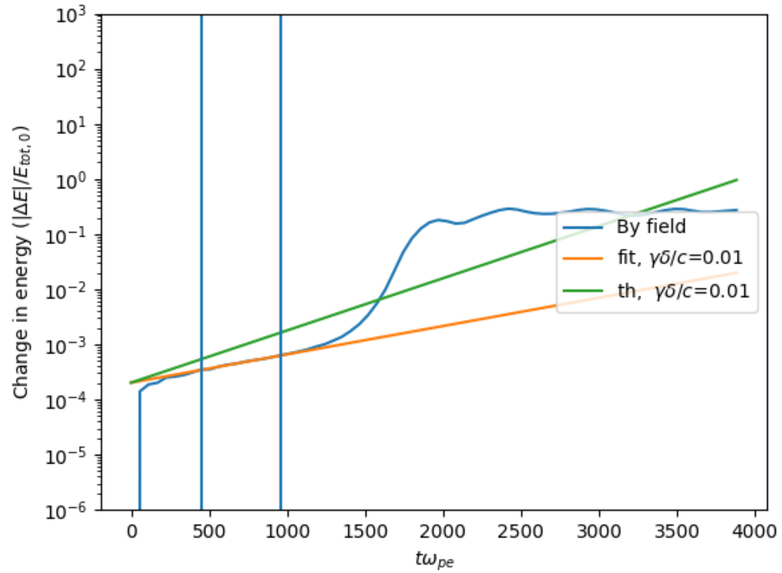
Because of numerical heating?

Implicit method recovers explicit result (for cheaper)

$\lambda/dx = 4$

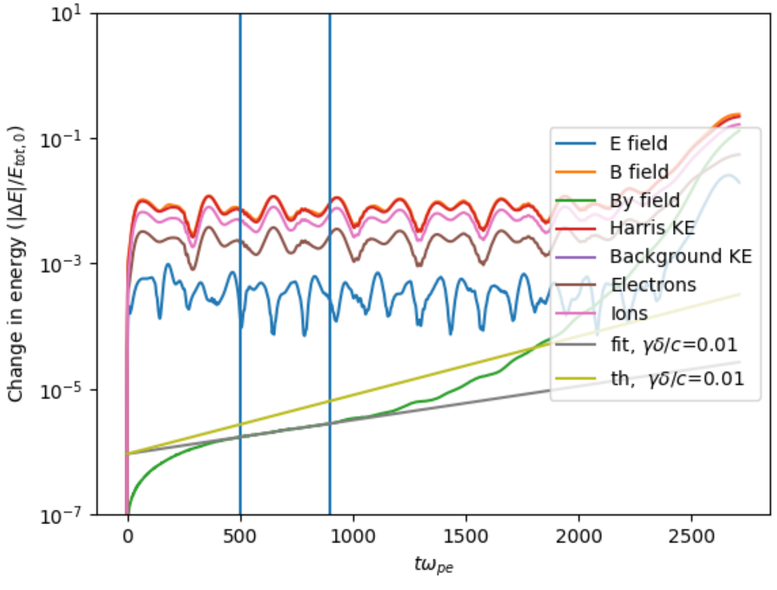
$ppg = 16$

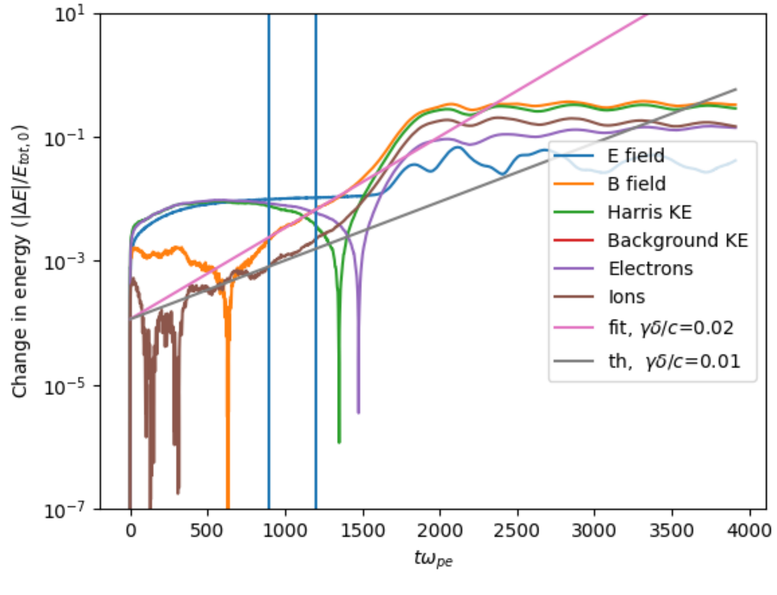


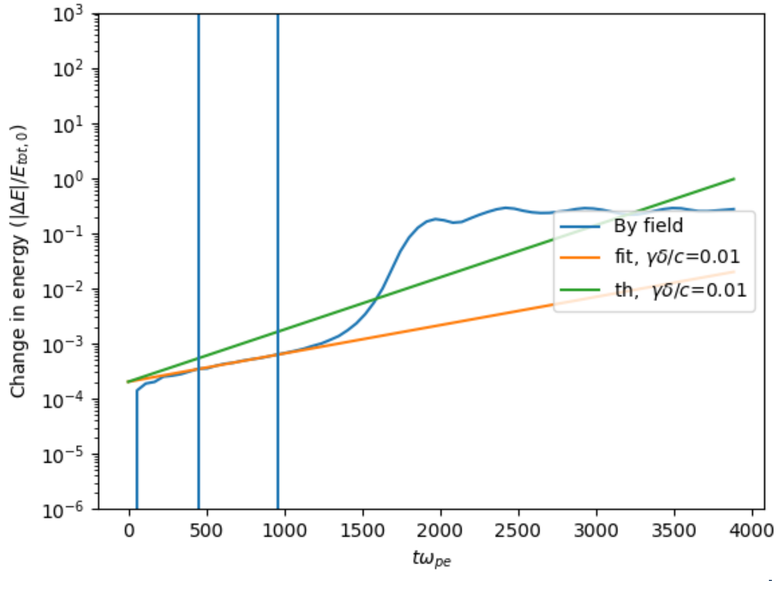


Good agreement with low resolution

Implicit method recovers explicit result (for cheaper)







