MODELING TIME-DEPENDENT DIFFUSIVE SHOCK ACCELERATION IN THE TRANSITION REGION

SFB 1491 GENERAL ASSEMBLY -

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A3 PROJECT: TRANSITION REGION FROM GALACTIC TO EXTRAGALACTIC COSMIC RAYS

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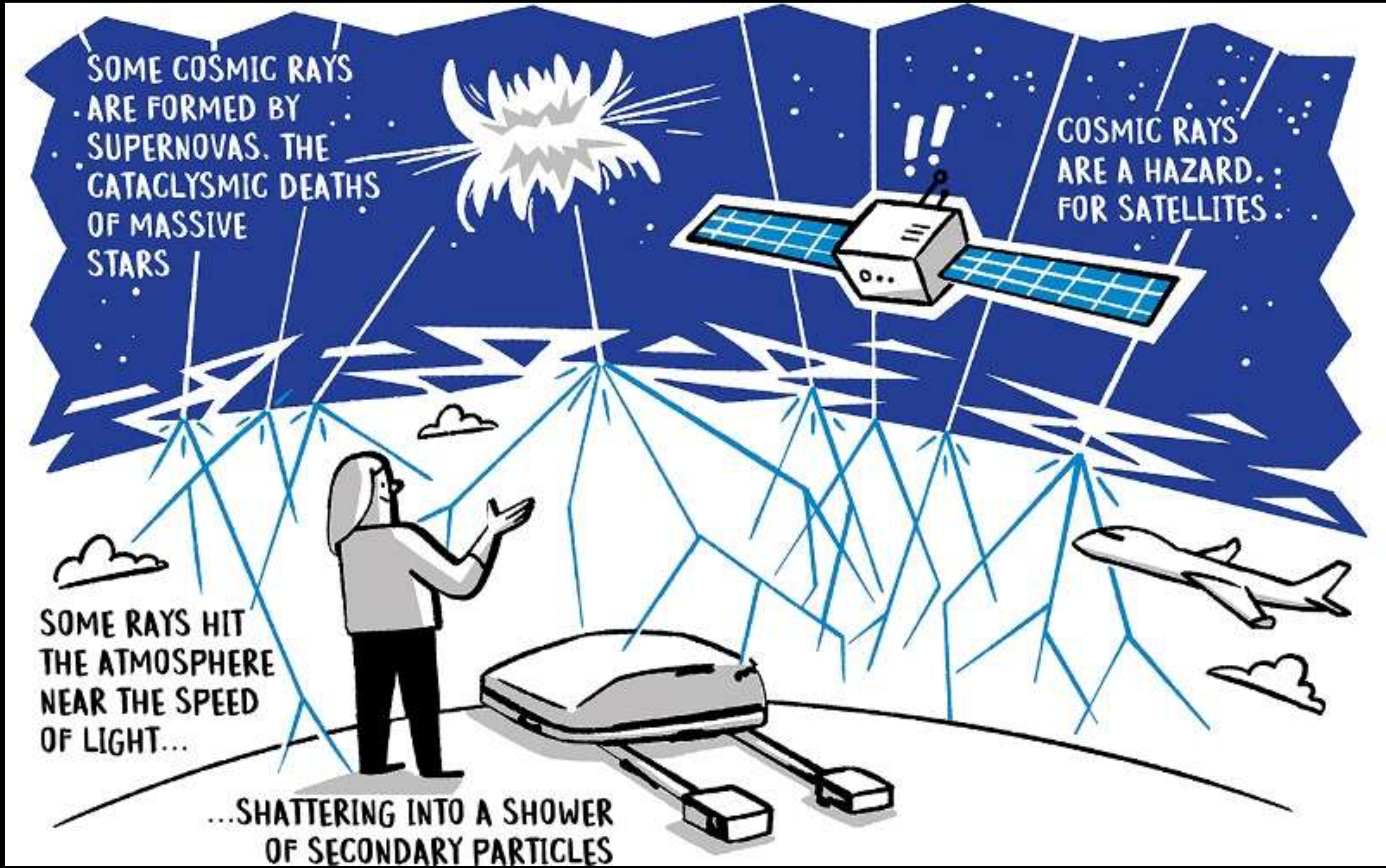
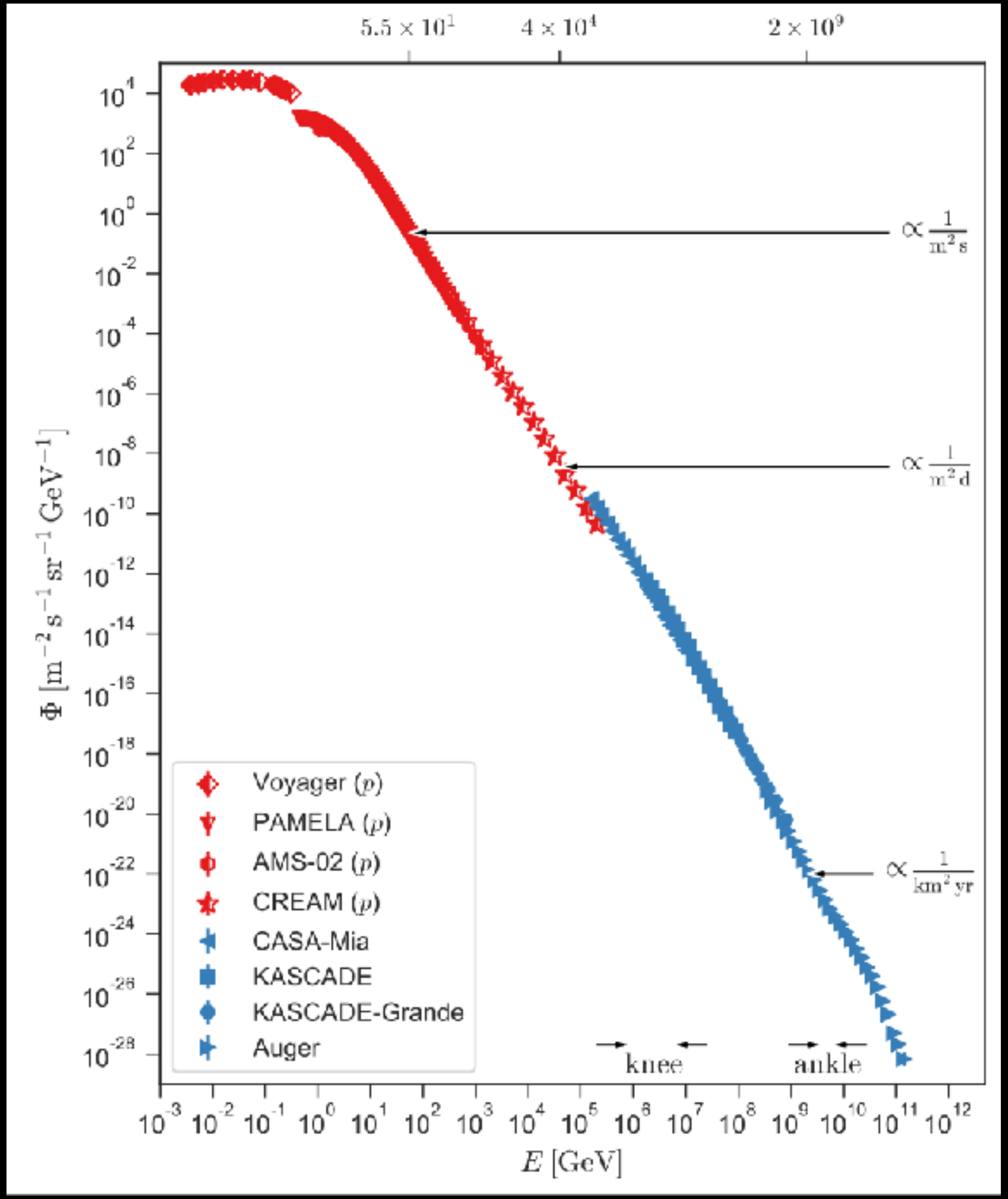


Image Credit: IOP



Becker Tjus, Merten, 2020

TRANSITION REGION…

FROM GALACTIC TO EXTRA-GALACTIC ORIGIN

Image Credit: NASA, ESA, Hubble

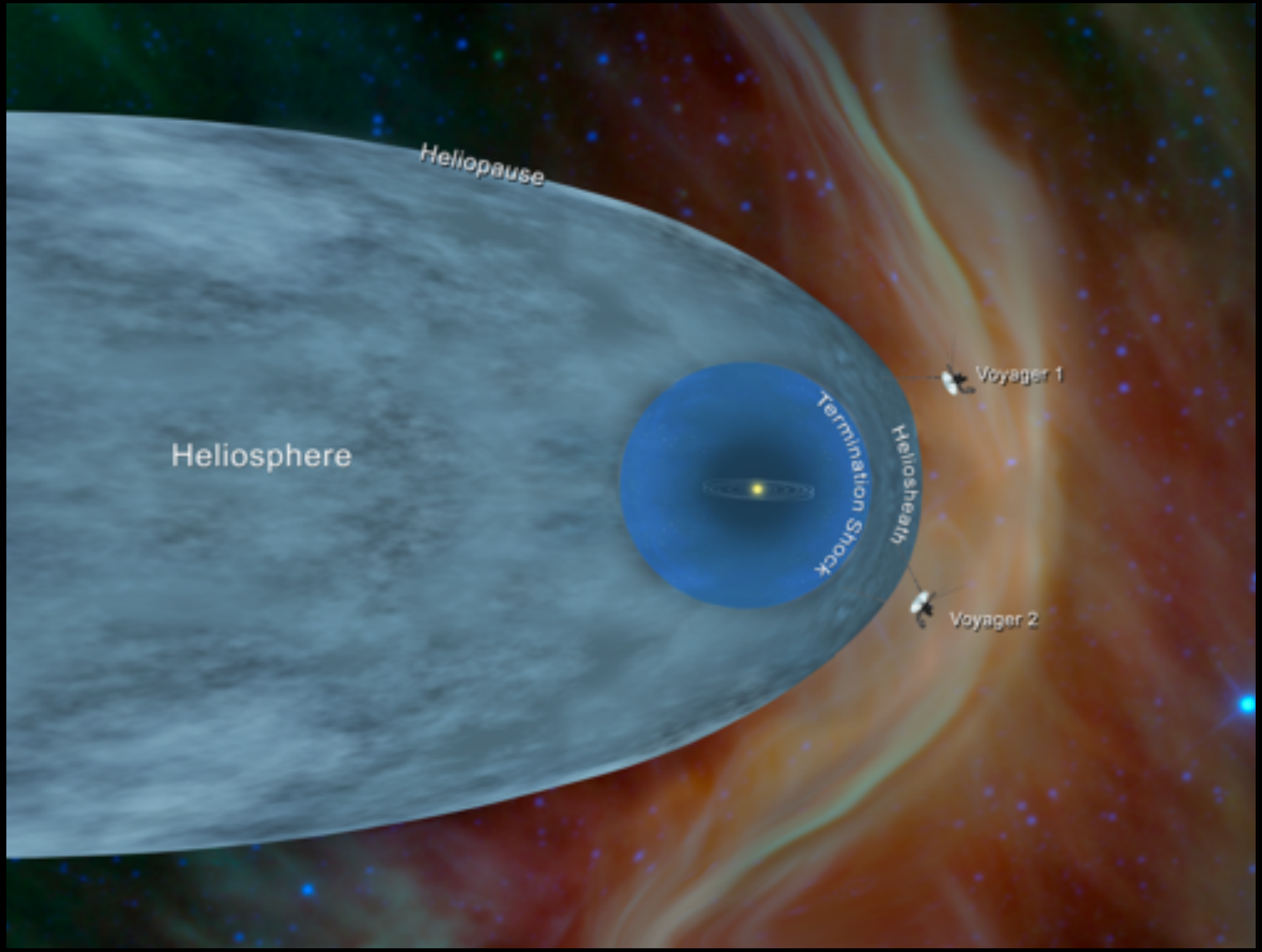
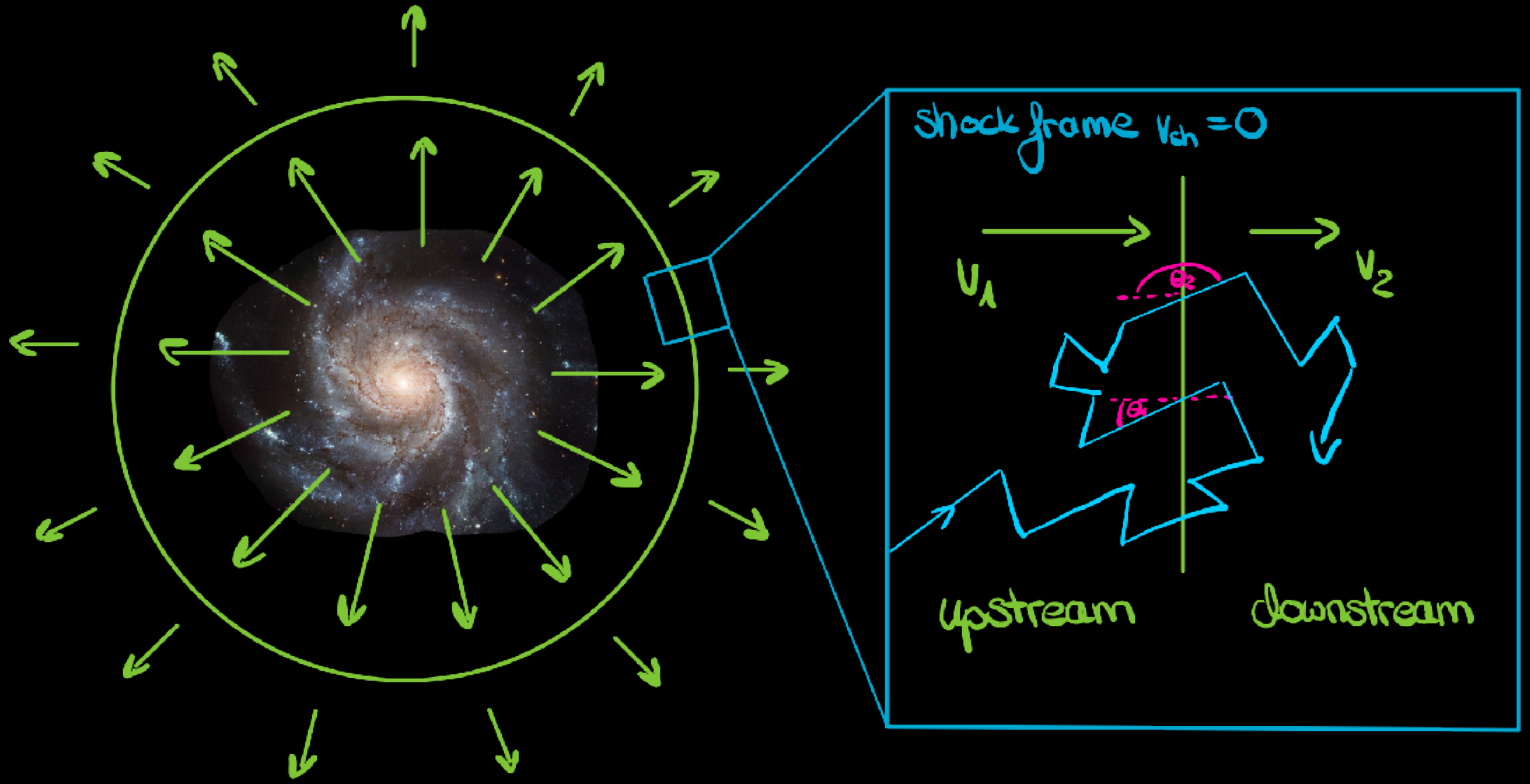


