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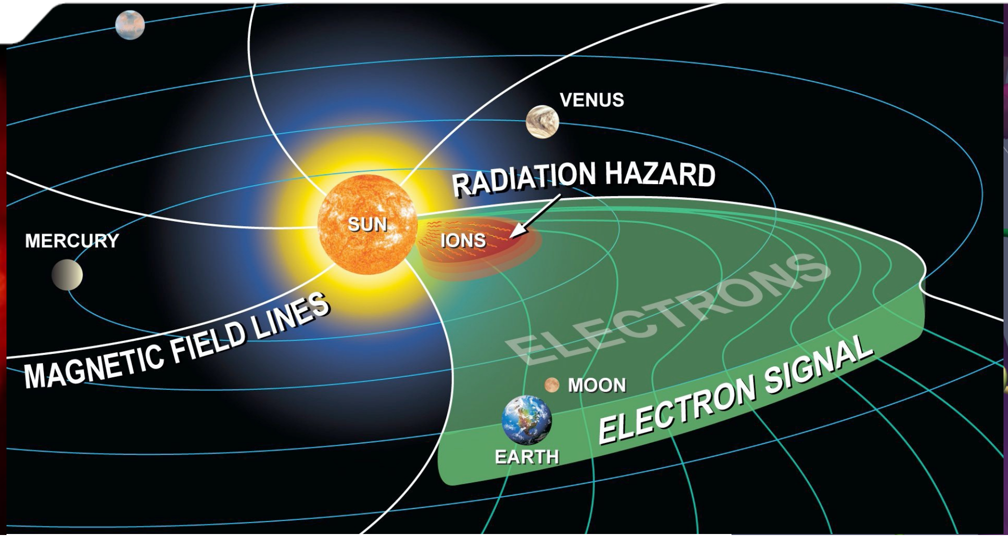
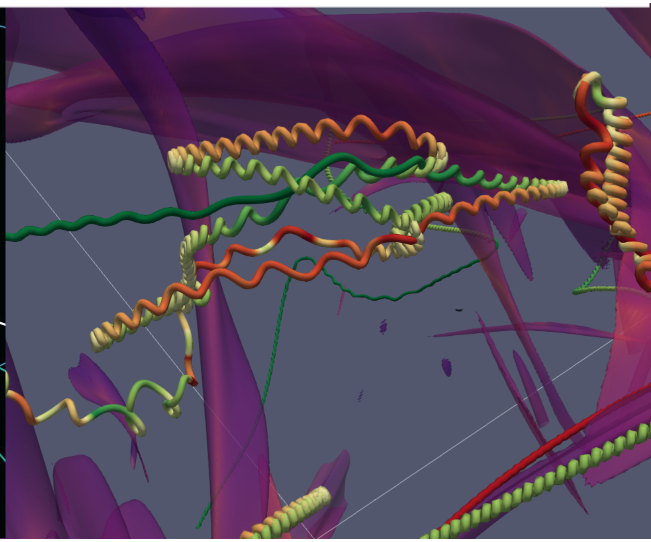




Investigating Charged Particle Transport in Non-Gaussian Magnetic Turbulence Models