OSIRIS

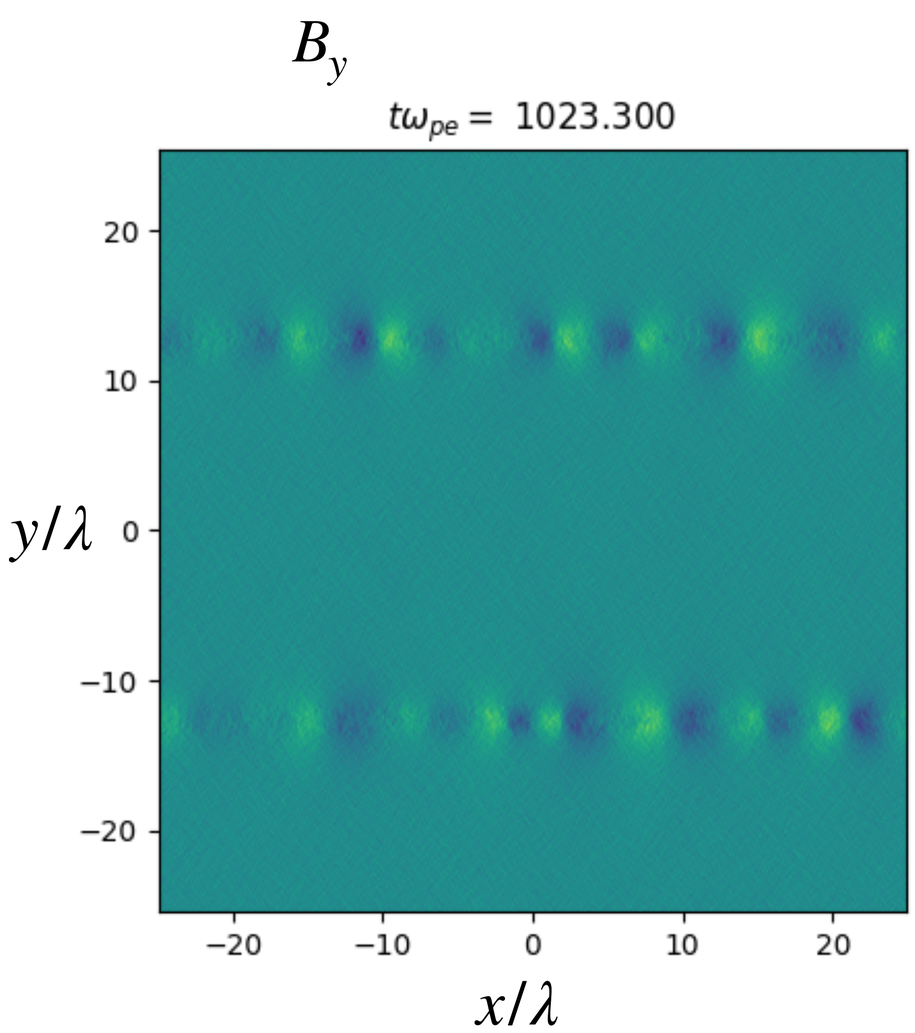
$\lambda/dx = 16$ $ppg = 4096$

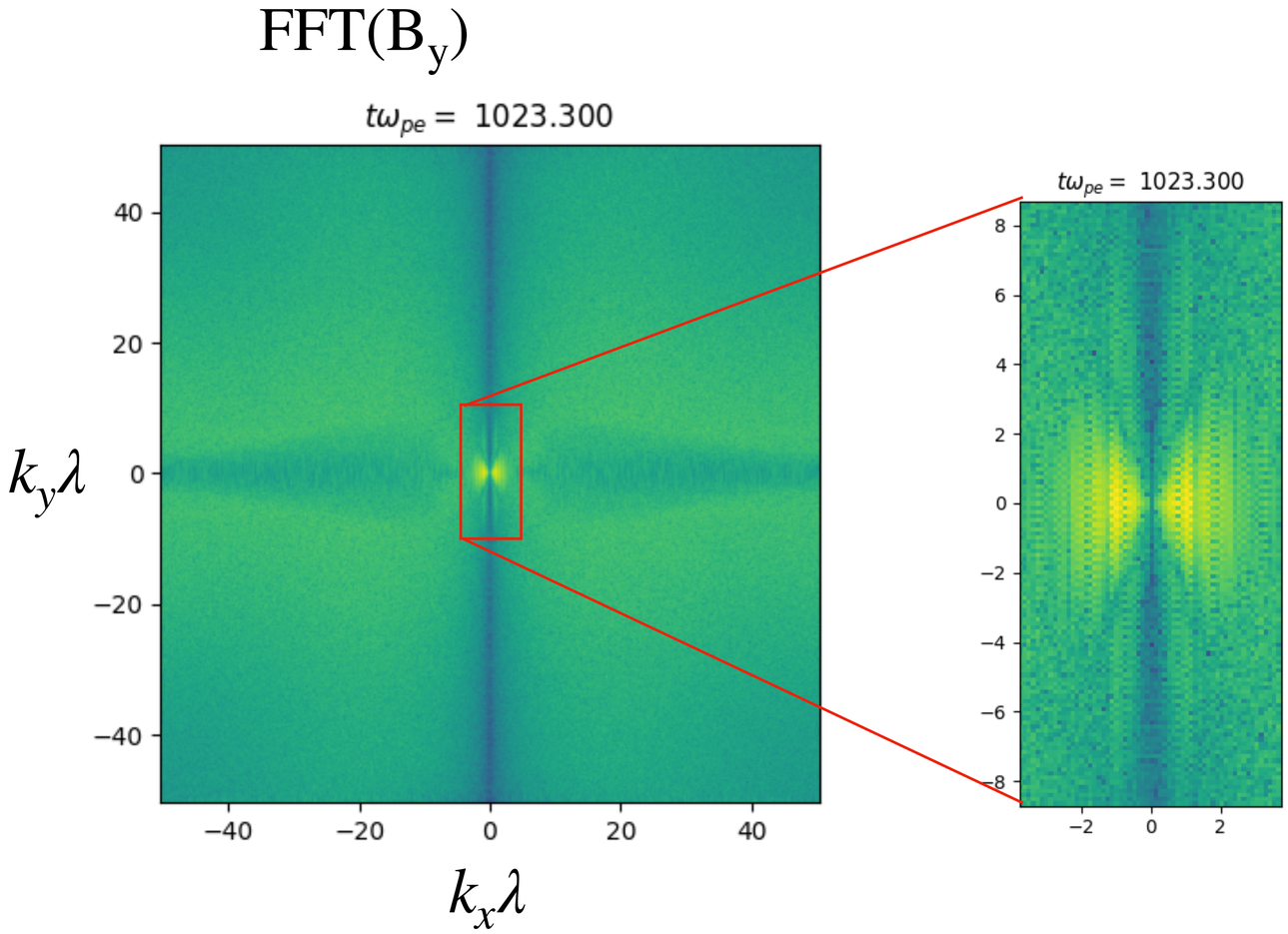
RelSIM

$\lambda/dx = 4$ $ppg = 16$

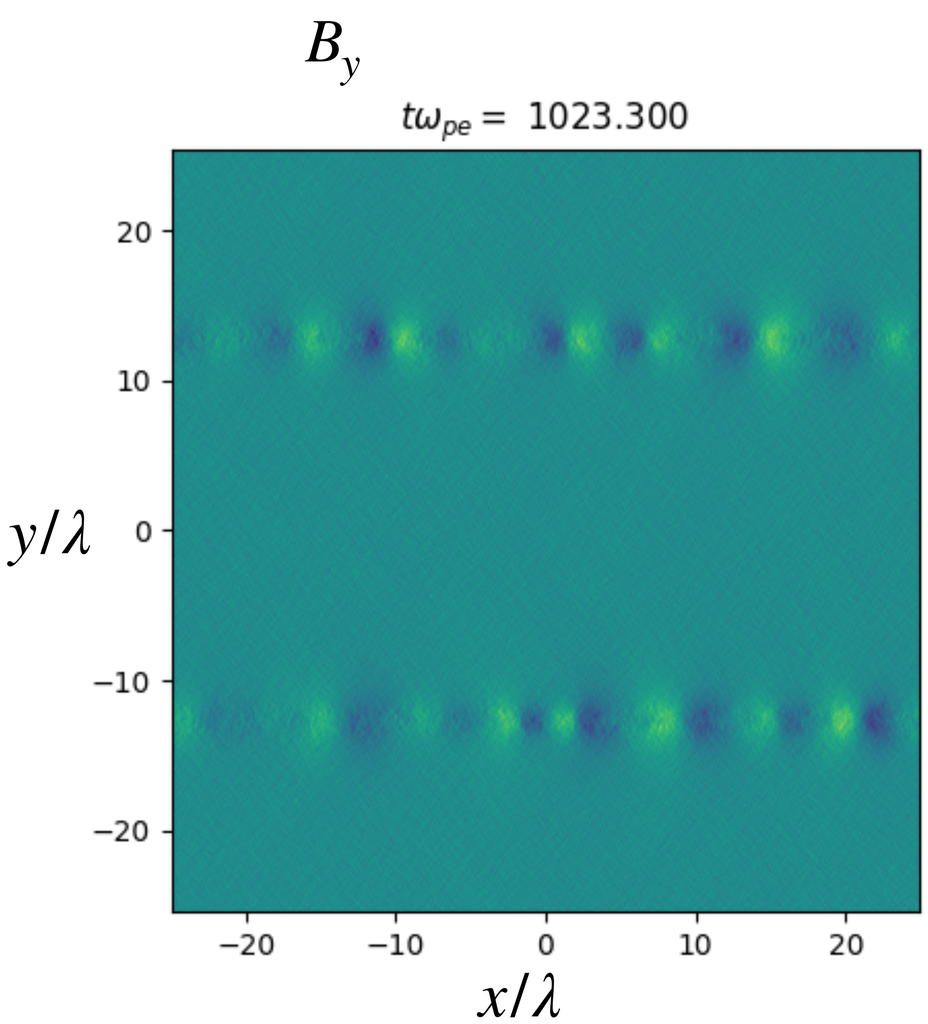