Image Credit: NASA

RE-ACCELERATION AT THE GALACTIC WIND TERMINATION SHOCK

- Diffusive Shock Acceleration (DSA) at the Galactic Wind Termination Shock (GWTS)
- CRs accelerated in the Galactic disk propagate outwards and are re-accelerated at the GWTS
- A fraction of re-accelerated CRs is able to propagate back to the Galaxy (Merten et al., 2018)



TRANSITION REGION...

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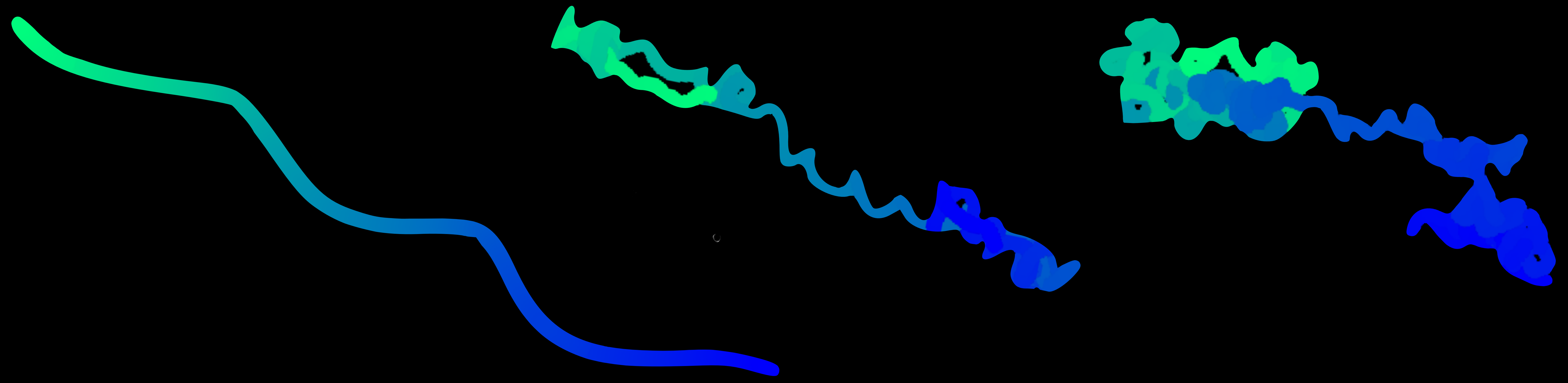
FROM BALLISTIC TO DIFFUSIVE PARTICLE TRANSPORT

CRPropa3.2

Cosmic Ray Propagation Framework



$$\frac{\partial n}{\partial t} + \underbrace{\vec{u} \cdot \nabla n}_{\text{advection}} = \underbrace{\nabla \cdot (\hat{\kappa} \nabla n)}_{\text{spatial diffusion}} + \underbrace{\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D \frac{\partial n}{\partial p}\right)}_{\text{momentum diffusion}} + \underbrace{\frac{1}{3}\left(\nabla \cdot \vec{u}\right)\frac{\partial n}{\partial \ln p}}_{\text{adiabatic energy change}} + S(\vec{x}, p, t)$$

$$\mathrm{d}\vec{x} = \left(\nabla \hat{\kappa} + \vec{u}\right) \mathrm{d}t + \sqrt{2\hat{\kappa}}\, \mathrm{d}\vec{\omega}_t, \qquad \mathrm{d}p = -\frac{p}{3} \nabla \cdot \vec{u}\, \mathrm{d}t$$

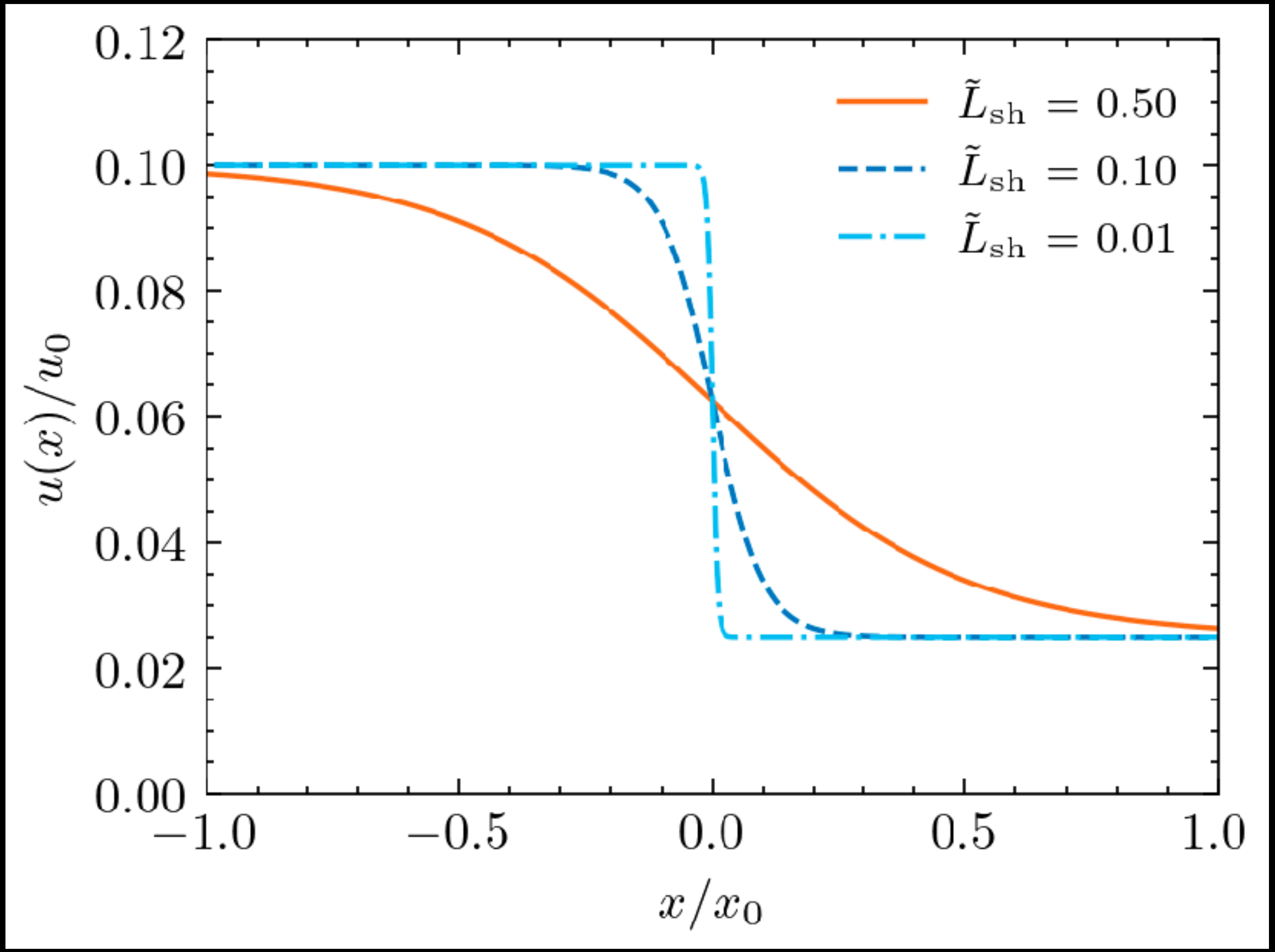
Pseudo-particles are propagated with Stochastic Differential Equations

MODELING DSA WITH STOCHASTIC DIFFERENTIAL EQUATIONS

- Interplay between diffusion, advection and adiabatic heating is responsible for energy gain at the shock:

$$\vec{x}_{t+1} = \vec{x}_t + \left[\nabla \cdot \hat{\kappa} + \vec{u}(\vec{x})\right] \Delta t + \sqrt{2\hat{\kappa}}\sqrt{\Delta t}\vec{\eta}_t$$

$$p_{t+1} = p_t - \frac{p}{3}\nabla \cdot \vec{u}\,\Delta t$$



One-dimensional wind profile with shock at $x = 0$, compression $q = u_1/u_2 = 4$