CRPropa Workshop, September 2023

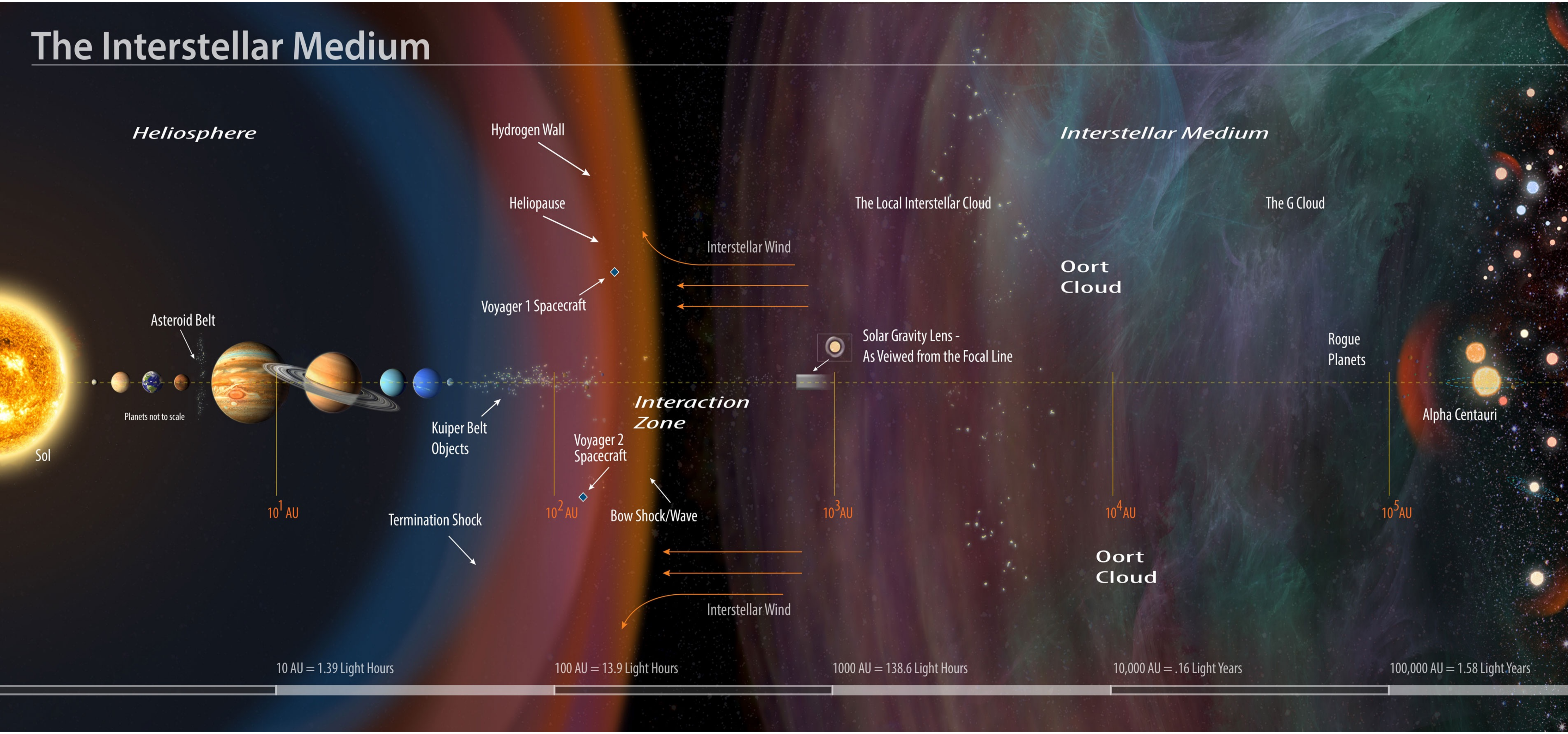
Frederic Effenberger, Jeremiah Lübke, Horst Fichtner, Rainer Grauer
Ruhr-University Bochum, Germany