Good agreement with low resolution

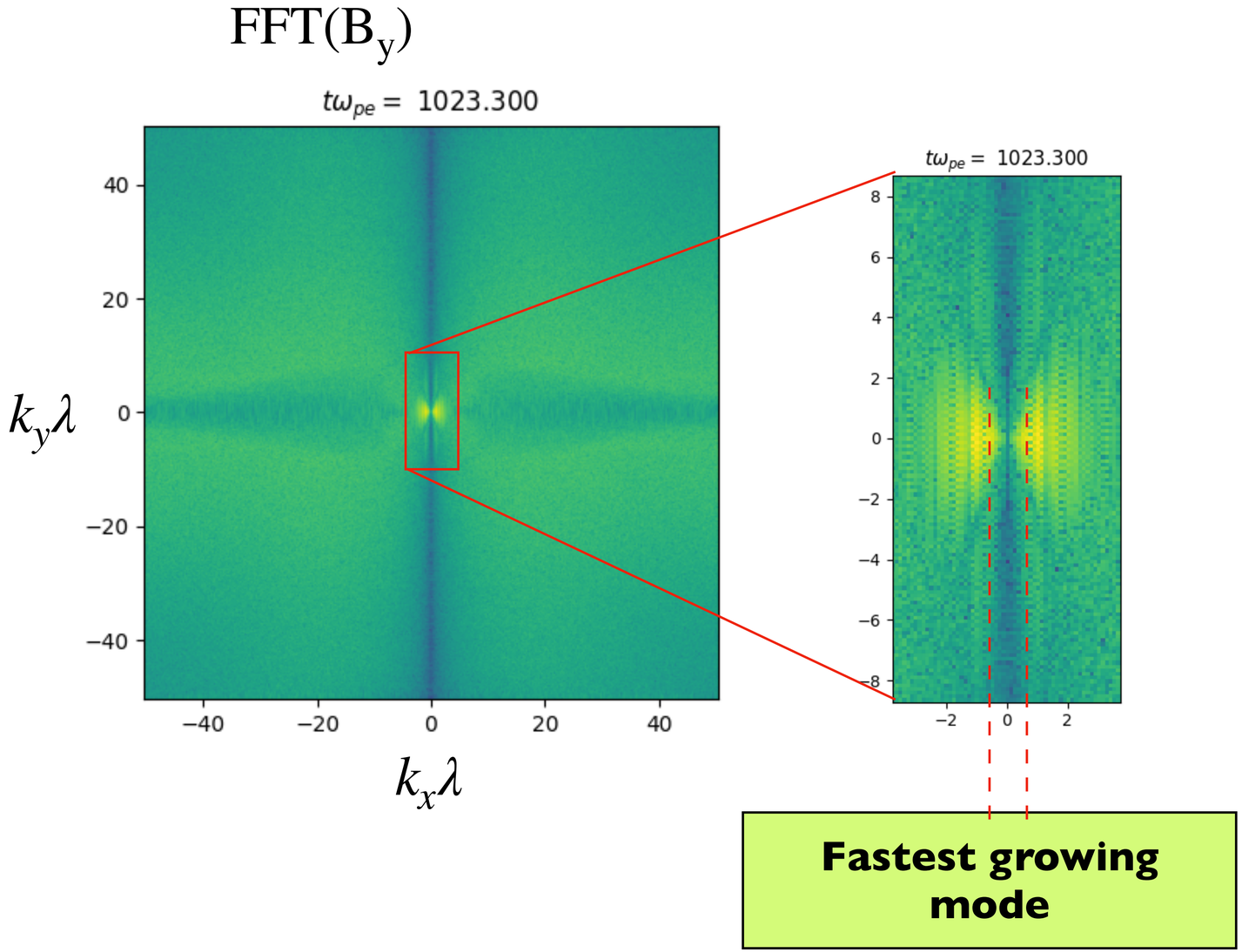
Early tearing (explicit high resolution)





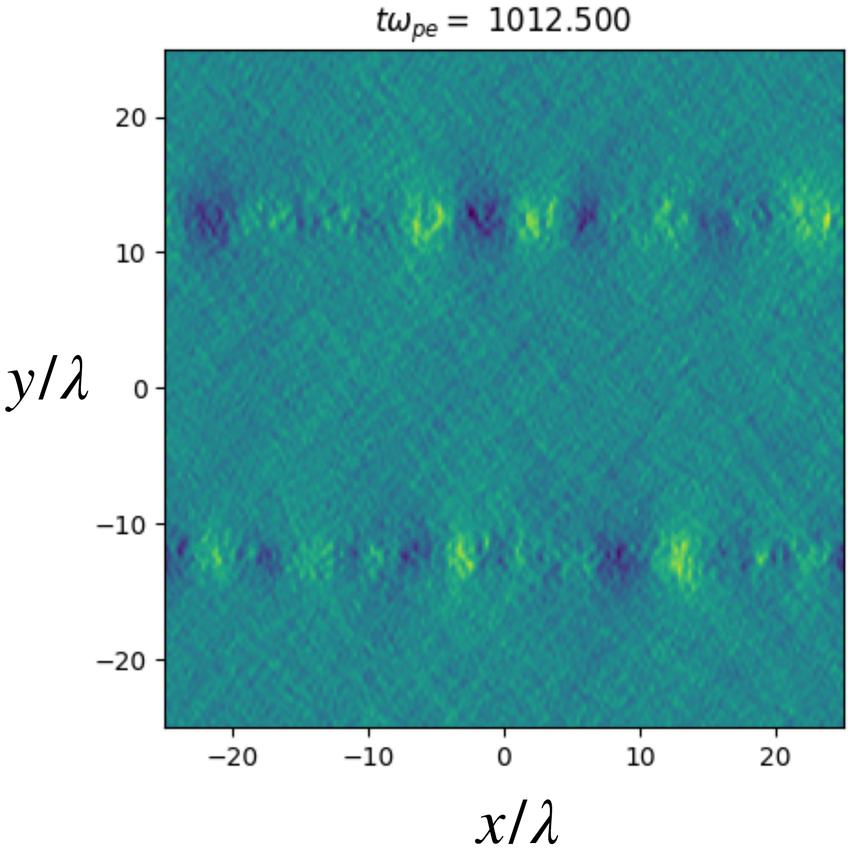
Early tearing (explicit high resolution)