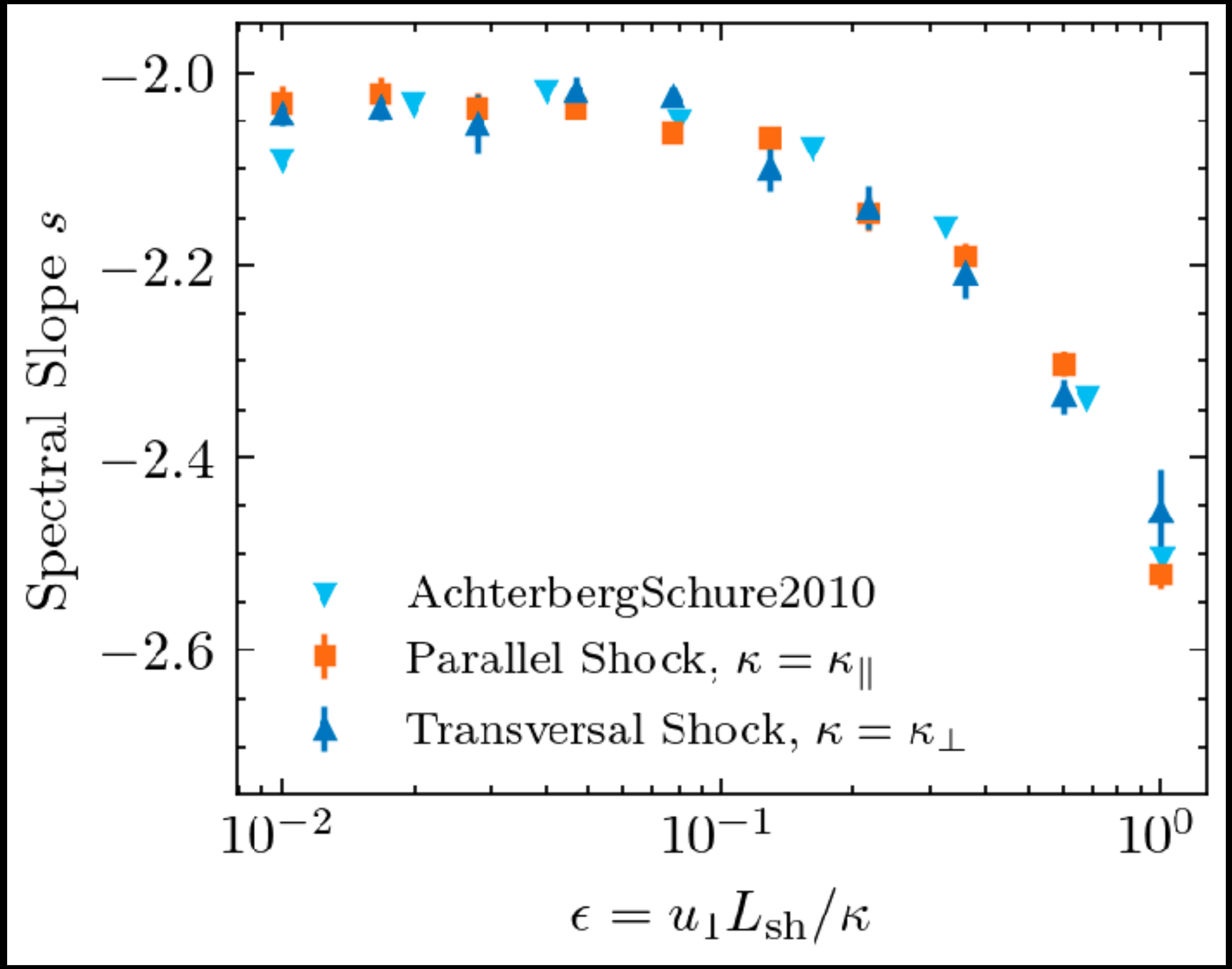
CONSTRAINTS

- Pseudo-particles have to encounter the diverging advection field to gain energy:
  - $\left[\frac{\partial\kappa}{\partial x} + u(x)\right]\Delta t$

Krülls & Achterberg, 1994

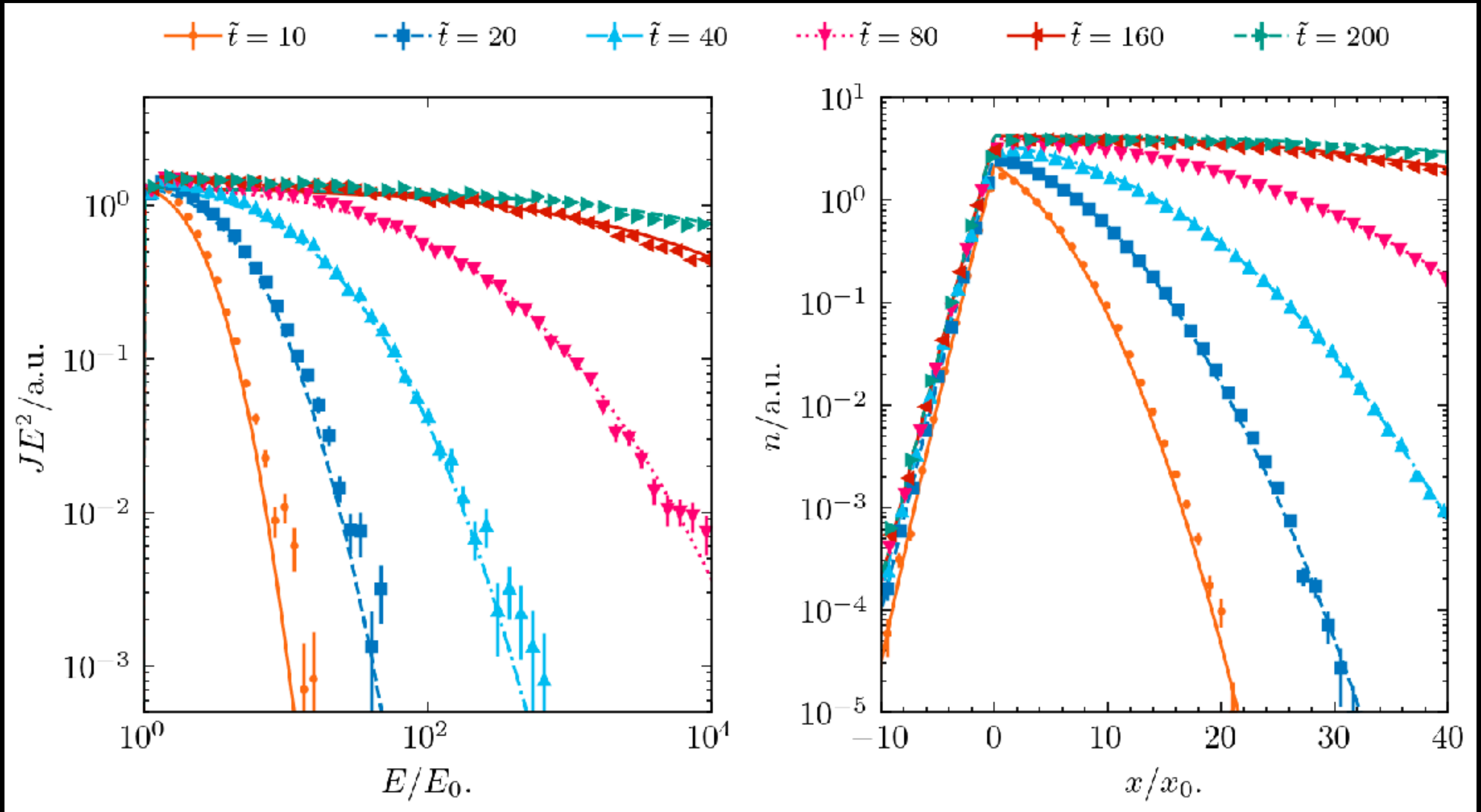
- Diffusion must be high enough to cross the shock front multiple times:
  - $L_{\mathrm{sh}} < \sqrt{2\kappa\Delta t}$
- Shock width must be small compared to advection and diffusion to model infinitely thin shock:
  - $\epsilon = u_1 L_{\mathrm{sh}}/\kappa_1$ sufficiently small

Achterberg & Schure, 2011



TIME-DEPENDENT DSA...

AT 1D PLANAR SHOCK WITH CONSTANT DIFFUSION

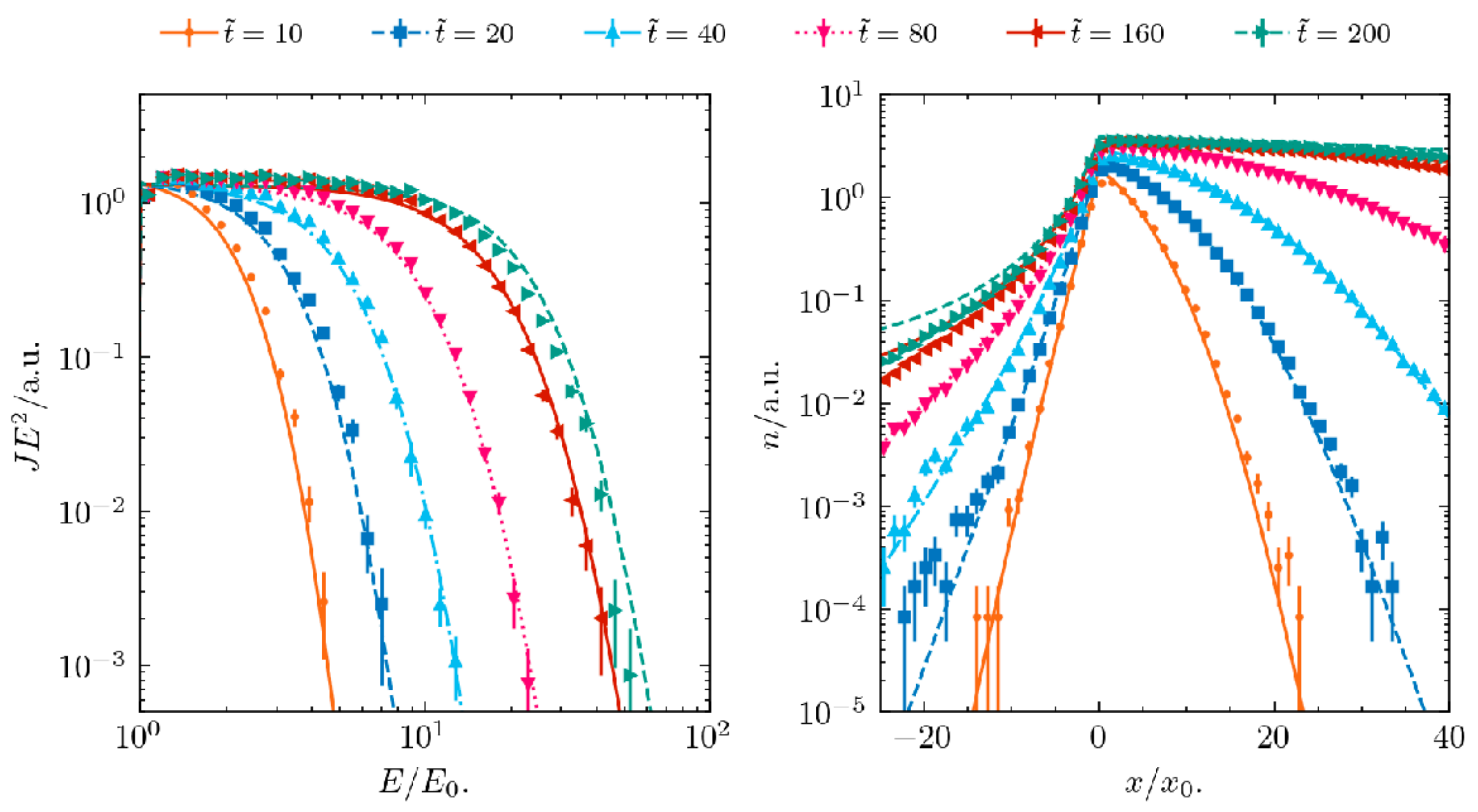


- SDE approach with CRPropa3.2
- Integrating transport eq. with VLUGR3
- Shock gets active at $\tilde{t} = 0$

Aerdker, Merten, Becker Tjus, Walter, Effenberger, Fichtner, submitted to JCAP

TIME-DEPENDENT DSA...

