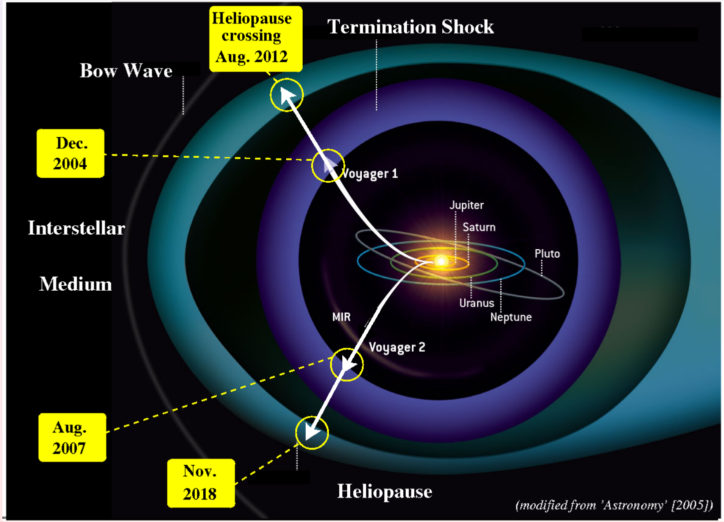
Diffusion of Energetic Particles in Turbulent Magnetic Fields

Horst Fichtner, Jens Kleimann, Sean Oughton
Klaus Scherer, Dustin Schröder, Tobias Wiengarten

Institut für Theoretische Physik IV, Ruhr-Universität Bochum, Germany
Department of Mathematics, University of Waikato, Hamilton, New Zealand

$$\overbrace{\delta B^2}^{1-comp.\ model} = \overbrace{\underbrace{\delta B^2_{2D}}_{\substack{\text{'quasi-2D'} \\ \text{low frequency} \\ k \approx k_\perp}} + \underbrace{\delta B^2_{sl}}_{\substack{\text{'slab/wave-like'} \\ \text{high frequency} \\ k \approx k_\parallel}}}^{2-comp.\ model}$$

The first in-situ exploration campaign of the heliosphere is completed



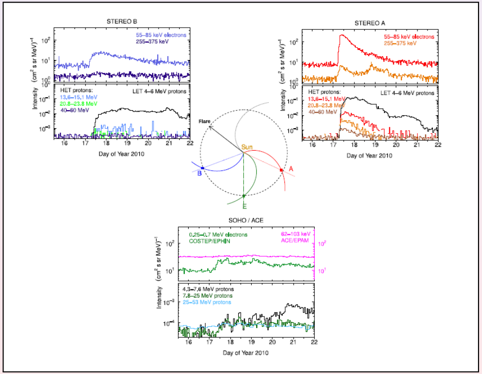
(modified from 'Astronomy' [2005])

The Significance of Knowing the Turbulence Conditions

Inner heliosphere:

- interpretation of **spacecraft data**
- understanding of **space weather**
- input to models of **the outer heliosphere**

multi-spacecraft observations:



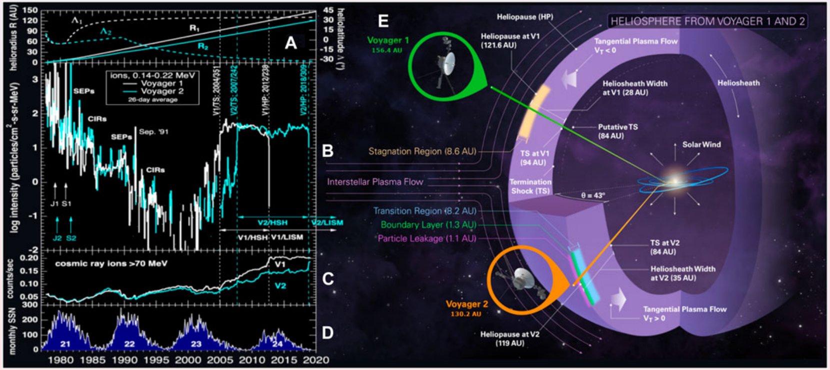
Dresing et al. [2012]

→ Of particular importance for the transport of energetic particles is the **turbulent inner heliospheric magnetic field**

The Significance of Knowing the Turbulence Conditions

Global heliosphere:

$$\frac{\partial F}{\partial t} = \nabla\cdot\left[\overleftrightarrow{\boldsymbol{\kappa}}\,\nabla F\right] + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D\,\frac{\partial F}{\partial p}\right] - \mathrm{u}\cdot\nabla F + \frac{p}{3}\left(\nabla\cdot\mathrm{u}\right)\frac{\partial F}{\partial p} + S(\mathrm{r}, p, t)$$