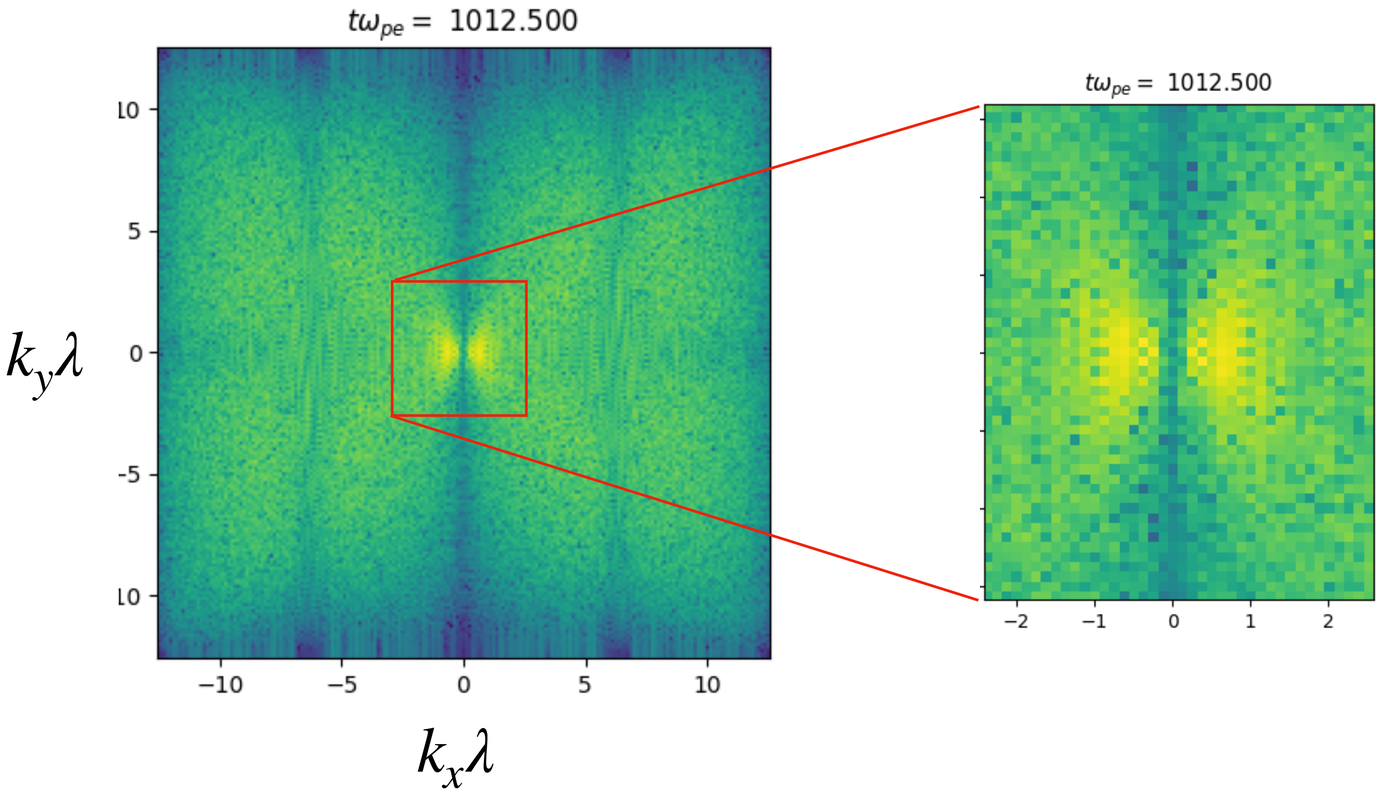


Early tearing (semi-implicit low resolution)

B_y

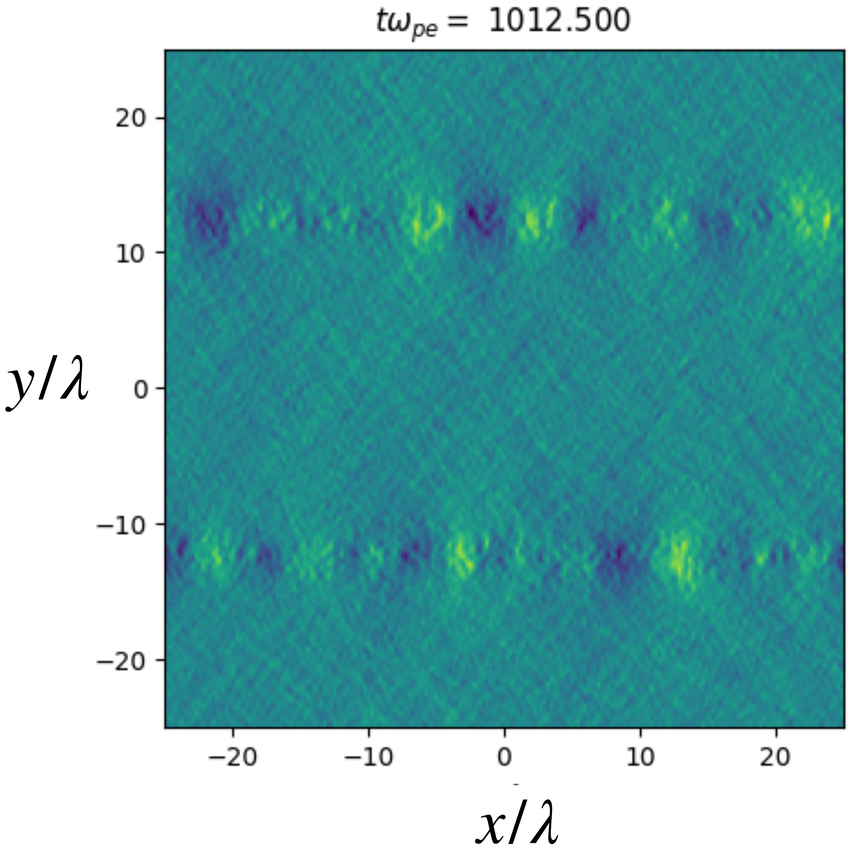


FFT(B_y)

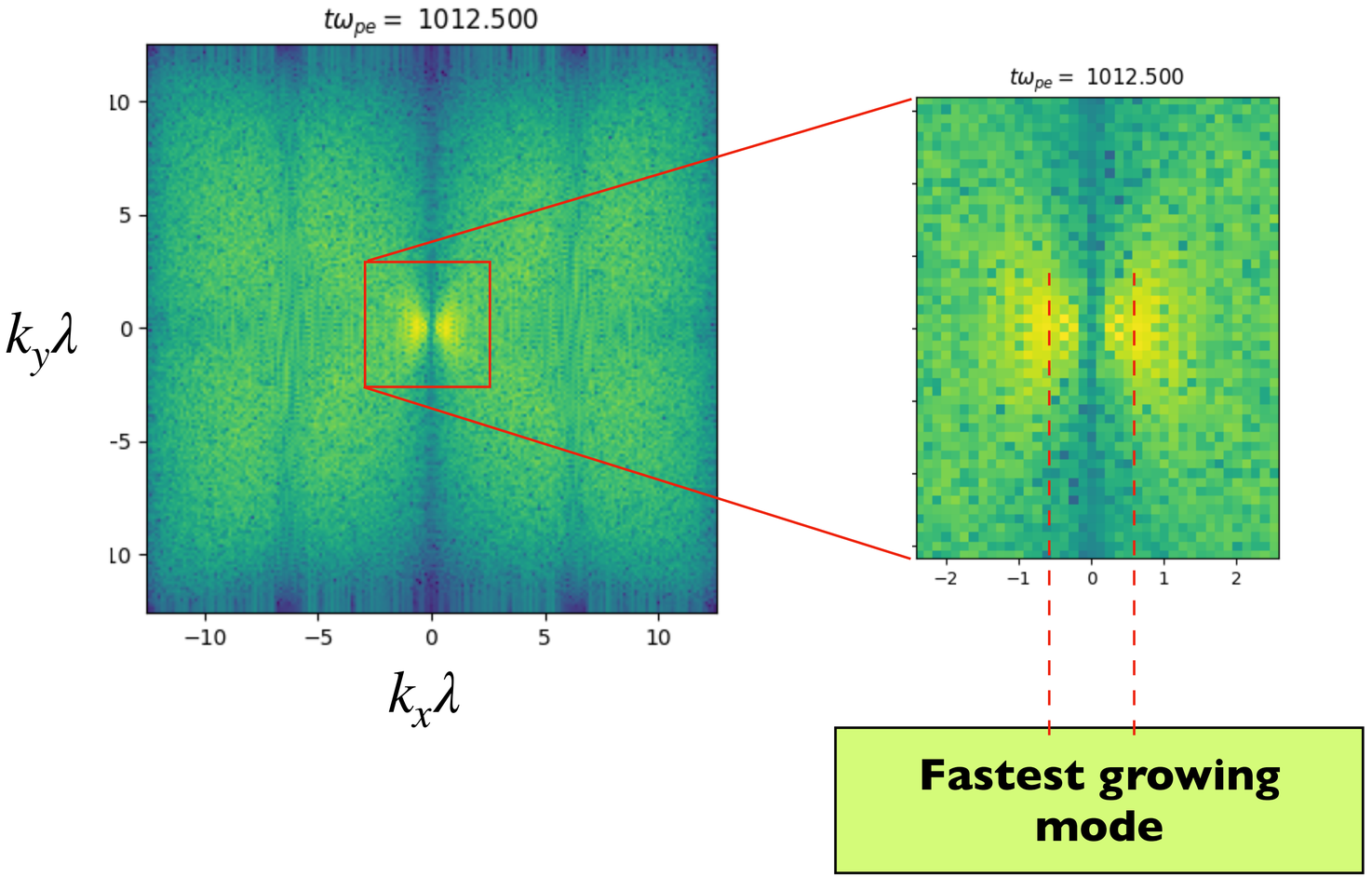


Early tearing (semi-implicit low resolution)