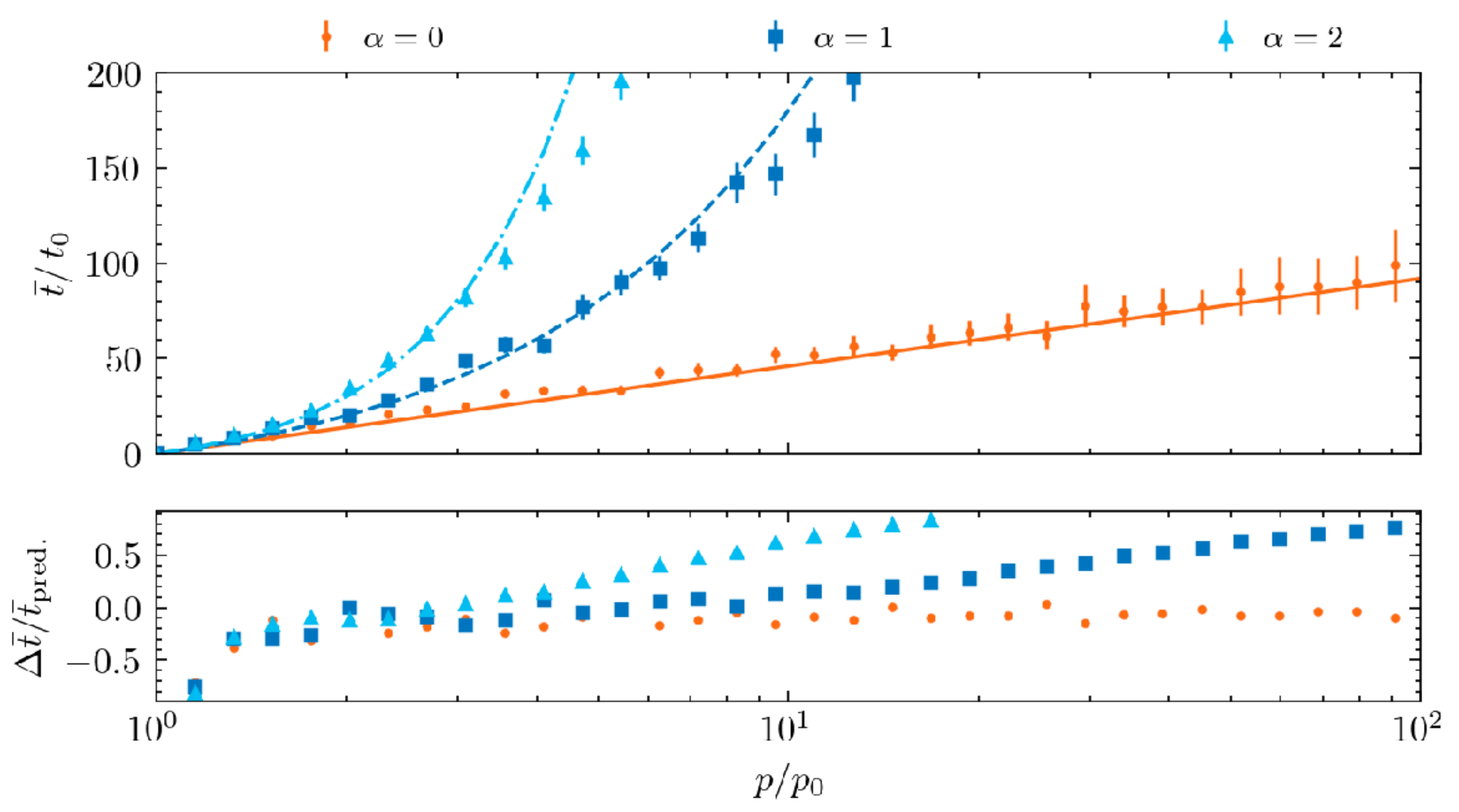
AT 1D PLANAR SHOCK WITH ENERGY-DEPENDENT DIFFUSION



- $\kappa = \kappa_0(E/E_0)^\alpha$, $\alpha = 1$
- Acceleration slows down over time
- More particles make it into upstream region

Aerdker, Merten, Becker Tjus, Walter, Effenberger, Fichtner, submitted to JCAP

ACCELERATION TIME SCALE



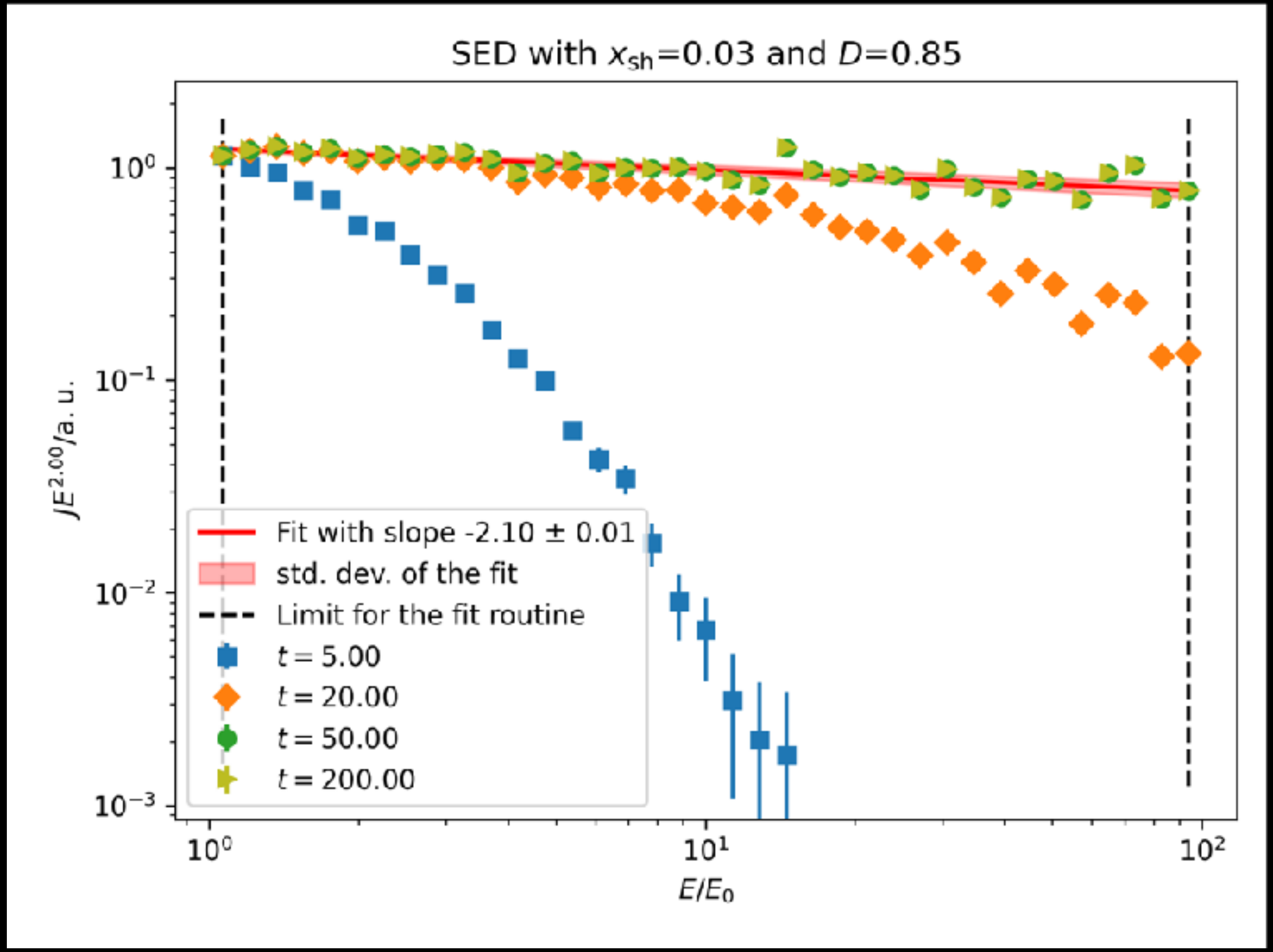
- Mean acceleration time to momentum p depends on:
- energy-dependence α of diffusion coefficient
- $\tau_{\rm acc} = \frac{3}{u_1 - u_2}\left(\frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2}\right)$

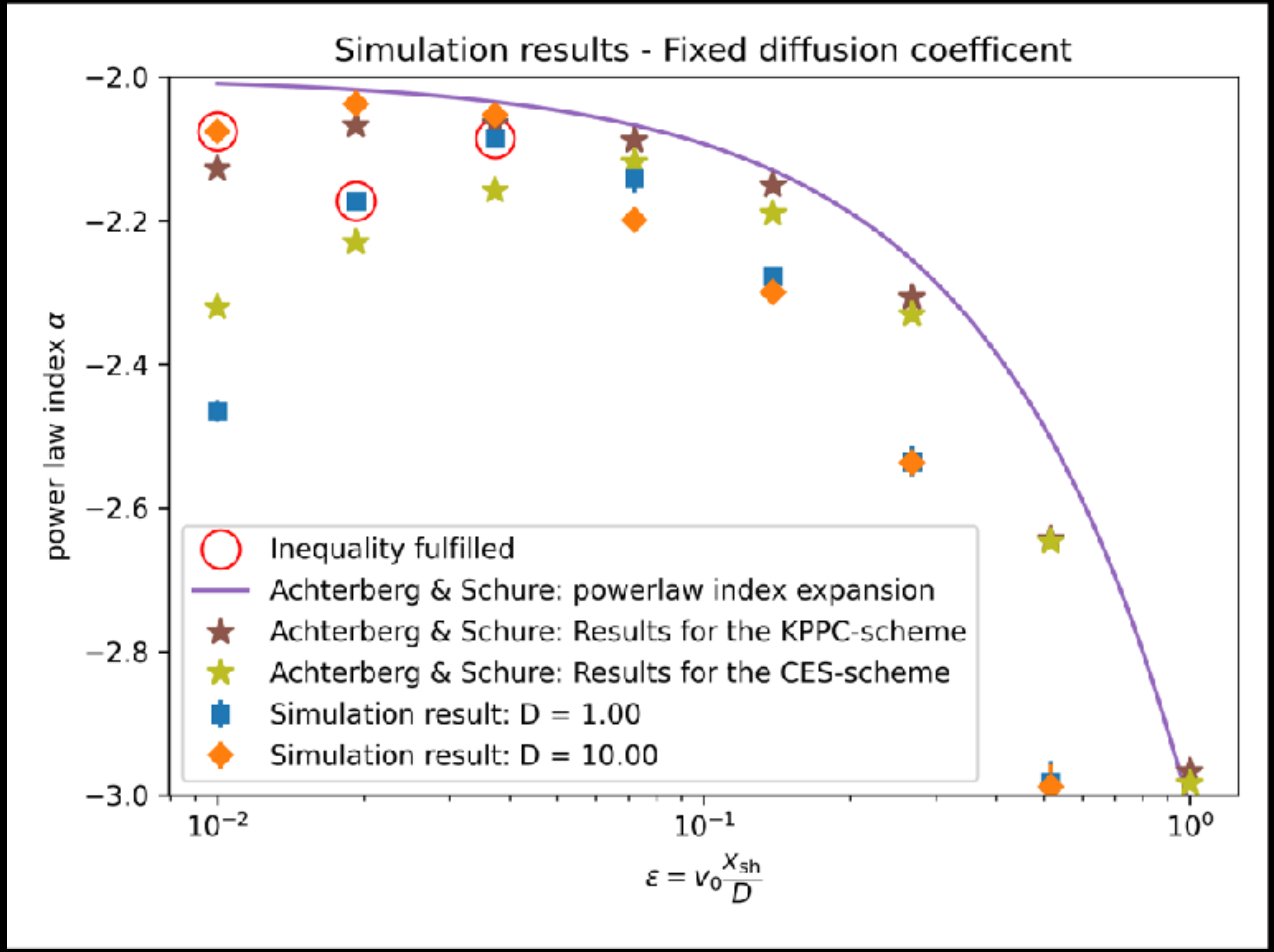
Aerdker, Merten, Becker Tjus, Walter, Effenberger, Fichtner, submitted to JCAP

TIME-DEPENDENT DSA...

AT 1D PLANAR SHOCK WITH SPATIAL-DEPENDENT DIFFUSION

$$\kappa/v^2 = \mathrm{const}\,.$$





Student Project, SOWAS, Jurek Völp

TIME-DEPENDENT DSA...