- anisotropic spatial diffusion: tensor $\overleftrightarrow{\boldsymbol{\kappa}}\,(\mathrm{r}, p, \mathrm{B}(t))$
- scalar momentum diffusion: coefficient $D(\mathrm{r}, p, \mathrm{B}(t)) \sim 1/\angle$
- convection and drift: velocity $\mathrm{u}(t)$
- adiabatic energy changes: $\nabla\cdot\mathrm{u}(t) \neq 0$
- sources/sinks: $S(\mathrm{r}, p, t)$

- modulation of cosmic ray energy spectra

The Significance of Knowing the Turbulence Conditions

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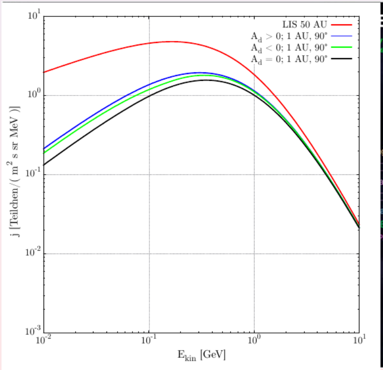

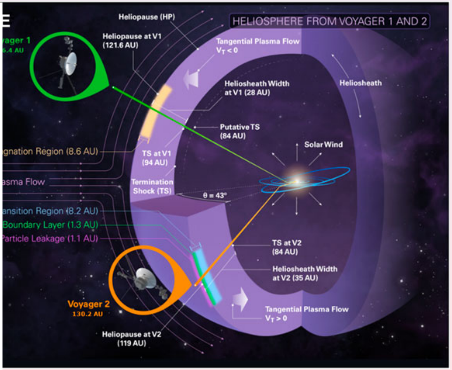

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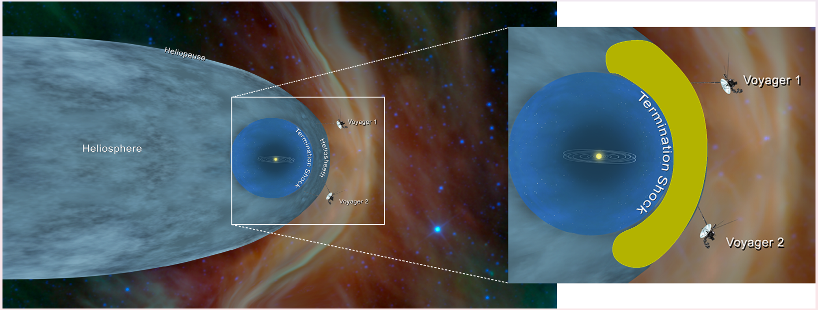
The Significance of Knowing the Turbulence Conditions

Outer heliosphere:



- only stellar wind downstream region observed in situ
- responsible for a significant decrease of cosmic ray flux (shielding)
- (apparently dominated by compressible turbulence)

The Diffusion Processes

Transport equation:

$$\frac{\partial F}{\partial t} = \nabla \cdot \left[\overleftrightarrow{\boldsymbol{\kappa}} \nabla F \right] + \frac{1}{p^2} \frac{\partial}{\partial p} \left[p^2 D \frac{\partial F}{\partial p} \right] - \mathrm{u} \cdot \nabla F + \frac{p}{3} \left(\nabla \cdot \mathrm{u} \right) \frac{\partial F}{\partial p} + S(\mathrm{r}, p, t)$$

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spatial diffusion (tensor)

$$\overleftrightarrow{\boldsymbol{\kappa}} = \begin{pmatrix} \kappa_\perp & 0 & 0 \\ 0 & \kappa_\perp & 0 \\ 0 & 0 & \kappa_\parallel \end{pmatrix}$$

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(scalar) momentum diffusion

$$D \sim \left(\langle \tfrac{1}{3}\kappa_\parallel + \tfrac{2}{3}\kappa_\perp \rangle\right)^{-1}$$

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$$D \sim \left(\langle \tfrac{1}{3}\kappa_\parallel + \tfrac{2}{3}\kappa_\perp \rangle\right)^{-1}$$

- $\kappa_\parallel$, $\kappa_\perp$ are mainly determined by the turbulence
- different ways to obtain these coefficients

Determining the Diffusion Coefficients

At least four methods:

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Phenomenolocically:

- fits to data (outdated...)