B_y

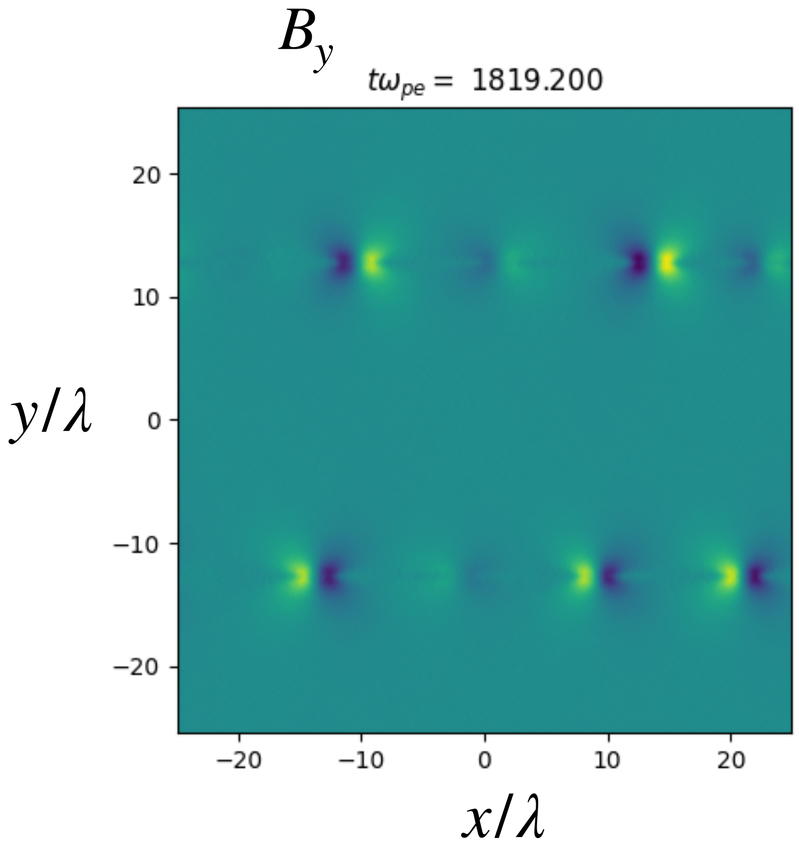


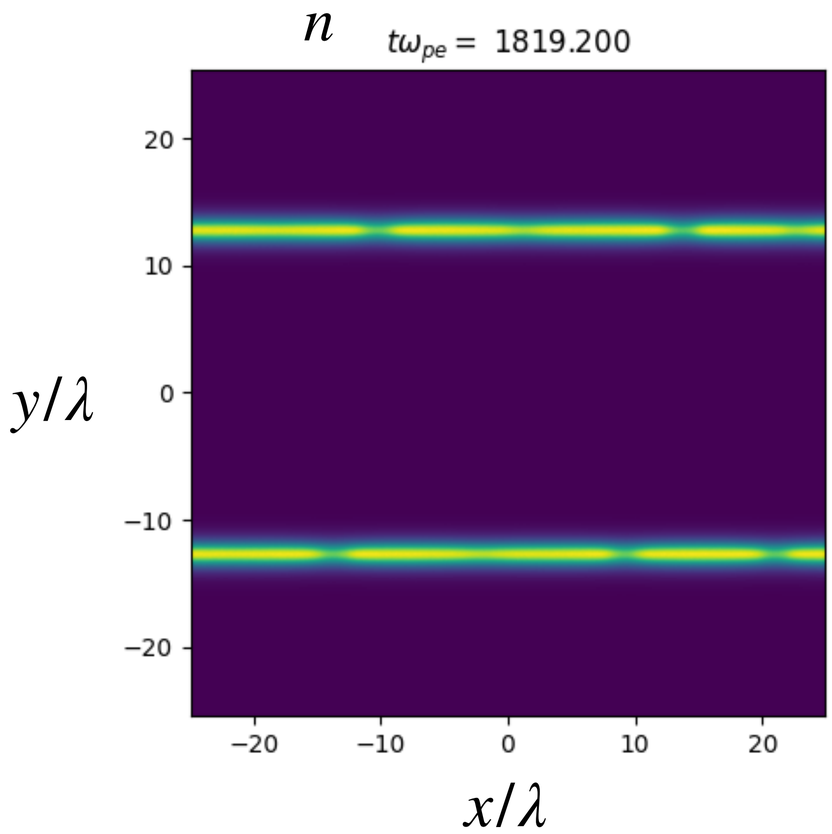
FFT(B_y)



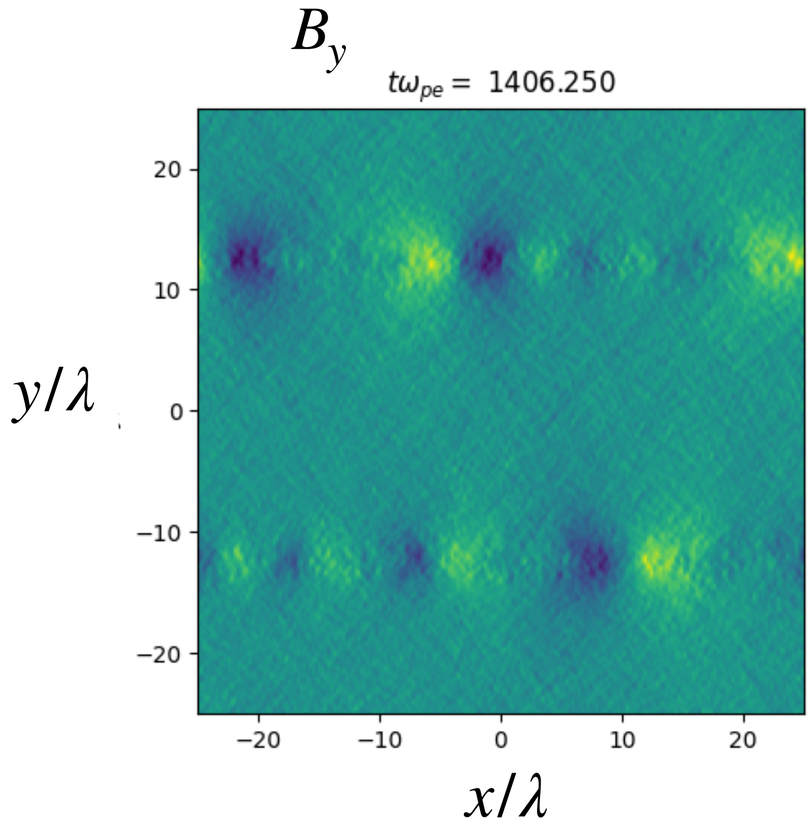
Fastest growing mode

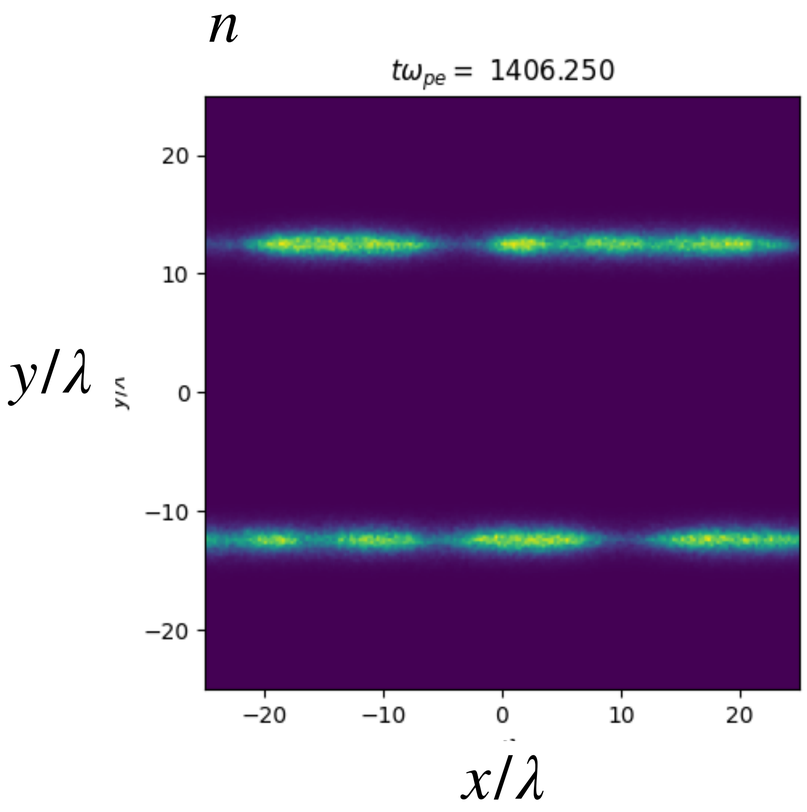
Nonlinear stage (explicit high resolution)