AT A SPHERICAL GWTS

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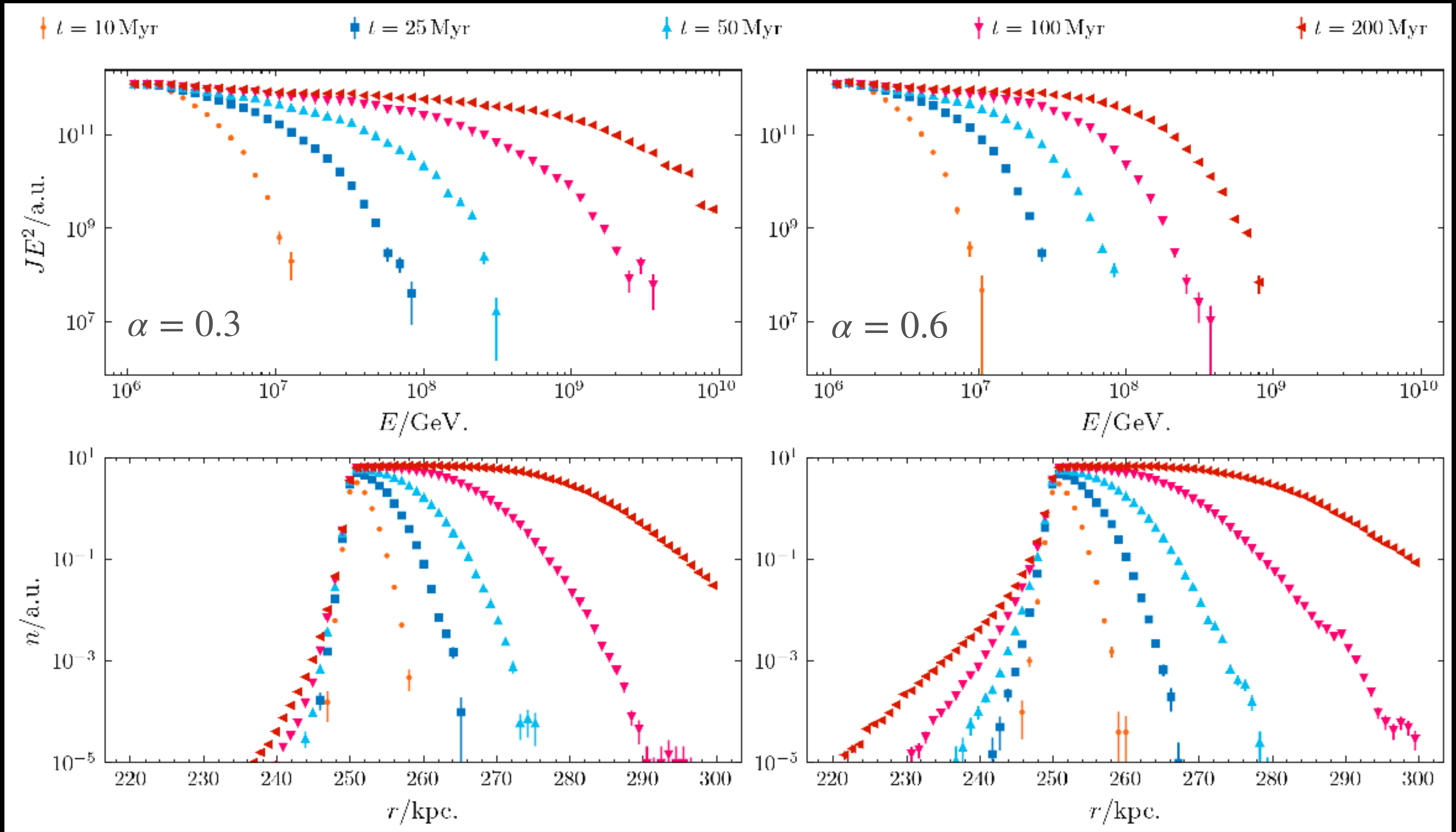
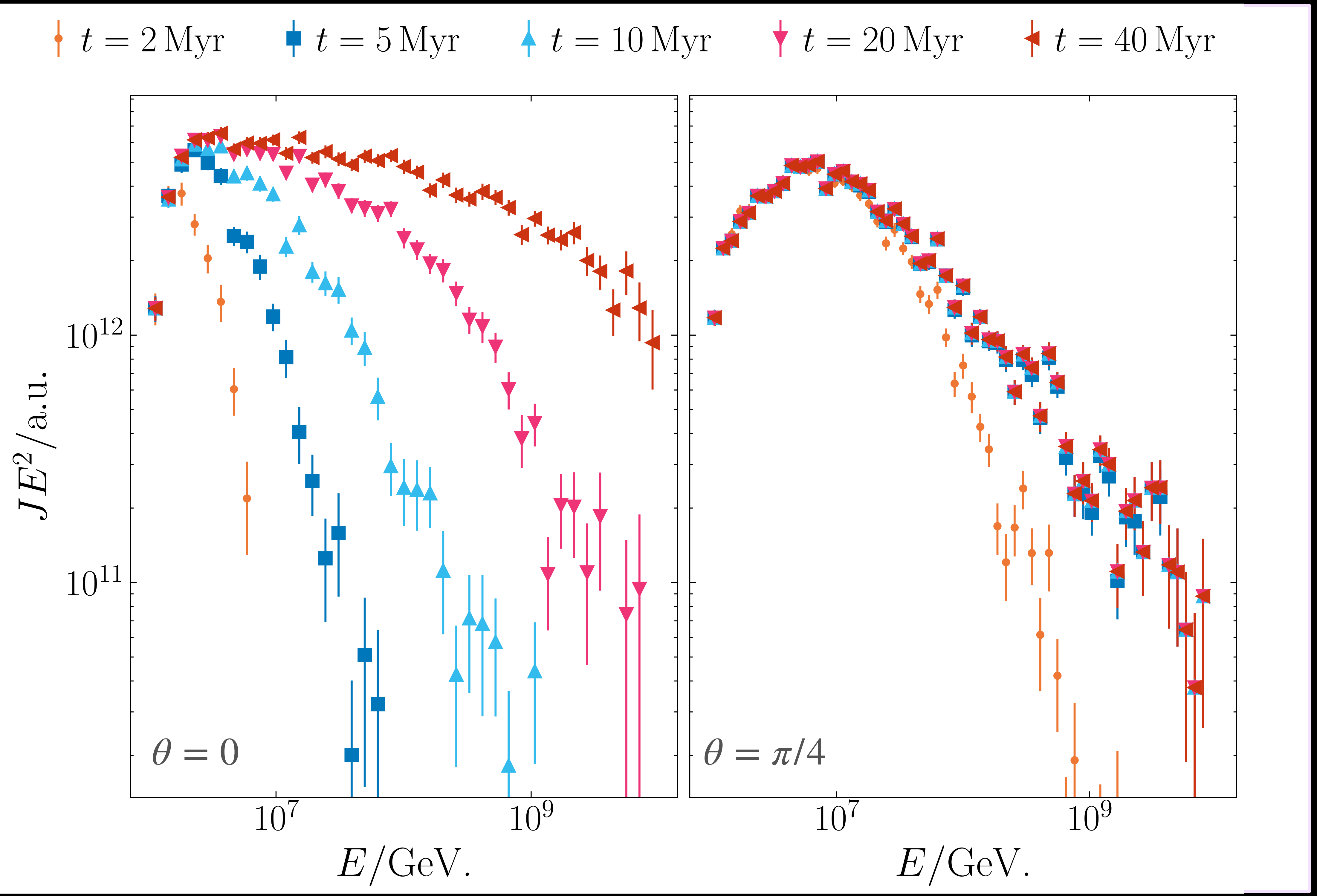


- $\kappa(E) = 5 \cdot 10^{24}\,\mathrm{m^2/s}\,\left(E/E_0\right)^{\alpha}$
- $E_0 = 10^6\,\mathrm{GeV}$
- Spectrum & number density at $R_{\mathrm{sh}} = 250\,\mathrm{kpc}$

Aerdker, Merten, Becker Tjus, Walter, Effenberger, Fichtner, PoS, ICRC 2023

3D TIME-DEPENDENT DSA...

AT A SPHERICAL SHOCK & SPIRAL MAGNETIC FIELD



Aerdker, Merten, Becker Tjus, Walter, Effenberger, Fichtner, PoS, ICRC 2023

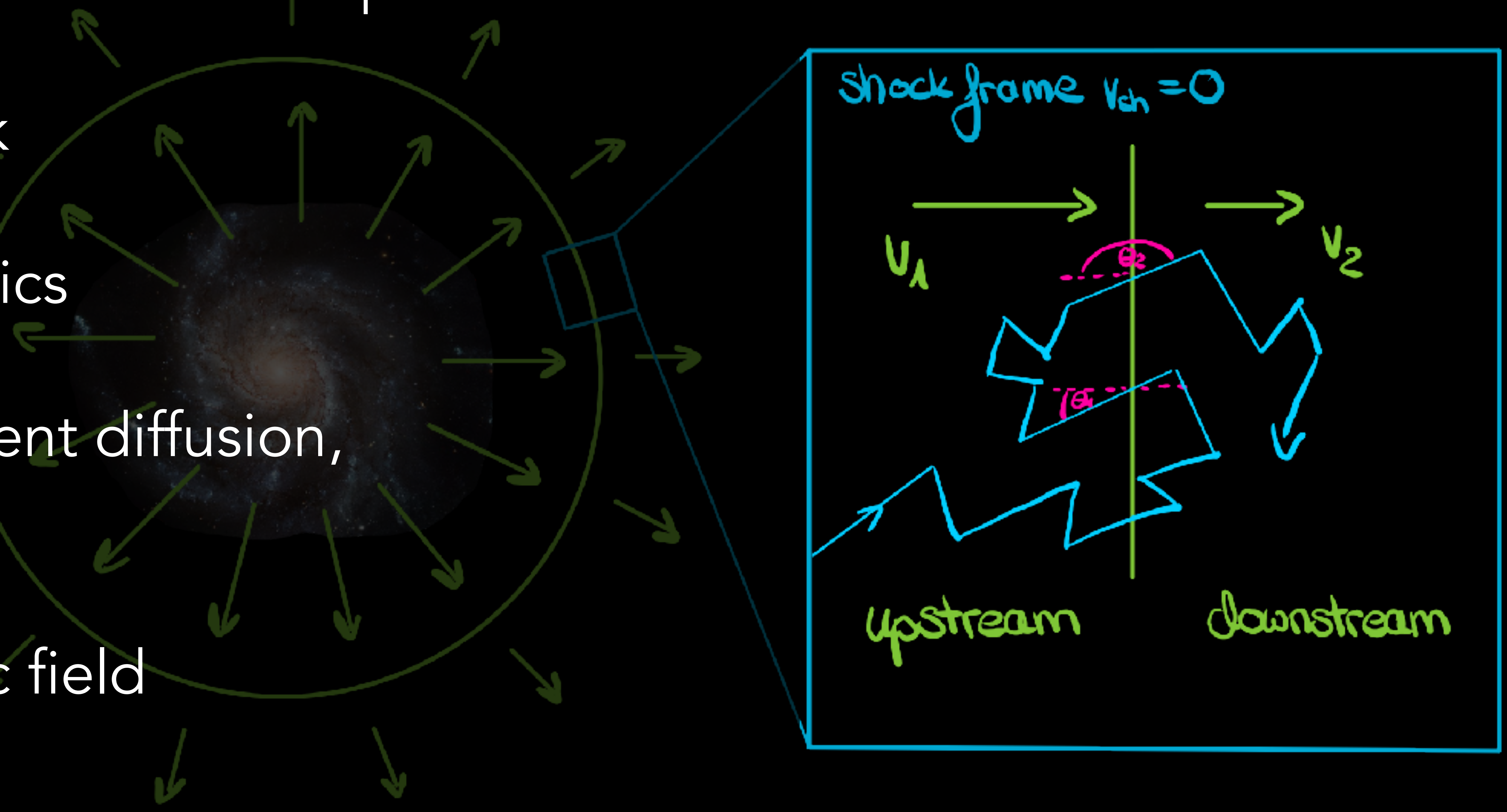
- Energy gain depends on effective diffusion over the shock front
- Angle between shock front and magnetic field
- Anisotropy of diffusion tensor

$$\hat{\kappa} = \begin{pmatrix} \kappa_\parallel \epsilon & 0 & 0 \\ 0 & \kappa_\parallel \epsilon & 0 \\ 0 & 0 & \kappa_\parallel \end{pmatrix}$$