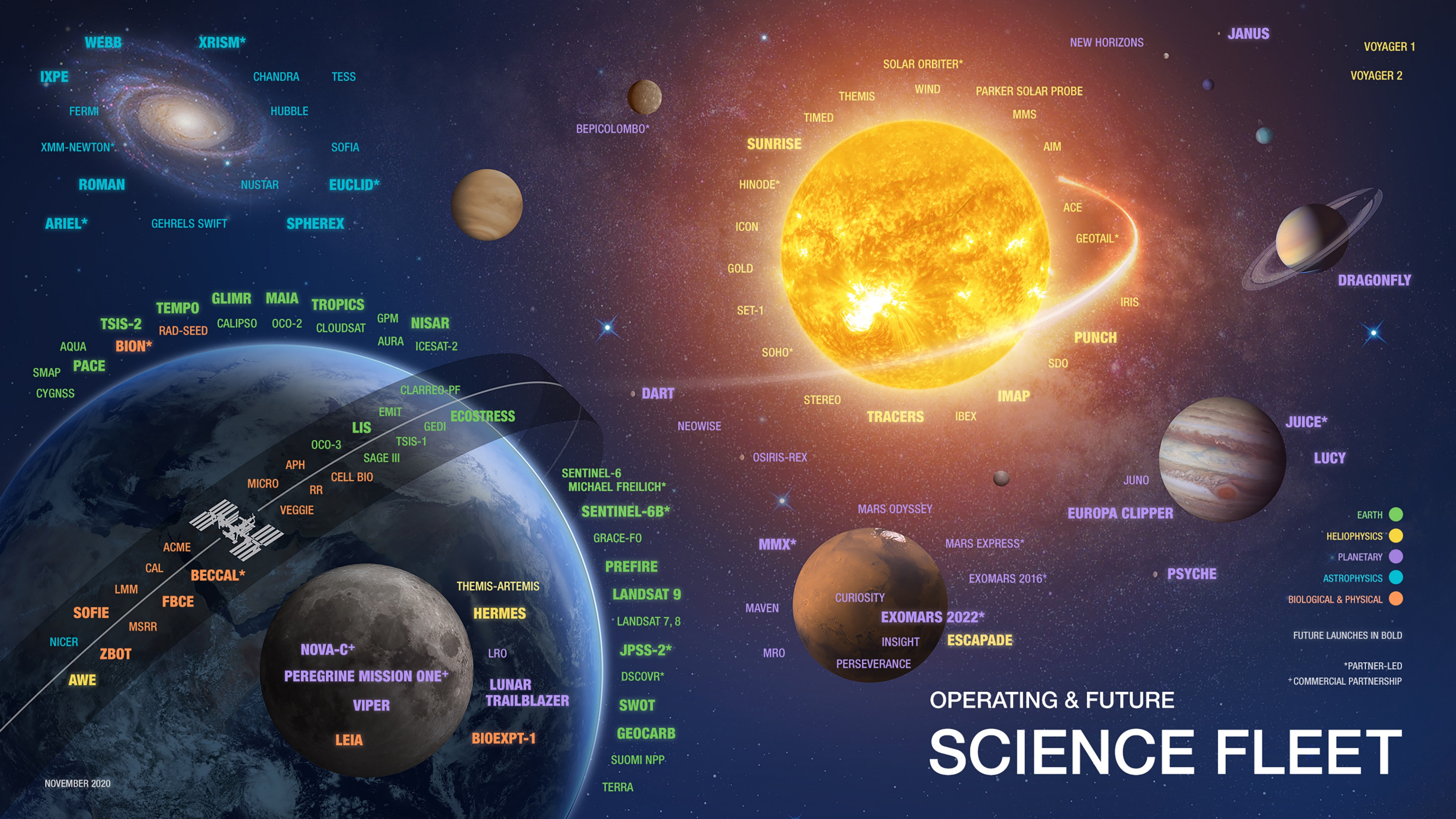
Overview

RUB

- Introduction
- Energetic particle transport
- Synthetic turbulence modelling
- Particles in turbulence
- Summary