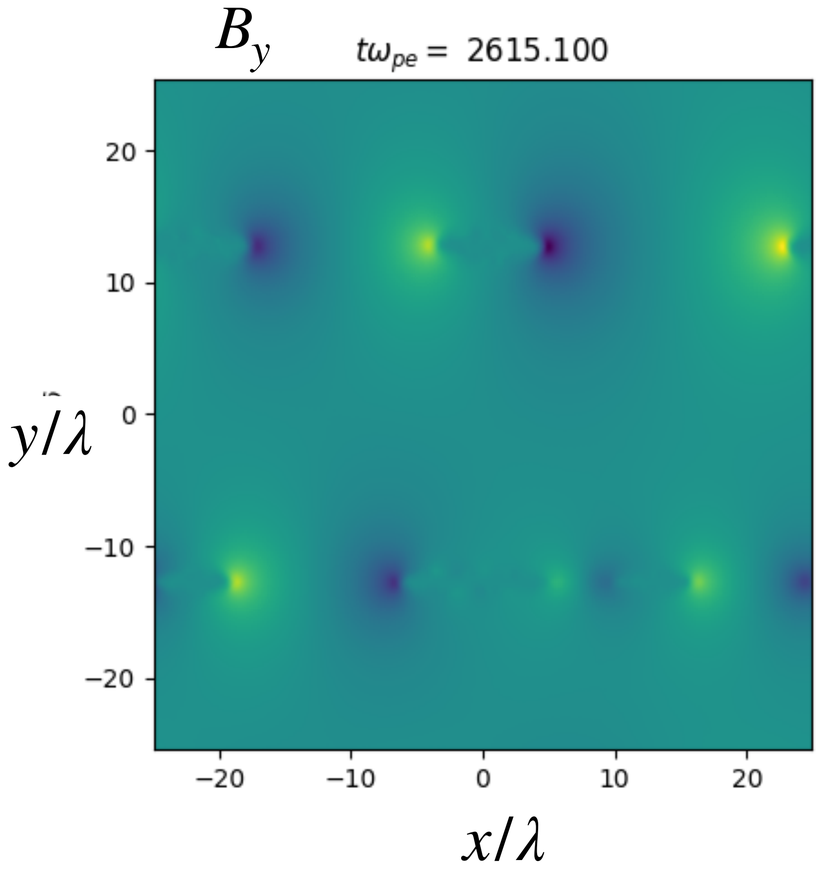


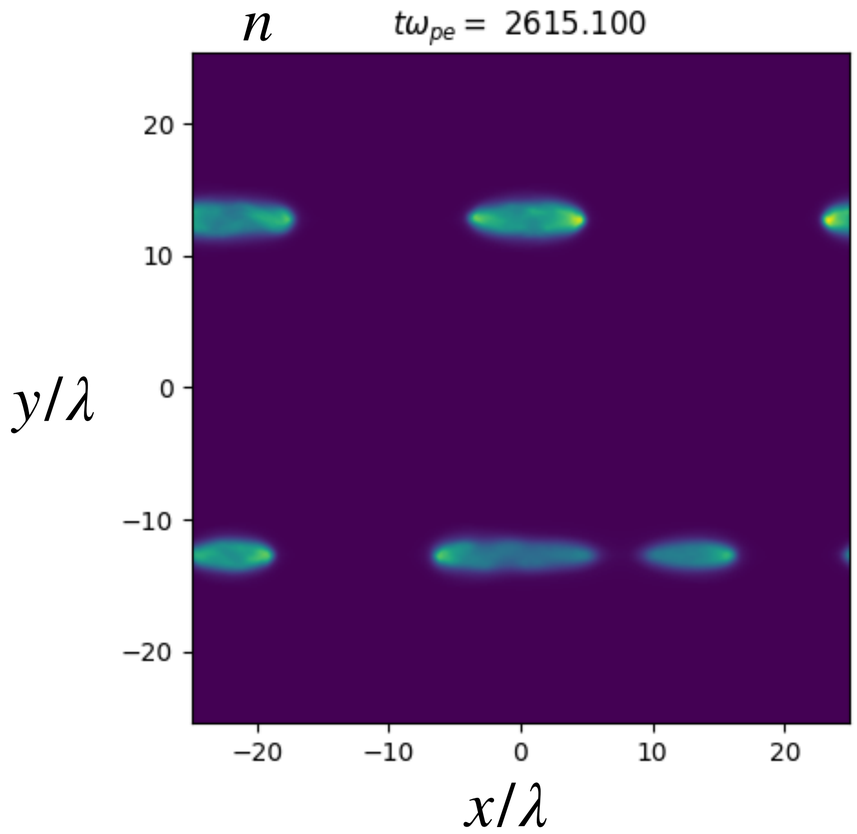
Nonlinear stage (semi-implicit low resolution)



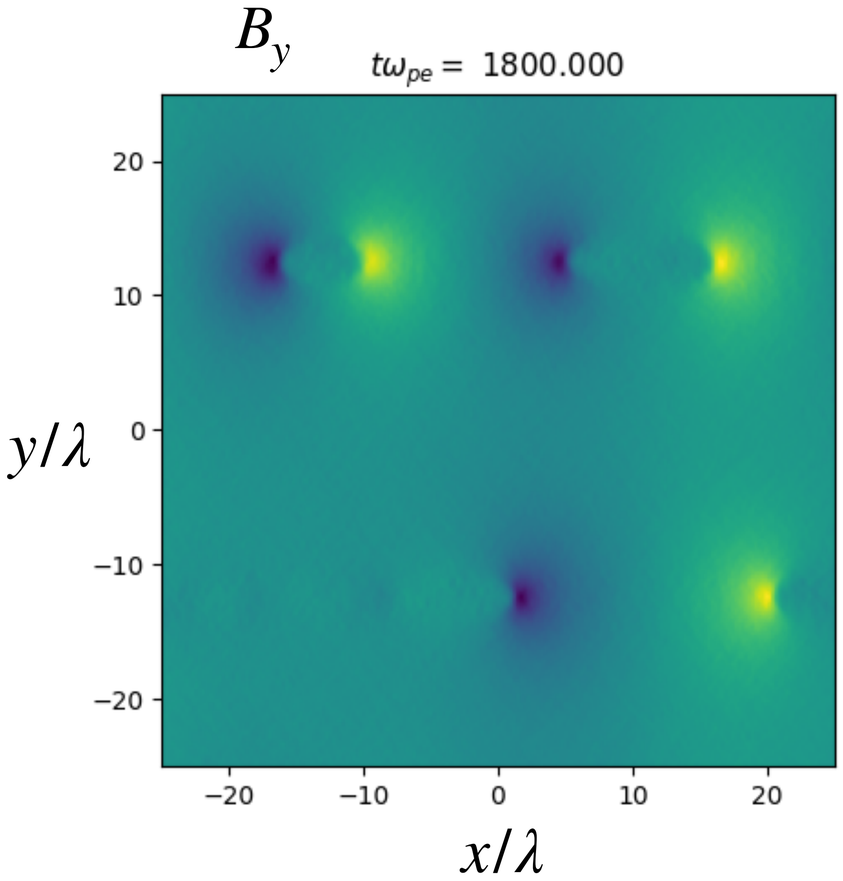


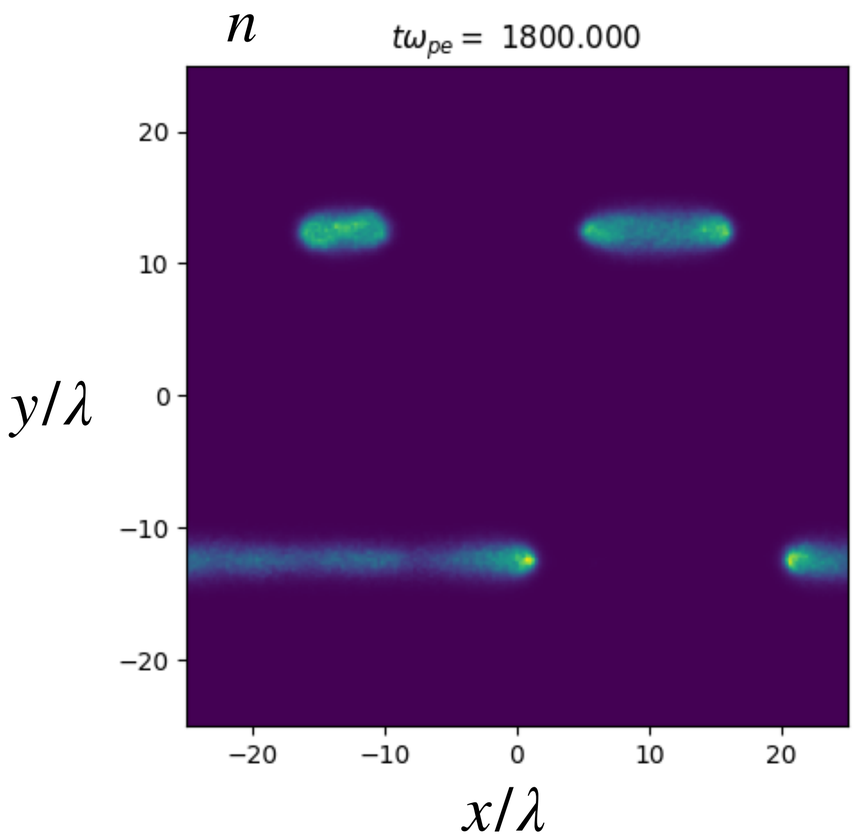
Saturation stage (explicit high resolution)