SUMMARY

MODELING TIME DEPENDENT DSA WITH CRPROPA

- DSA modeled with *DiffusionSDE* module of CRPropa3.2
- Time-dependent spectra at the shock
- *CandidateSplitting* to enhance statistics
- Energy-dependent & spatial-dependent diffusion, anisotropic diffusion
- 3D spherical GWTS & spiral magnetic field
- Acceleration time scale



OUTLOOK

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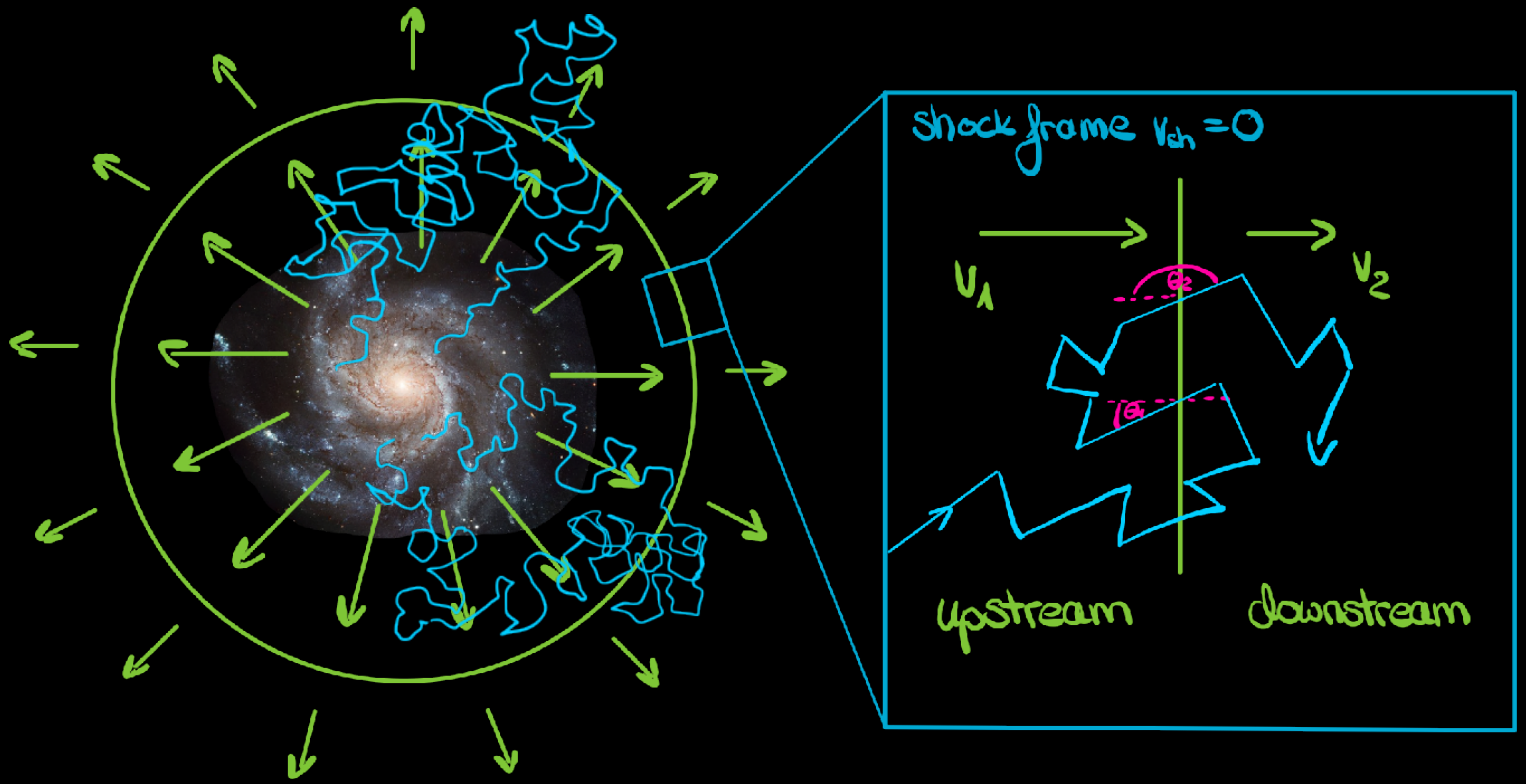
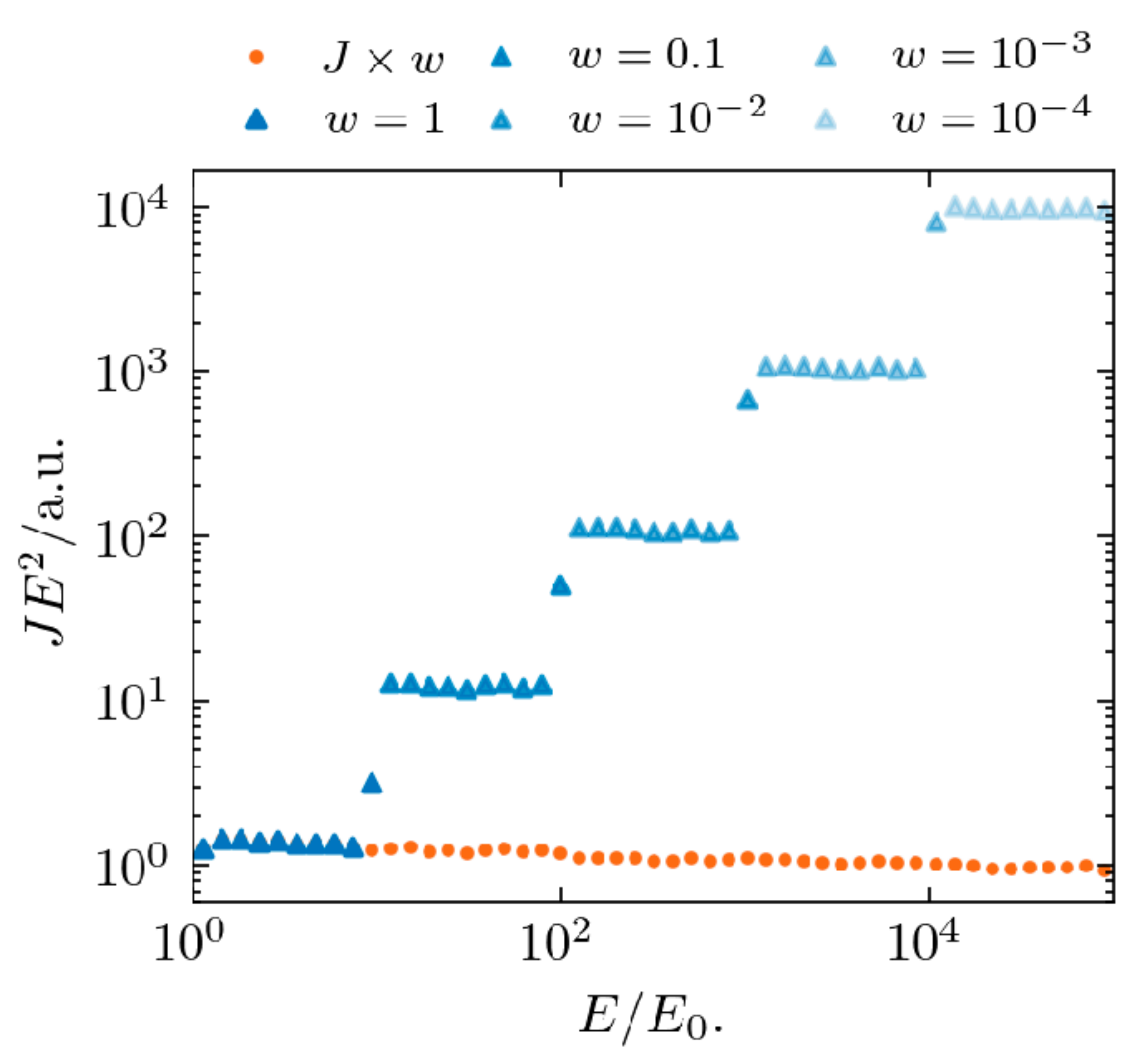
PROPAGATION OUT OF & BACK TO THE GALAXY