The Heliosphere & ISM





WEBB
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XMM-NEWTON*
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EUCLID*
ARIEL*
GEHRELS SWIFT
SPHEREX
TEMPO
GLIMR
MAIA
TROPICS
TSIS-2
RAD-SEED
CALIPSO
OCO-2
CLOUDSAT
GPM
NISAR
AQUA
BION*
AURA
ICESAT-2
SMAP
PACE
CYGNSS
CLARREO-PF
EMIT
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OCO-3
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HERMES
NOVA-C+
LRO
PEREGRINE MISSION ONE+
LUNAR TRAILBLAZER
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SENTINEL-6 MICHAEL FREILICH*
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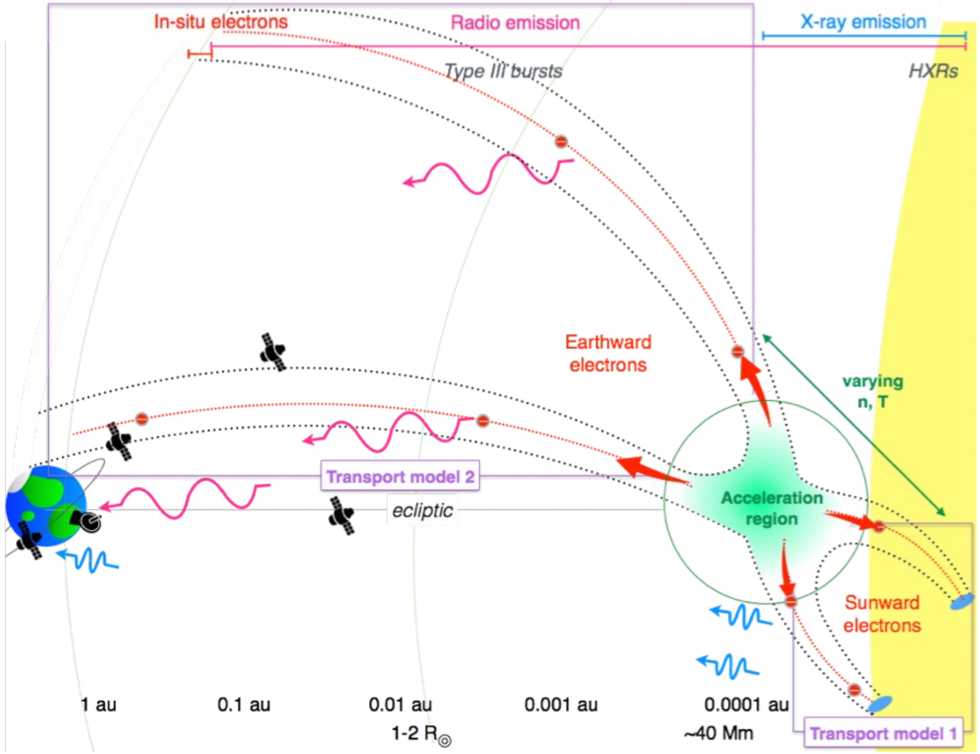
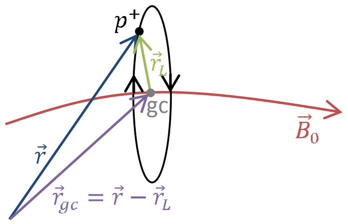
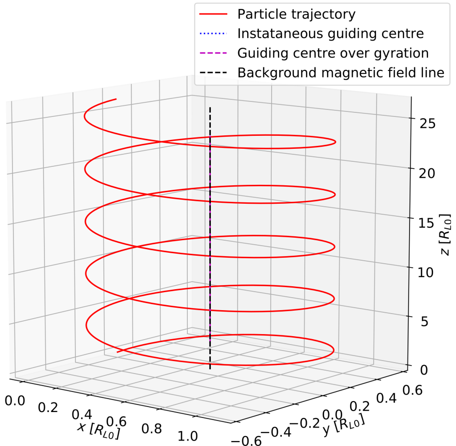
Particle Transport in the Heliosphere

How do particles travel from Sun to Earth?