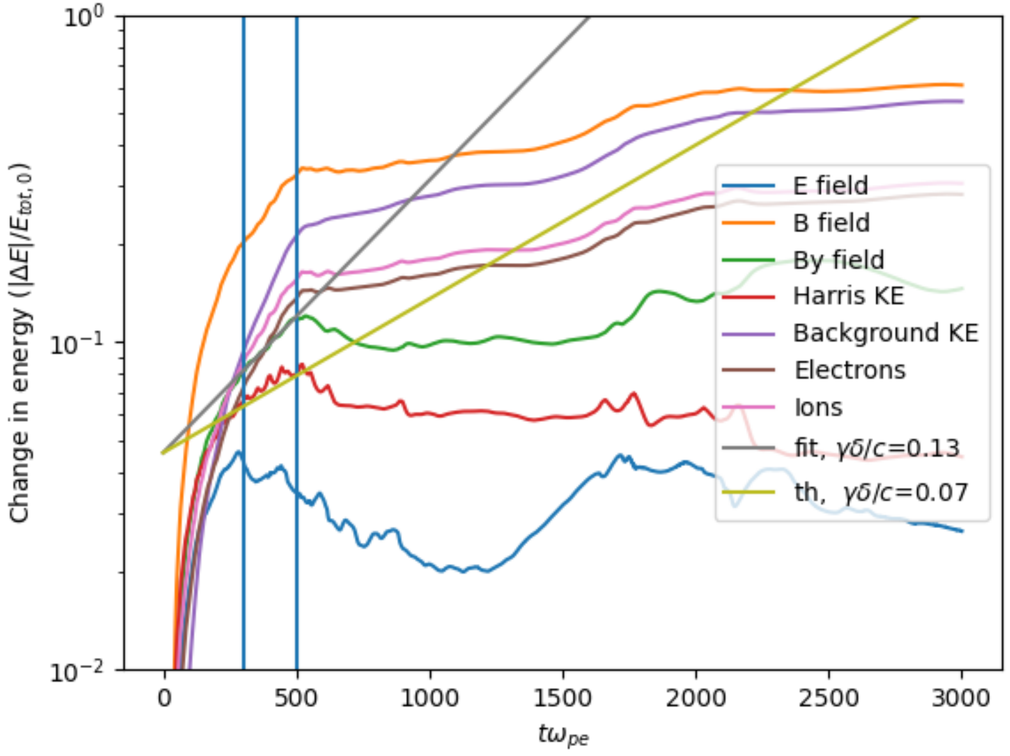
Saturation stage (semi-implicit low resolution)





Large scale tearing/reconnection (explicit simulations)

$$\frac{L_x}{\lambda} = 1000$$



$$\frac{T_{e,H}}{m_e c^2} = 10 \qquad \frac{u_d}{c} = 0.4$$

$$\frac{\lambda}{d_e} = 2.0$$

$$\frac{T_{e,b}}{m_e c^2} = 10 \qquad \frac{n_0}{n_b} = 270$$

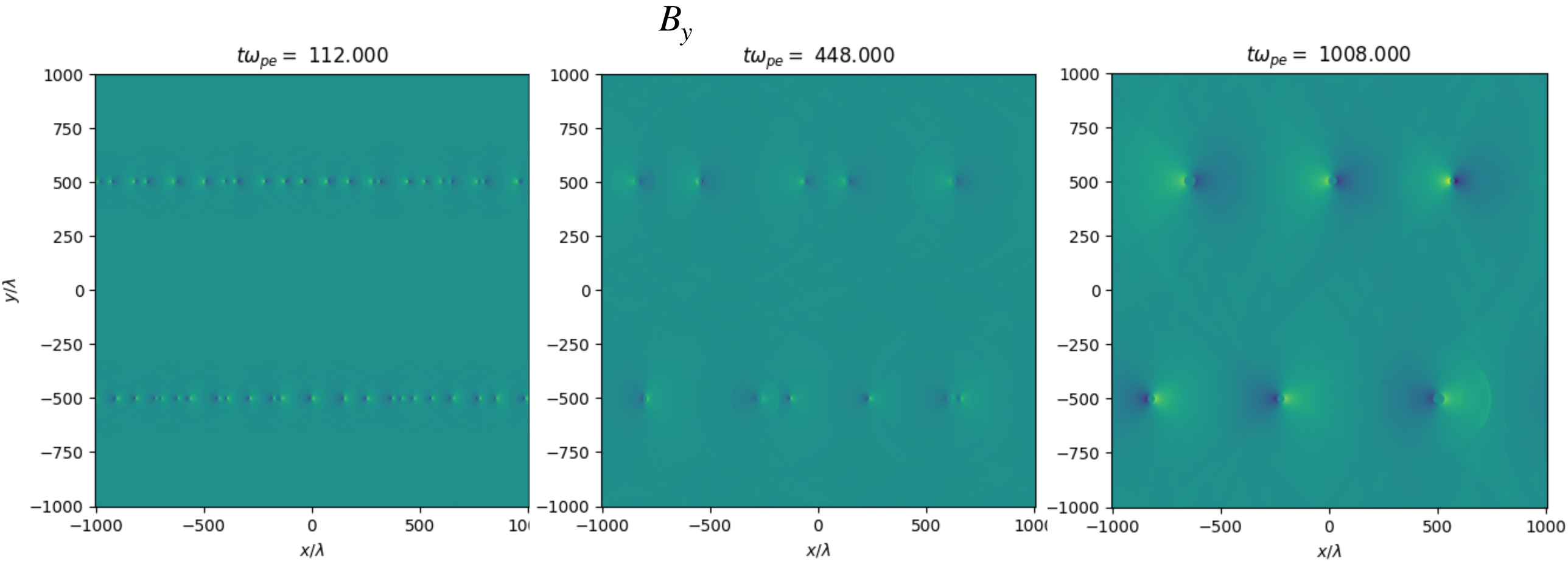
$$\sigma_c = 10000$$

$$\frac{m_i}{m_e} = 100$$

$$\frac{T_i}{T_e} = 1$$

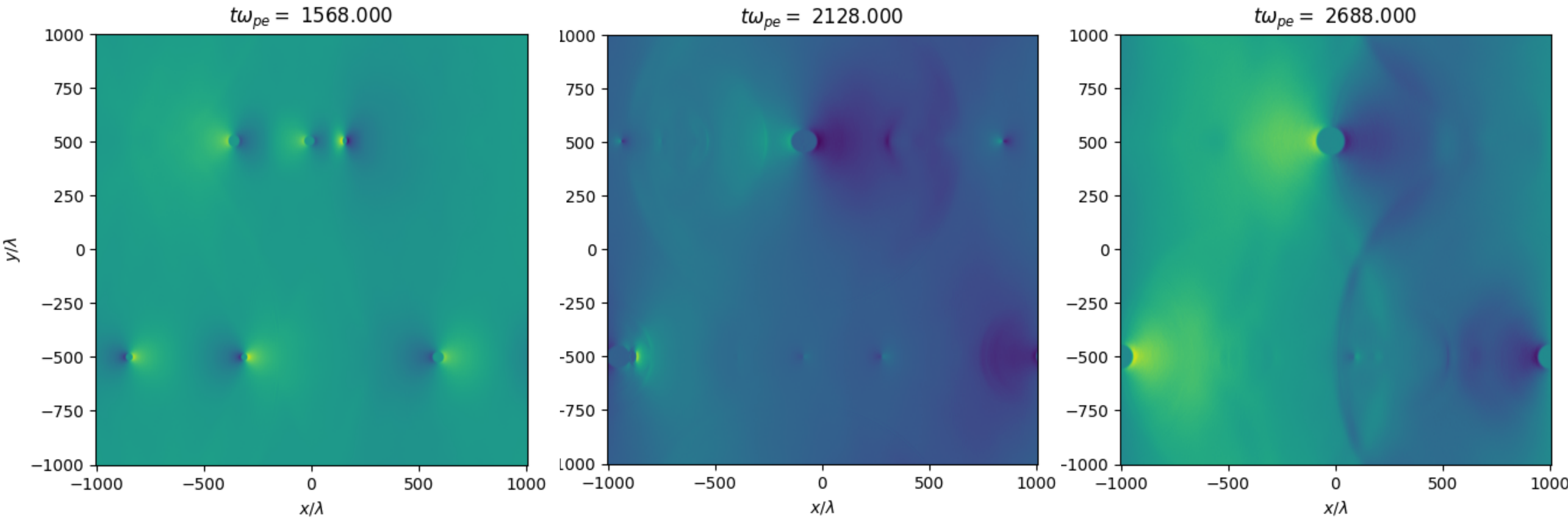
We can essentially ignore background initially