(Moraal, Potgieter)

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Analytically:

- quasilinear theory ($\kappa_{||}$)
- nonlinear guiding center theory ($\kappa_{\perp}$)

(Matthaeus, Shalchi)

Determining the Diffusion Coefficients

At least four methods:

Phenomenolocically:	**Analytically:**	**MHD simulations:**
• fits to data (outdated...)	• quasilinear theory ($\kappa_{\|\|}$) • nonlinear guiding center theory ($\kappa_{\perp}$)	• scale separation • self-consistent evolution
(Moraal, Potgieter)	*(Matthaeus, Shalchi)*	*(Adhikari,Wiengarten)*

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Phenomenolocically:	**Analytically:**	**MHD simulations:**	**Full-Orbit simul.**
• fits to data (outdated...)	• quasilinear theory ($\kappa_{\parallel}$) • nonlinear guiding center theory ($\kappa_{\perp}$)	• scale separation • self-consistent evolution	• prescribe turbulence • compute particle trajectories
(Moraal, Potgieter)	*(Matthaeus, Shalchi)*	*(Adhikari,Wiengarten)*	*($\rightarrow$ F. Effenberger)*

Determining the Diffusion Coefficients

At least four methods:

this talk	next talk
MHD simulations: • scale separation • self-consistent evolution *(Adhikari, Wiengarten)*	**Full-Orbit simul.** • prescribe turbulence • compute particle trajectories *($\rightarrow$ F. Effenberger)*

Outline of the Talk

- Introduction (✓)
- Large-Scale Plasma Transport
- One-Component Turbulence Transport
- Two-Component Turbulence Transport
- Application: Cosmic Ray Modulation
- Summary

Large-Scale Plasma Transport

Theoretical Framework I: Large-Scale Plasma Transport

MHD equations:

$$
\begin{aligned}
\partial_t \rho + \nabla \cdot (\rho \mathrm{v}) &= 0 \\
\partial_t (\rho \mathrm{v}) + \nabla \cdot \left[\rho \mathrm{v}\mathrm{v} + (p + |\mathrm{B}|^2/2)\, \mathbb{1} - \mathrm{B}\mathrm{B}\right] &= \mathrm{f} \\
\partial_t e + \nabla \cdot \left[(e + p + |\mathrm{B}|^2/2)\, \mathrm{v} - (\mathrm{v} \cdot \mathrm{B})\mathrm{B}\right] &= \mathrm{v} \cdot \mathrm{f} \\
\partial_t \mathrm{B} + \nabla \times \mathrm{E} &= 0
\end{aligned}
$$

$+$ **boundary conditions at 0.1 AU**

$=$ **'heliobase'**

(Zhao & Hoeksema [2010])

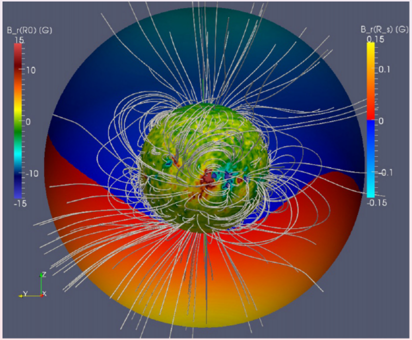
with

$$
\begin{aligned}
e &= \frac{\rho\, |\mathrm{v}|^2}{2} + \frac{|\mathrm{B}|^2}{2} + \frac{p}{\gamma - 1} \\
\mathrm{E} + \mathrm{v} \times \mathrm{B} &= 0 \\
\nabla \cdot \mathrm{B} &= 0\,,
\end{aligned}
$$

Theoretical Framework I: Large-Scale Plasma Transport

Inner boundary conditions:

(a) potential-field modelling (FDIPS model) for $1 - 2.5R_\odot$:

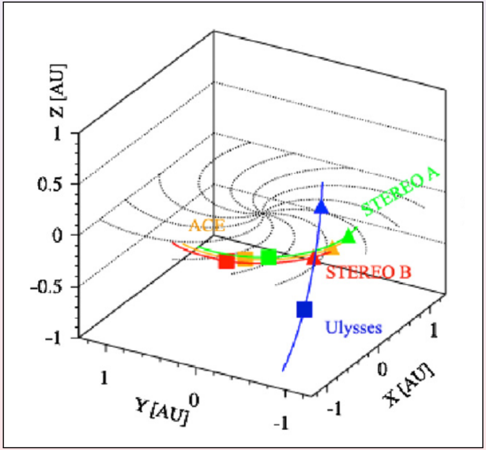


'source surface' at $2.5R_\odot$

Wiengarten et al. [2014]
(using GONG maps)

(b) WSA + PFSS + 'Detman et al. [2006,2011]'-construction of solar wind values for $2.5 - 21R_\odot$

Validation: Comparison to Spacecraft Data (Year 2007)



Dresing et al. [2009]

Reconstruction of measurements simultaneously obtained with three spacecraft:

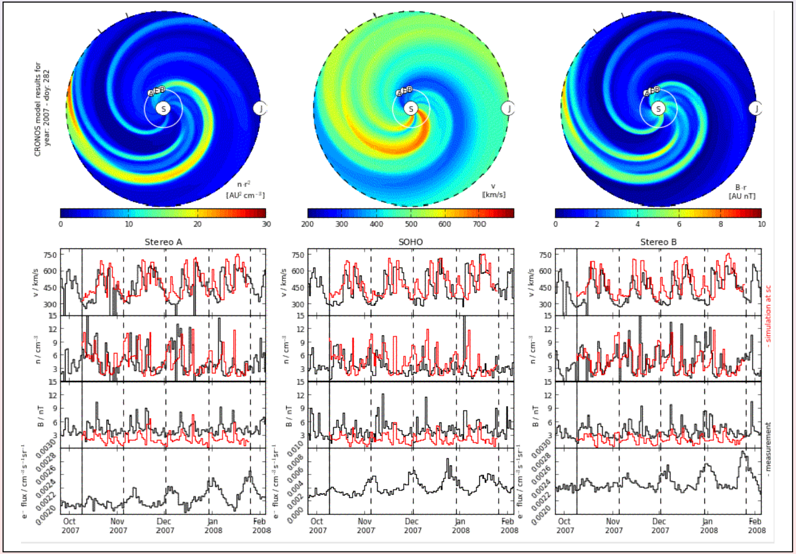
STEREO A at $1\ AU$

STEREO B at $1\ AU$

Ulysses at $\geq 1.2\ AU$

= 4-D data set

Validation: Comparison to Spacecraft Data (Year 2007)



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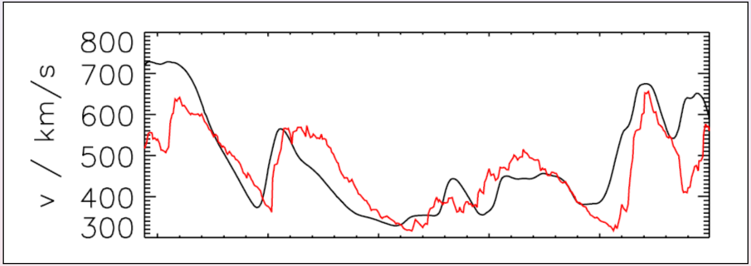

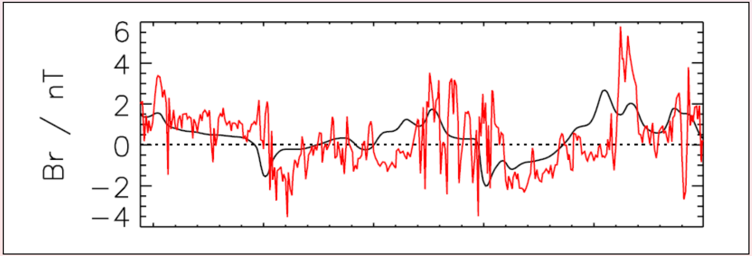

Detail:

speed

and

radial magnetic field component

at Ulysses spacecraft

One-Component Turbulence Transport

Reynolds Decomposition

Basic idea:

- split fluctuation quantities in average and fluctuating part:

$$\mathrm{v} = \mathrm{U} + \delta\mathrm{u} \quad ; \quad \mathrm{U} = \langle\mathrm{v}\rangle$$ and $$\mathrm{B} = \langle\mathrm{B}\rangle + \delta\mathrm{b}$$

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- limit analysis to incompressible fluctuations:

  $\delta\rho = 0$

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- re-write equations for fluctuating components in terms of *Elsasser variables*:

  $\mathrm{z}^{\pm} = \delta \mathrm{u} \pm \delta \mathrm{b} / \sqrt{\rho}$

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- re-write equations for fluctuating components in terms of *Elsasser variables*:

  $\mathrm{z}^{\pm} = \delta\mathrm{u} \pm \delta\mathrm{b}/\sqrt{\rho}$

- combine the latter into

$$
\begin{aligned}
Z^2 &:= \frac{\langle \mathrm{z}^+ \cdot \mathrm{z}^+ \rangle + \langle \mathrm{z}^- \cdot \mathrm{z}^- \rangle}{2} = \langle (\delta u)^2 \rangle + \langle (\delta b)^2/\rho \rangle && \text{norm. energy} \\
Z^2 \sigma_C &:= \frac{\langle \mathrm{z}^+ \cdot \mathrm{z}^+ \rangle - \langle \mathrm{z}^- \cdot \mathrm{z}^- \rangle}{2} = 2\langle \delta\mathrm{u} \cdot \delta\mathrm{b}/\sqrt{\rho} \rangle && \text{cross helicity} \\
Z^2 \sigma_D &:= \langle \mathrm{z}^+ \cdot \mathrm{z}^- \rangle = \langle (\delta u)^2 \rangle - \langle (\delta b)^2/\rho \rangle && \text{norm. energy difference}
\end{aligned}
$$