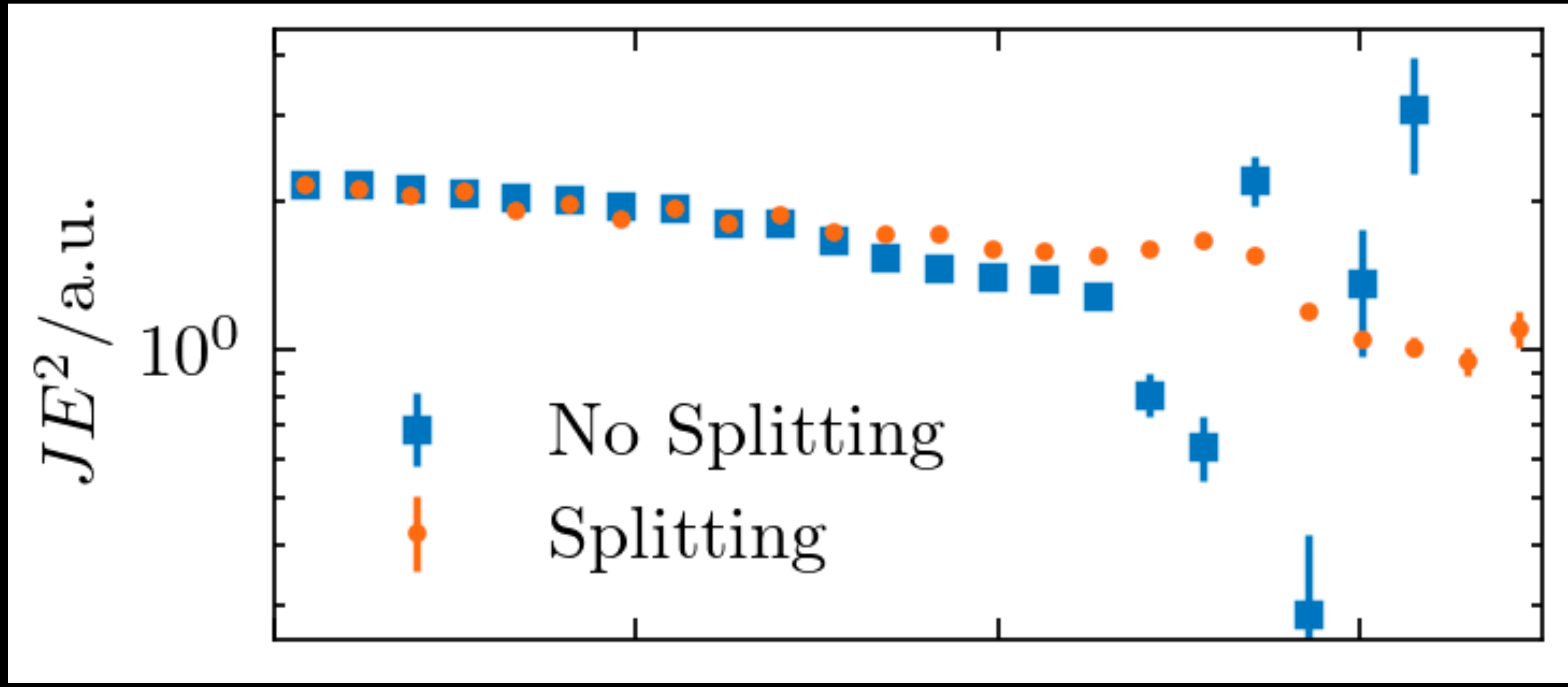


CANDIDATE SPLITTING

TO ENHANCE STATISTICS AT HIGH ENERGIES

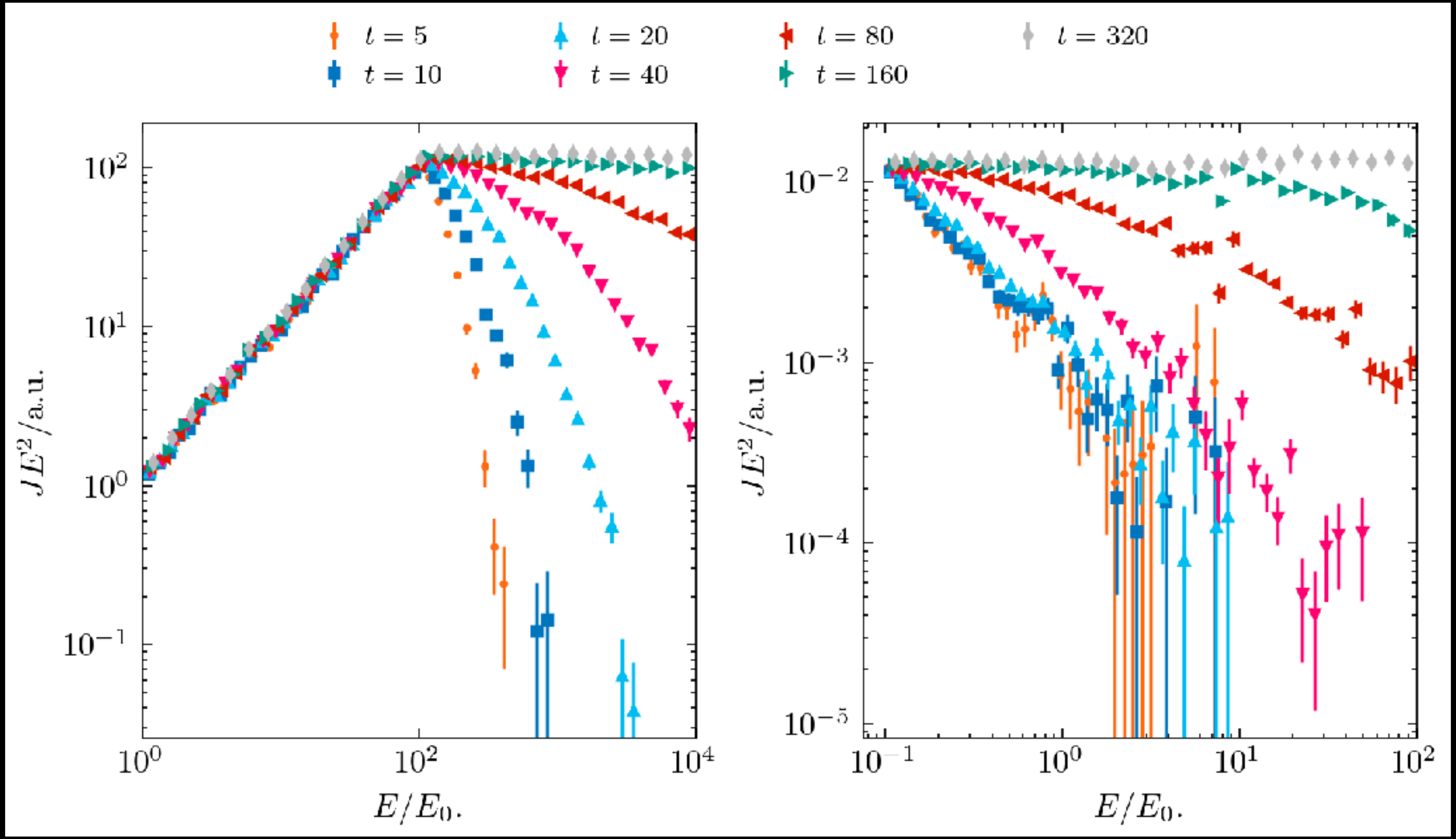


Split candidates in n copies depending on spectral **slope**, when crossing energy bins and assign **weights**



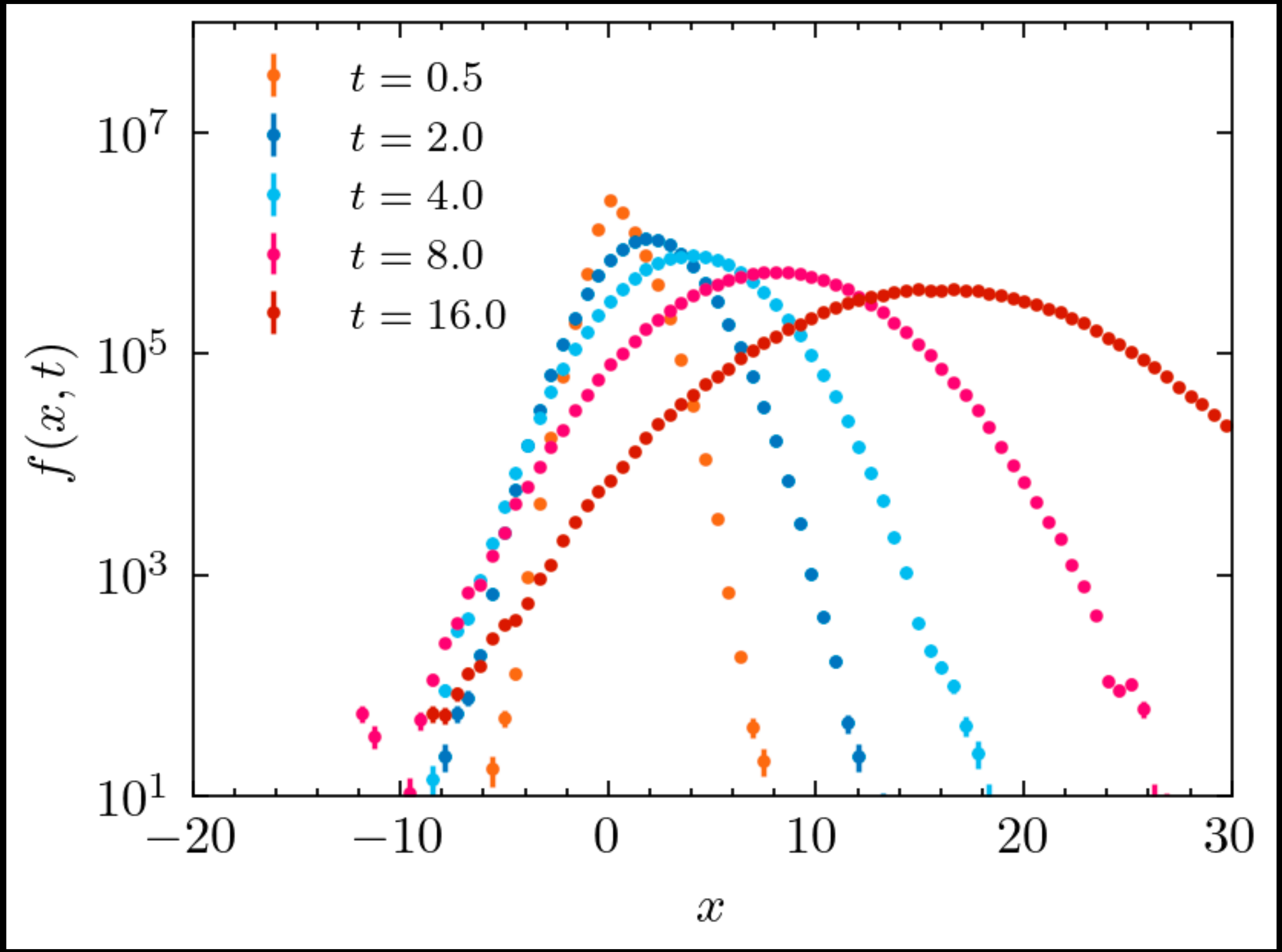
TIME-DEPENDENT DSA...

INJECTING A PRE-ACCELERATED SPECTRUM

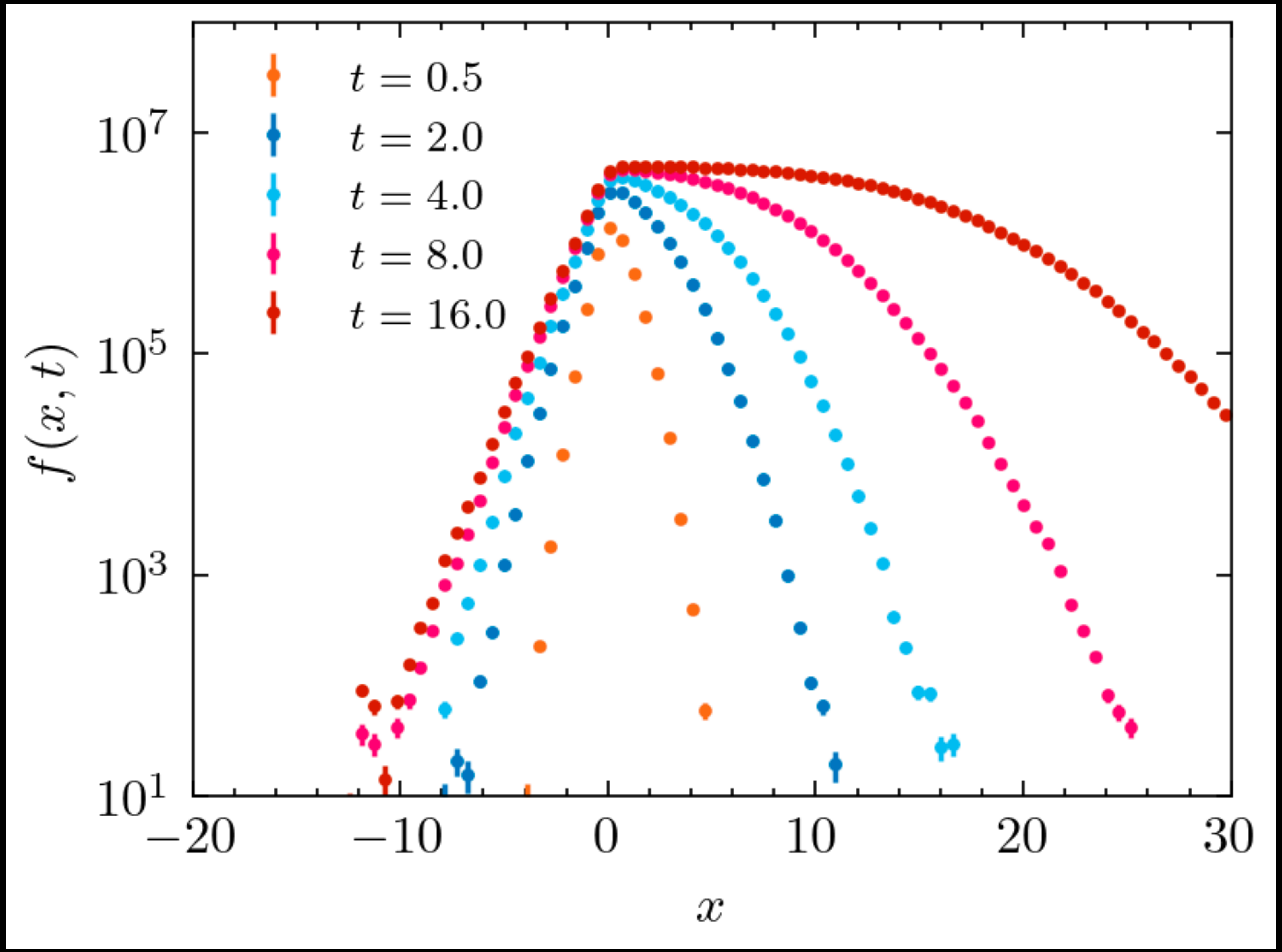


Aerdker, Merten, Becker Tjus, Walter, Effenberger, Fichtner, JCAP (under review)