ISSI Team Jeffrey & Effenberger

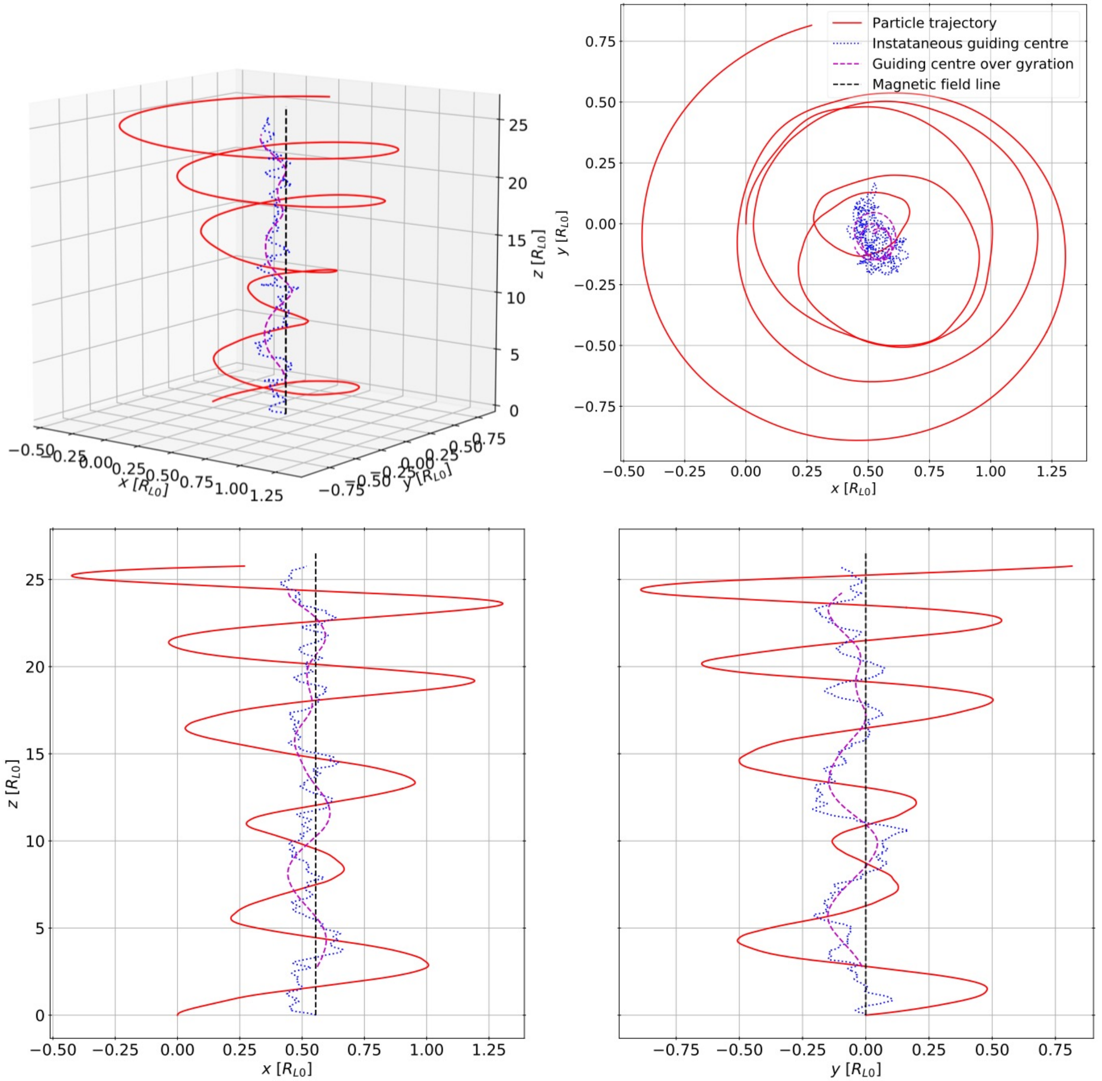




A Primer on Focused Solar Energetic Particle Transport

Basic physics and recent modelling results

Jabus van den Berg · Du Toit Strauss · Frederic Effenberger



1D Solar Energetic Particle Modelling

Basic 1D focused transport equation

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial s}\left[\mu v f\right] + \frac{\partial}{\partial \mu}\left[\frac{(1-\mu^2)v}{2L(s)} f\right] = \frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu}\right]$$

$$D_{\mu\mu}^{\mathrm{QLT}}(\mu) = D_0(1-\mu^2)|\mu|^{q-1}$$

If S and M represents the stochastic variables corresponding to s and μ, respectively, then the two first order SDEs equivalent to the Roelof equation (Eq. 9) are