1-Component Turbulence Transport

- **evolution of large-scale solar wind get new terms due to fluctuations:**

$$\partial_t \rho + \nabla \cdot (\rho \mathrm{V}) = 0 \qquad ; \qquad \mathrm{V} = \mathrm{U} - \Omega \times \mathrm{r}$$

$$\partial_t (\rho \mathrm{U}) + \nabla \cdot \left[\rho \mathrm{V}\mathrm{U} + p + |\mathrm{B}|^2/2) \, \mathbb{1} - \mathrm{B}\mathrm{B} \qquad \right]$$

$$= -\rho(\mathrm{g} + \Omega \times \mathrm{U})$$

$$\partial_t e + \nabla \cdot \left[e\mathrm{V} + (p + |\mathrm{B}|^2/2) \, \mathrm{U} - (\mathrm{U} \cdot \mathrm{B})\mathrm{B} \right] + \mathrm{q}_H$$

$$= -\rho \mathrm{V} \cdot \mathrm{g}$$

$$\partial_t \mathrm{B} + \nabla \cdot (\mathrm{V}\mathrm{B} - \mathrm{B}\mathrm{V}) = 0$$

Wiengarten et al. [2015]

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$$\partial_t(\rho \mathrm{U}) + \nabla \cdot \left[\rho \mathrm{VU} + p + |\mathrm{B}|^2/2)\ \mathbb{1} - \mathrm{BB} + p_w \mathbb{1} - \sigma_D \rho Z^2/(2B^2)\mathrm{BB}\right]$$

$$= -\rho(\mathrm{g} + \Omega \times \mathrm{U}) \qquad ; \qquad p_w = (\sigma_D + 1)\rho Z^2/4$$

$$\partial_t e + \nabla \cdot \left[e\mathrm{V} + (p + |\mathrm{B}|^2/2)\ \mathrm{U} - (\mathrm{U} \cdot \mathrm{B})\mathrm{B}\right] + \mathrm{q}_H - \mathrm{V}_A \rho Z^2 \sigma_C/2$$

$$= -\rho \mathrm{V} \cdot \mathrm{g} - \mathrm{U} \cdot \nabla p_w - (\mathrm{V}_A \cdot \nabla \rho) Z^2 \sigma_C/2 + \rho Z^3 f^+(\sigma_C)/(2\lambda)$$

$$+ \mathrm{U} \cdot (\mathrm{B} \cdot \nabla)[\sigma_D \rho Z^2/(2B^2)\mathrm{B}] - \rho \mathrm{V}_A \cdot \nabla(Z^2 \sigma_C)$$

$$\partial_t \mathrm{B} + \nabla \cdot (\mathrm{VB} - \mathrm{BV}) = 0$$

Wiengarten et al. [2015]

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Wiengarten et al. [2015]

1-Component Turbulence Transport

- **evolution of fluctuations:**

$$\partial_t Z^2 + \nabla\cdot(\mathrm{U}Z^2 + \mathrm{V}_A Z^2\sigma_C) = \frac{Z^2(1-\sigma_D)}{2}\nabla\cdot\mathrm{U} + 2\mathrm{V}_A\cdot\nabla(Z^2\sigma_C) + Z^2\sigma_D\hat{\mathrm{B}}\cdot(\hat{\mathrm{B}}\cdot\nabla)\mathrm{U} - \frac{\alpha Z^3 f^+}{\lambda} + \langle \mathrm{z}^+\cdot\mathrm{S}^+\rangle + \langle \mathrm{z}^-\cdot\mathrm{S}^-\rangle$$

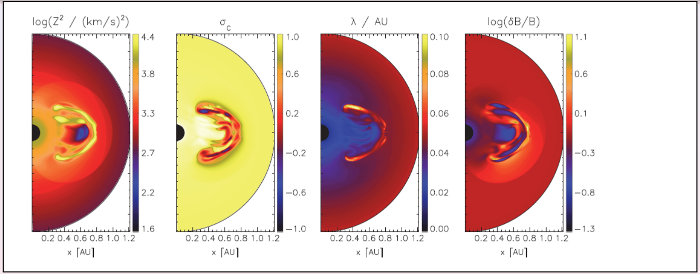
$$\partial_t(Z^2\sigma_C) + \nabla\cdot(\mathrm{U}Z^2\sigma_C + \mathrm{V}_A Z^2) = \frac{Z^2\sigma_C}{2}\nabla\cdot\mathrm{U} + 2\mathrm{V}_A\cdot\nabla Z^2 + Z^2\sigma_D\nabla\cdot\mathrm{V}_A - \frac{\alpha Z^3 f^-}{\lambda} + \langle \mathrm{z}^+\cdot\mathrm{S}^+\rangle - \langle \mathrm{z}^-\cdot\mathrm{S}^-\rangle$$