DIFFUSION-ADVECTION EQUATION
APPROXIMATE STATIONARY STATE WITH CRPROPA

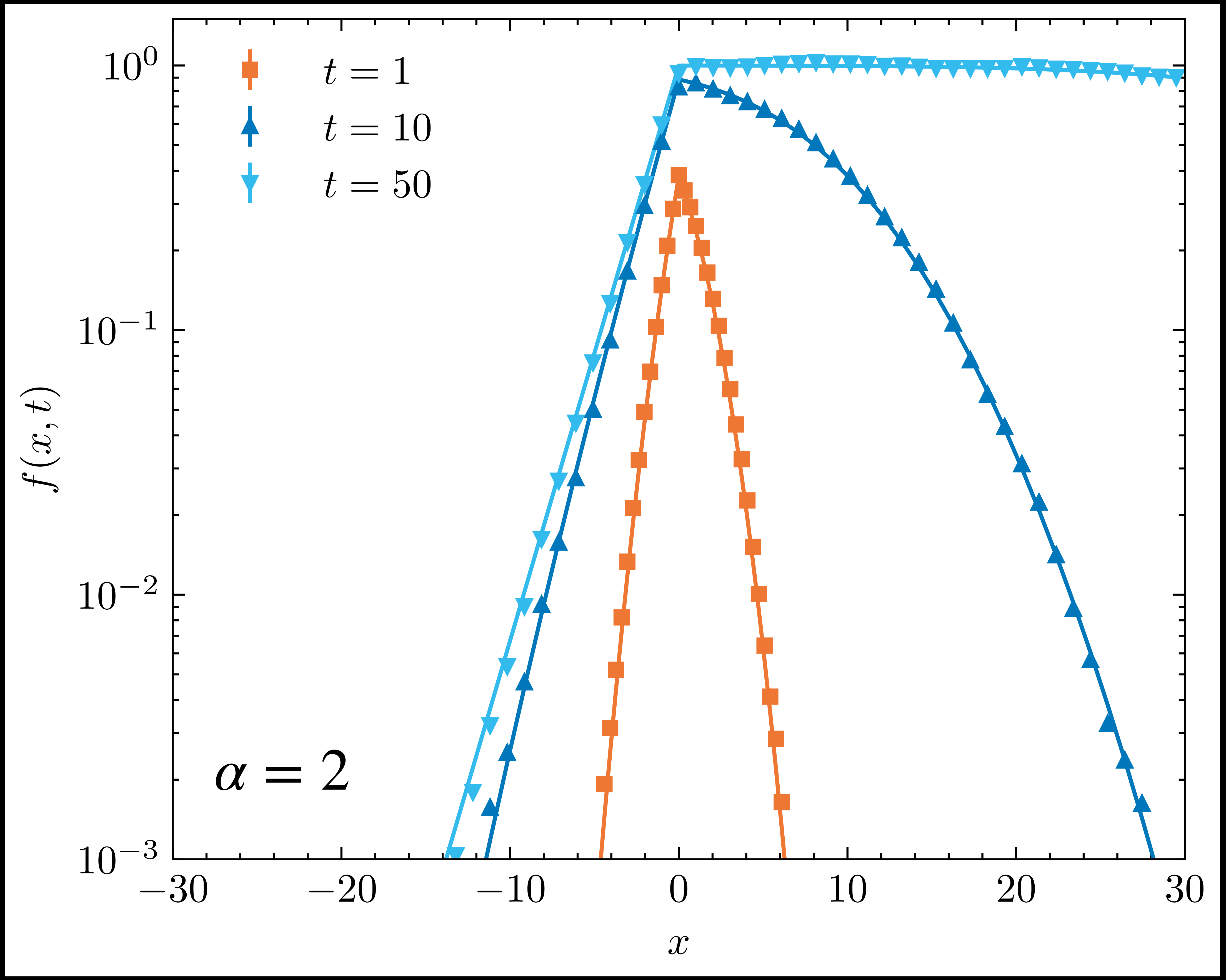


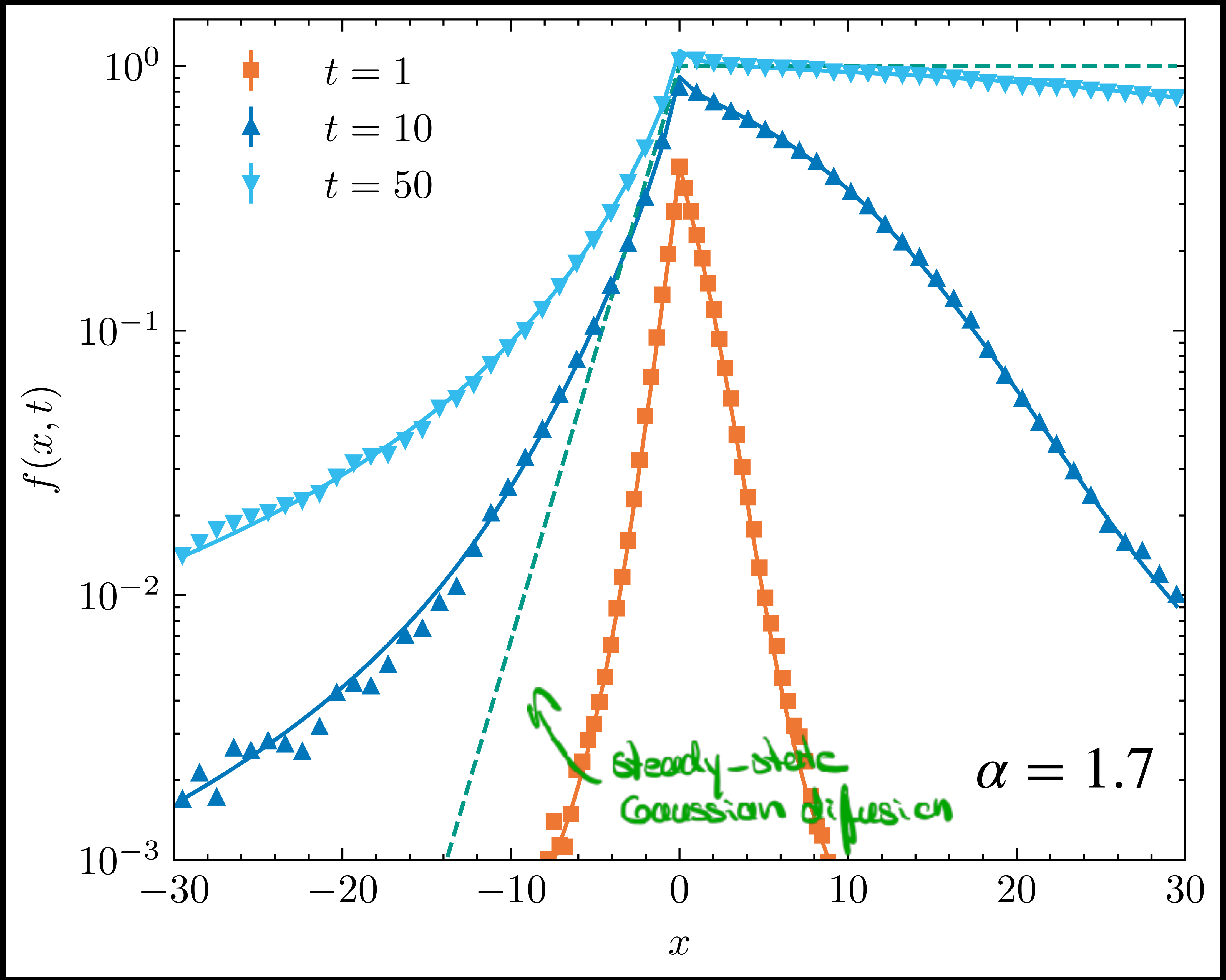
Distribution $f_t(x, t)$ of pseudo-particles at time t



Summed distribution of pseudo-particles $f(x, t) = \sum_i f_t(x, t)\Delta T_i$

2. TESTCASE: 1D SUPERDIFFUSION
DIFFUSION-ADVECTION AT SHOCK





Effenberger et al. (in preparation)

SUPERDIFFUSION

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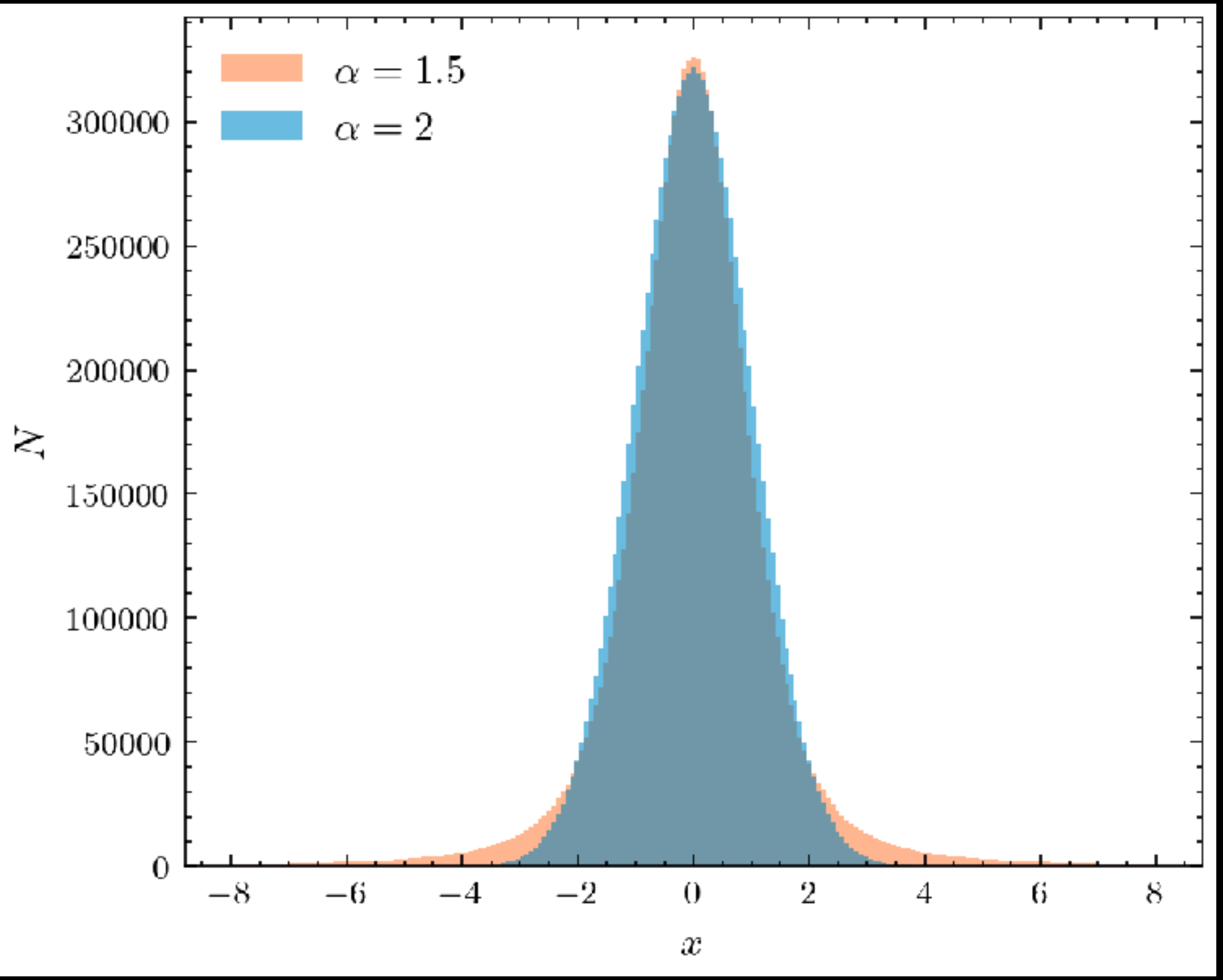
STOCHASTIC DIFFERENTIAL EQUATION: LEVY FLIGHTS

$$\mathrm{d}x = u(x)\mathrm{d}t + \sqrt{2}\kappa^{1/2}\,\mathrm{d}W_t$$

⇓

$$\mathrm{d}x = u(x)\mathrm{d}t + \sqrt{2}\kappa^{1/\alpha}\,\mathrm{d}L_{\alpha,t}$$

- Wiener process $\mathrm{d}W_t \propto \eta_W t^{1/2}$ is exchanged by Lévy process $\mathrm{d}L_\alpha \propto \eta_L t^{1/\alpha}$
- Random numbers η_L are drawn from α-stable Lévy distribution.



Sample of 10^7 random numbers drawn from a α-stable Lévy distribution

SUPERDIFFUSION

STOCHASTIC DIFFERENTIAL EQUATION: LEVY FLIGHTS

MODELING DSA WITH STOCHASTIC DIFFERENTIAL EQUATIONS

- SDE is integrated with Euler-Maruyama Scheme:

$$\vec{x}_{t+1} = \vec{x}_t + \left[\nabla \cdot \hat{\kappa} + \vec{u}(\vec{x}) \right] \Delta t + \sqrt{2\hat{\kappa}} \sqrt{\Delta t} \vec{\eta}_t$$

$$\hat{\kappa} = \begin{pmatrix} \kappa_{\|}\epsilon & 0 & 0 \\ 0 & \kappa_{\|}\epsilon & 0 \\ 0 & 0 & \kappa_{\|} \end{pmatrix}$$