Tearing and merging



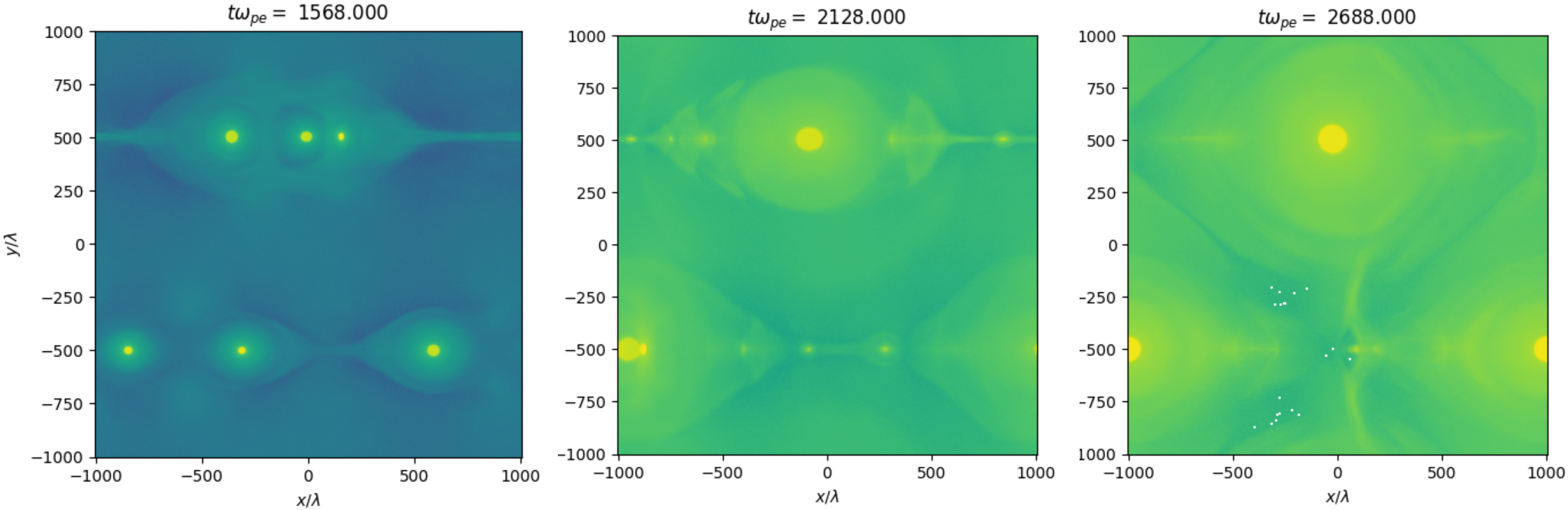
Late stage merging/ secondary islands

B_y

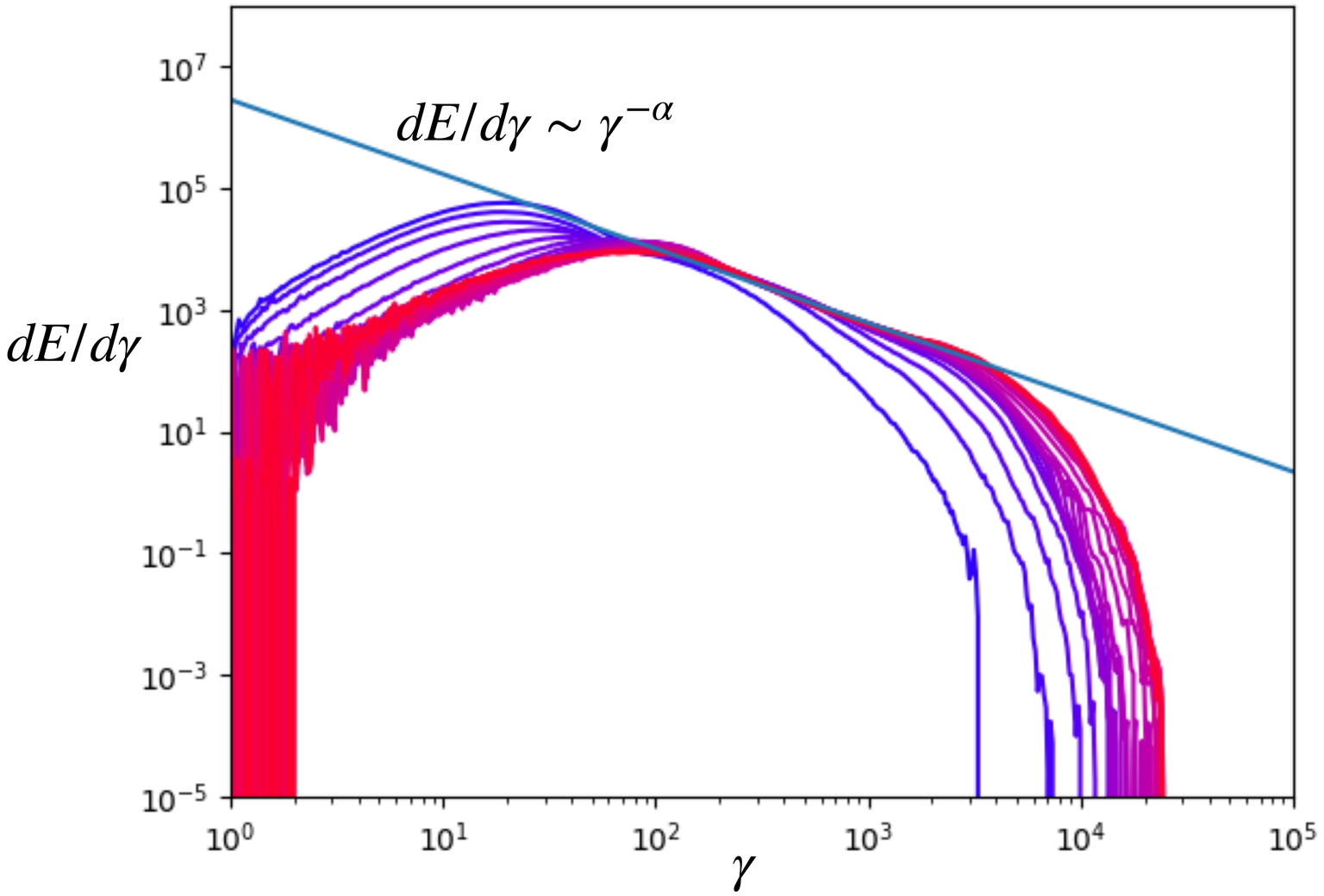


Late stage merging/ secondary islands (density)

n



Particle spectra



$$\sigma_c = 10000$$

$$\frac{m_i}{m_e} = 100$$

$$\frac{T}{m_e c^2} = 10$$

Spectral slope

$$\alpha \approx -1.221$$

Comparable with:
Werner 2016
Guo 2016

Conclusions

RUHR UNIVERSITÄT BOCHUM RUB

Semi-Implicit methods
- Allow for underresolution
- Free from numerical instabilities present in explicit codes

Relativistic EcSIM (RelSIM)
- Seems to do a good job on tearing for low resolution
- Could help find particles spectra in relativistic reconnection

SFB1491