$$\partial_t(\rho\lambda) + \nabla\cdot(\mathrm{U}\rho\lambda) = \rho\beta\left[Zf^+ - \frac{\lambda}{\alpha Z^2}\left(\langle \mathrm{z}^+\cdot\mathrm{S}^+\rangle(1-\sigma_C) + \langle \mathrm{z}^-\cdot\mathrm{S}^-\rangle(1+\sigma_C)\right)\right]$$

Wiengarten et al. [2015]

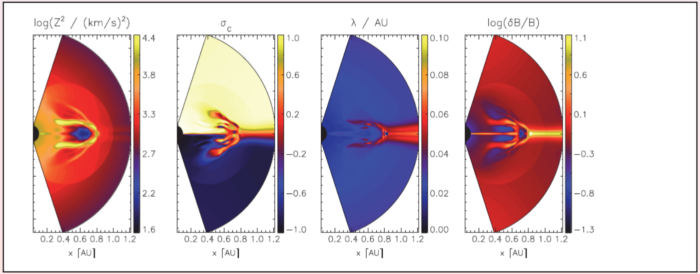
($S^{\pm}\,\hat{=}$ (pick-up ion) source terms; α,β Karman-Taylor constants; $f^{\pm} = f^{\pm}(\sigma_C)$)

Results: One-Component Turbulence Evolution



CME-Disturbed
Solar Wind:

near ecliptic

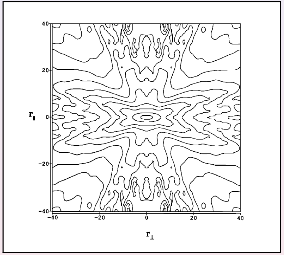


meridional plane

Wiengarten et al. [2015]

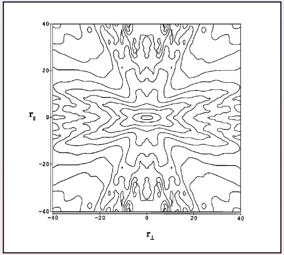
The Need for a Two-Component Turbulence Model

solar wind turbulence is anisotropic:



Matthaeus et al. [1990]: $R(\mathrm{x}) = \langle \mathrm{B}(\mathrm{r})\mathrm{B}(\mathrm{r}+\mathrm{x})\rangle$ *(two-point correlation function)*

The Need for a Two-Component Turbulence Model



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transport process	*1-comp. model*	
parallel diffusion $\kappa_\parallel$	$\sim B^2/\delta B^2$	
perpendicular diffusion $\kappa_\perp$	$\sim \delta B^2/B^2$	
drifts κ_A	$= \kappa_A\left(\delta B^2/B^2\right)$	
momentum diffusion D_{pp}	$= D_{pp}\left(\delta B^2/B^2\right)$	

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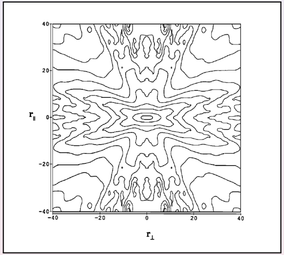

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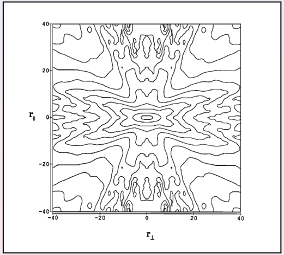

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transport process	*1-comp. model*	*2-comp. model*
parallel diffusion $\kappa_\parallel$	$\sim B^2/\delta B^2$	$\sim B^2/\delta B^2_{sl}$
perpendicular diffusion $\kappa_\perp$	$\sim \delta B^2/B^2$	$\sim \delta B^2_{2D}/B^2$
drifts κ_A	$= \kappa_A\left(\delta B^2/B^2\right)$	$= \kappa_A\left(\delta B^2_{sl}/B^2, \delta B^2_{2D}/B^2\right)$
momentum diffusion D_{pp}	$= D_{pp}\left(\delta B^2/B^2\right)$	$= D_{pp}\left(\delta B^2_{sl}/B^2, \delta B^2_{2D}/B^2\right)$