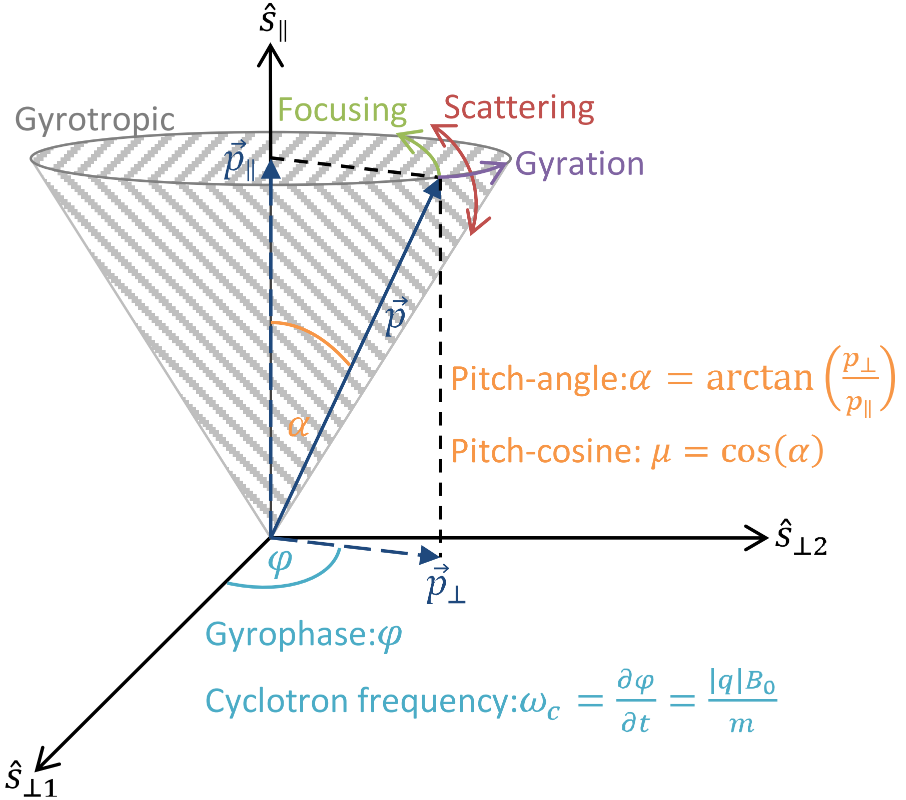
$$\mathrm{d}S = \mu v\,\mathrm{d}t$$

$$\mathrm{d}M = \left[\frac{(1-\mu^2)v}{2L(s)} + \frac{\partial D_{\mu\mu}}{\partial \mu}\right]\mathrm{d}t + \sqrt{2D_{\mu\mu}}\mathrm{d}W_\mu(t),$$

where $\mathrm{d}W_\mu(t)$ is a Wiener process. These SDEs are solved using the Euler-Maruyama scheme,

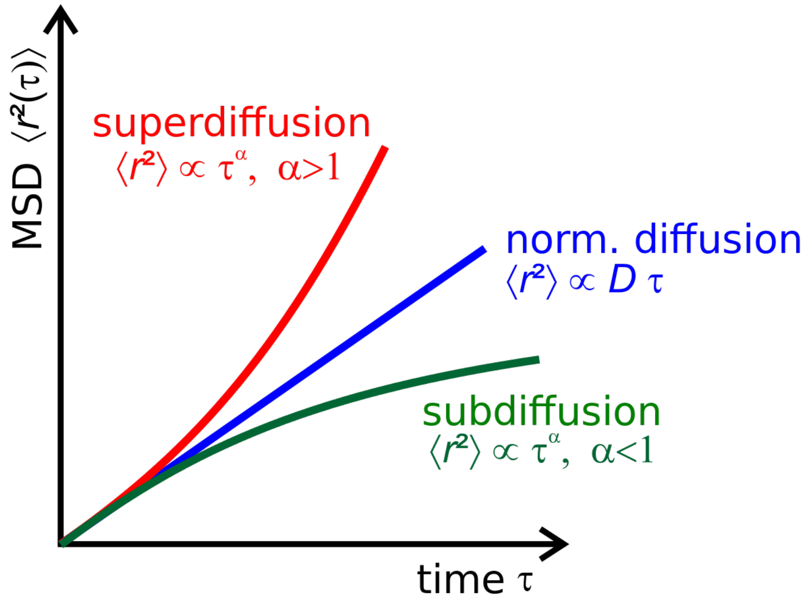
$$S(t+\Delta t) = S(t) + M(t)v\Delta t$$

$$M(t+\Delta t) = M(t) + \left[\frac{(1-M^2(t))v}{2L(S(t))} + \left.\frac{\partial D_{\mu\mu}}{\partial \mu}\right|_{\mu=M(t)}\right]\Delta t + \sqrt{2D_{\mu\mu}(M(t))\Delta t}\Lambda,$$