Two-Component Turbulence Transport

2-Component Turbulence Transport

quasi-2D fluctuations	$\longleftarrow$ distinction of $\longrightarrow$ $\boldsymbol{z}^{\pm}(\boldsymbol{r},\boldsymbol{x}) = \boldsymbol{q}^{\pm} + \boldsymbol{w}^{\pm}$	**wave-like fluctuations**
$Z_{\pm}^2 = \langle \boldsymbol{q}_{\pm} \cdot \boldsymbol{q}_{\pm} \rangle$	Elsasser 'energies'	$W_{\pm}^2 = \langle \boldsymbol{w}_{\pm} \cdot \boldsymbol{w}_{\pm} \rangle$
$2Z^2 = Z_{+}^2 + Z_{-}^2$	total 'energies'	$2W^2 = W_{+}^2 + W_{-}^2$
$2H_c^z = Z_{+}^2 - Z_{-}^2$	cross helicities	$2H_c^w = W_{+}^2 - W_{-}^2$
$\sigma_{c,z} = \dfrac{Z_{+}^2 - Z_{-}^2}{Z_{+}^2 + Z_{-}^2}$	normalized cross helicities	$\sigma_{c,w} = \dfrac{W_{+}^2 - W_{-}^2}{W_{+}^2 + W_{-}^2}$
$\sigma_D^z = \dfrac{\langle \boldsymbol{q}_{+} \cdot \boldsymbol{q}_{-} \rangle}{Z^2}$	normalized energy differences	$\sigma_D^w = \dfrac{\langle \boldsymbol{w}_{+} \cdot \boldsymbol{w}_{-} \rangle}{W^2}$

$$\delta B_{\mathrm{2D}}^2 = \frac{\mu_0 \rho}{r_A + 1} Z^2 \quad ; \quad \delta B_{\mathrm{sl}}^2 = \frac{\mu_0 \rho}{r_A + 1} W^2 \quad ; \quad r_A = \frac{1 + \sigma_D^{z,w}}{1 - \sigma_D^{z,w}}$$

2-Component Turbulence Transport

$$\frac{\partial Z^2}{\partial t} = -\nabla\cdot(\boldsymbol{V}Z^2+\boldsymbol{V}_A H_c^z)+2\boldsymbol{V}_A\cdot\nabla H_c^z+\frac{1}{2}(\nabla\cdot\boldsymbol{U})Z^2-\sigma_D^z Z^2\left[\frac{\nabla\cdot\boldsymbol{U}}{2}-\hat{\boldsymbol{B}}\cdot(\hat{\boldsymbol{B}}\cdot\nabla)\boldsymbol{U}\right]$$
$$-\alpha_z\left[\frac{Z^3}{\ell}f_{zz}^+ + \frac{2WZ^2}{\ell}\frac{f_{zw}^+}{1+Z/W}\right]+\alpha_z X^+ + \frac{Z^2}{r}C_{\mathrm{sh}}^Z|\boldsymbol{U}|,$$

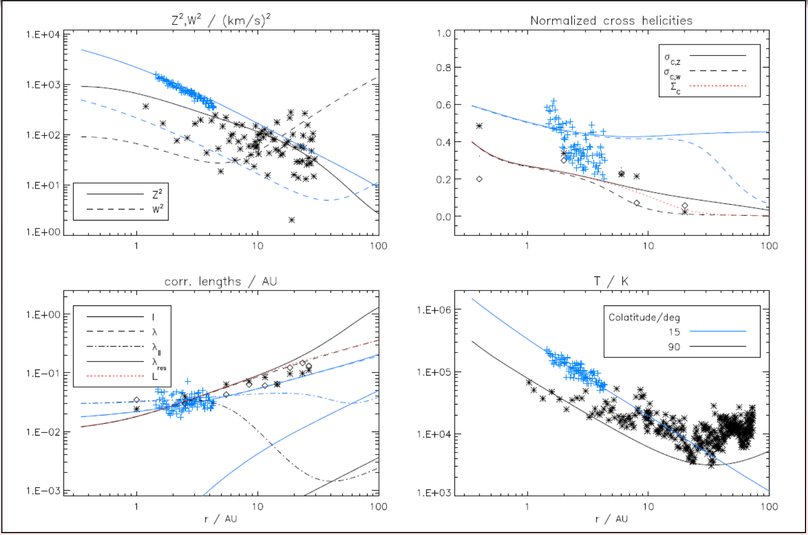
$$\frac{\partial W^2}{\partial t} = -\nabla\cdot(\boldsymbol{V}W^2+\boldsymbol{V}_A H_c^w)+2\boldsymbol{V}_A\cdot\nabla H_c^w+\frac{1}{2}(\nabla\cdot\boldsymbol{U})W^2-\sigma_D^w W^2\left[\frac{\nabla\cdot\boldsymbol{U}}{2}-\hat{\boldsymbol{B}}\cdot(\hat{\boldsymbol{B}}\cdot\nabla)\boldsymbol{U}\right]$$
$$-\alpha_w\left[\frac{2W^2Z}{\lambda}\frac{f_{wz}^+}{1+\lambda/\ell}+\frac{2W^4\lambda_\parallel}{\lambda^2 V_A}(1-\sigma_{c,w}^2)\right]-\alpha_z X^+ + \frac{W^2}{r}C_{\mathrm{sh}}^W|\boldsymbol{U}| + \dot{E}_{\mathrm{PI}},$$

$$\frac{\partial H_c^z}{\partial t} = -\nabla\cdot(\boldsymbol{V}H_c^z+\boldsymbol{V}_A Z^2)+2\boldsymbol{V}_A\cdot\nabla Z^2+\frac{1}{2}(\nabla\cdot\boldsymbol{U})H_c^z+\sigma_D^z Z^2\left[\nabla\cdot\boldsymbol{V}_A+\frac{\hat{\boldsymbol{B}}\cdot(\hat{\boldsymbol{B}}\cdot\nabla)\boldsymbol{B}}{\sqrt{4\pi\rho}}\right]$$
$$-\alpha_z\left[\frac{Z^3}{\ell}f_{zz}^- + \frac{2WZ^2}{\ell}\frac{f_{zw}^-}{1+Z/W}\right]+\alpha_z X^-,$$

$$\frac{\partial H_c^w}{\partial t} = -\nabla\cdot(\boldsymbol{V}H_c^w+\boldsymbol{V}_A W^2)+2\boldsymbol{V}_A\cdot\nabla W^2+\frac{1}{2}(\nabla\cdot\boldsymbol{U})H_c^w+\sigma_D^w W^2\left[\nabla\cdot\boldsymbol{V}_A+\frac{\hat{\boldsymbol{B}}\cdot(\hat{\boldsymbol{B}}\cdot\nabla)\boldsymbol{B}}{\sqrt{4\pi\rho}}\right]$$
$$-\alpha_w\left[\frac{2W^2Z}{\lambda}\frac{f_{wz}^-}{1+\lambda/\ell}\right]-\alpha_z X^-,$$

$$\frac{\partial\ell}{\partial t}=\ldots\quad;\quad\frac{\partial\lambda}{\partial t}=\ldots\quad;\quad\frac{\partial\lambda_\parallel}{\partial t}=\ldots$$

Validation: Comparison to Spacecraft Data

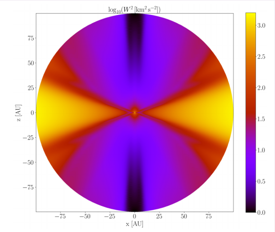


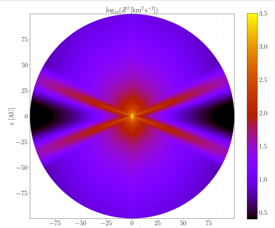
Wiengarten et al. [2016]:

- blue/black lines = colatitudes of 15°/90°
- blue/black symbols = Ulysses/Voyager 2

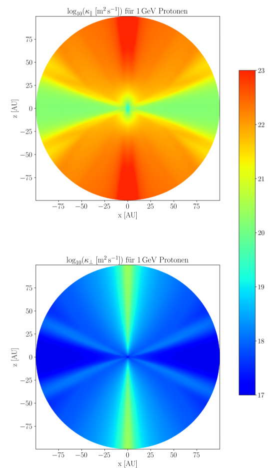
Application: Ab-initio modulation of CR proton spectra

wave-like/quasi-2D fluctuations (top/botton):



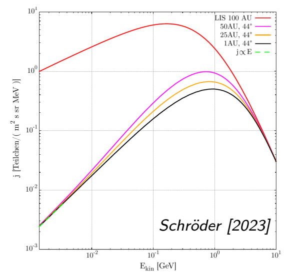


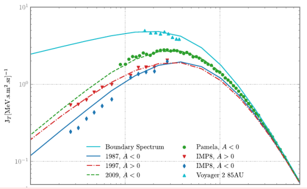
para./perp. diffusion coefficient at 1 GeV:



$\Longrightarrow$

modulated CR energy spectra





Moloto et al. [2018]

Summary

Results:

- MHD modelling of turbulence transport:
  - *self-consistent computation of large- and small-scale quantities*
  - *one- and two-component turbulence transport*
- kinetic modelling of energetic particle transport:
  - *analytical theories provide transport (diffusion, drift) coefficients*
  - *solution of kinetic transport equations (Parker & focused-transport eq.)*
  - *confrontation with observed energy spectra in the heliosphere*

Improvements (see following talk by Frederic Effenberger):

- prescribe turbulence with given properties (structure functions, intermittency)
- compute transport coefficients from full-orbit simulations (particle trajectories)