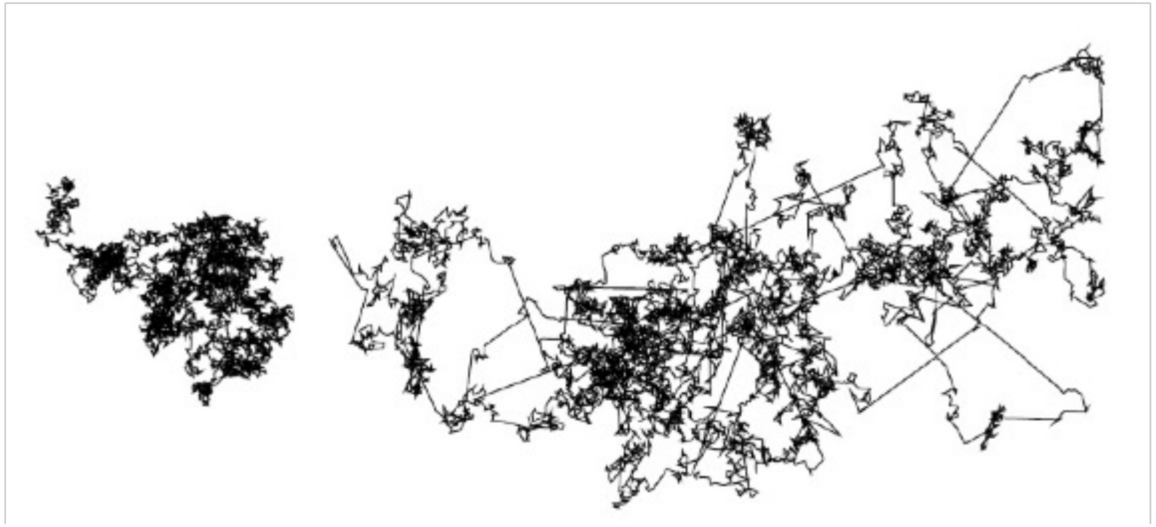


Anomalous Diffusion

Gaussian	Anomalous	
$(\Delta x)^2 \propto t$	$(\Delta x)^2 \propto t^\zeta$	Superdiffusion: $1 < \zeta < 2$ Subdiffusion: $0 < \zeta < 1$



Subdiffusion
(extended
waiting times)



Superdiffusion
(Lévy-Flights)

Anomalous Diffusion

Gaussian

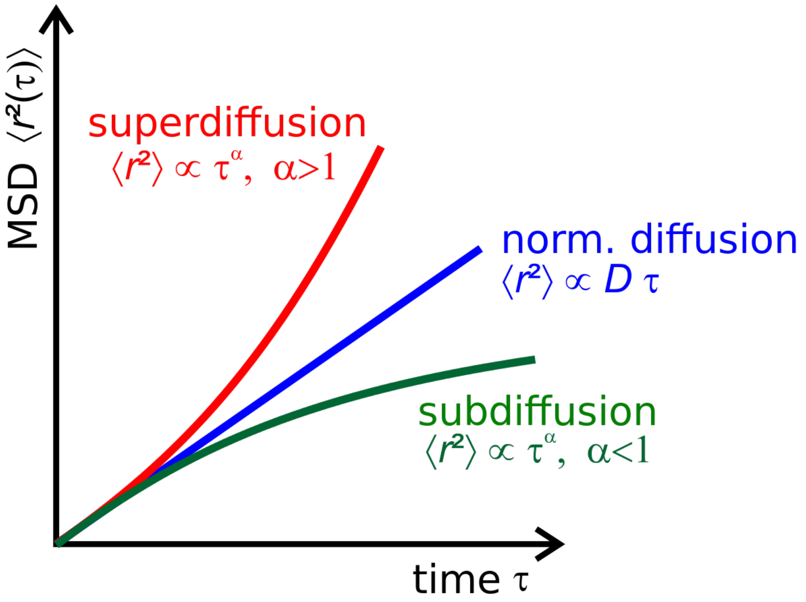
$$(\Delta x)^2 \propto t$$

Anomalous

$$(\Delta x)^2 \propto t^\zeta$$

Superdiffusion: $1 < \zeta < 2$

Subdiffusion: $0 < \zeta < 1$



Idea: Generalize Diffusion Equation to non-integer derivatives

$$\frac{\partial f}{\partial t} = \kappa \frac{\partial^\alpha f}{\partial |x|^\alpha} + a \frac{\partial f}{\partial x} + \delta(x)$$

Using symmetric fractional Riesz derivative (generalized Laplacian)

$$\frac{\partial^\alpha f(x,t)}{\partial |x|^\alpha} = \frac{1}{\pi} \sin\left(\frac{\pi}{2}\alpha\right) \Gamma(1+\alpha) \times \int_0^\infty \frac{f(x+\xi) - 2f(x) + f(x-\xi)}{\xi^{1+\alpha}} d\xi$$

The Complexity of Physics Based CR/SEP Models

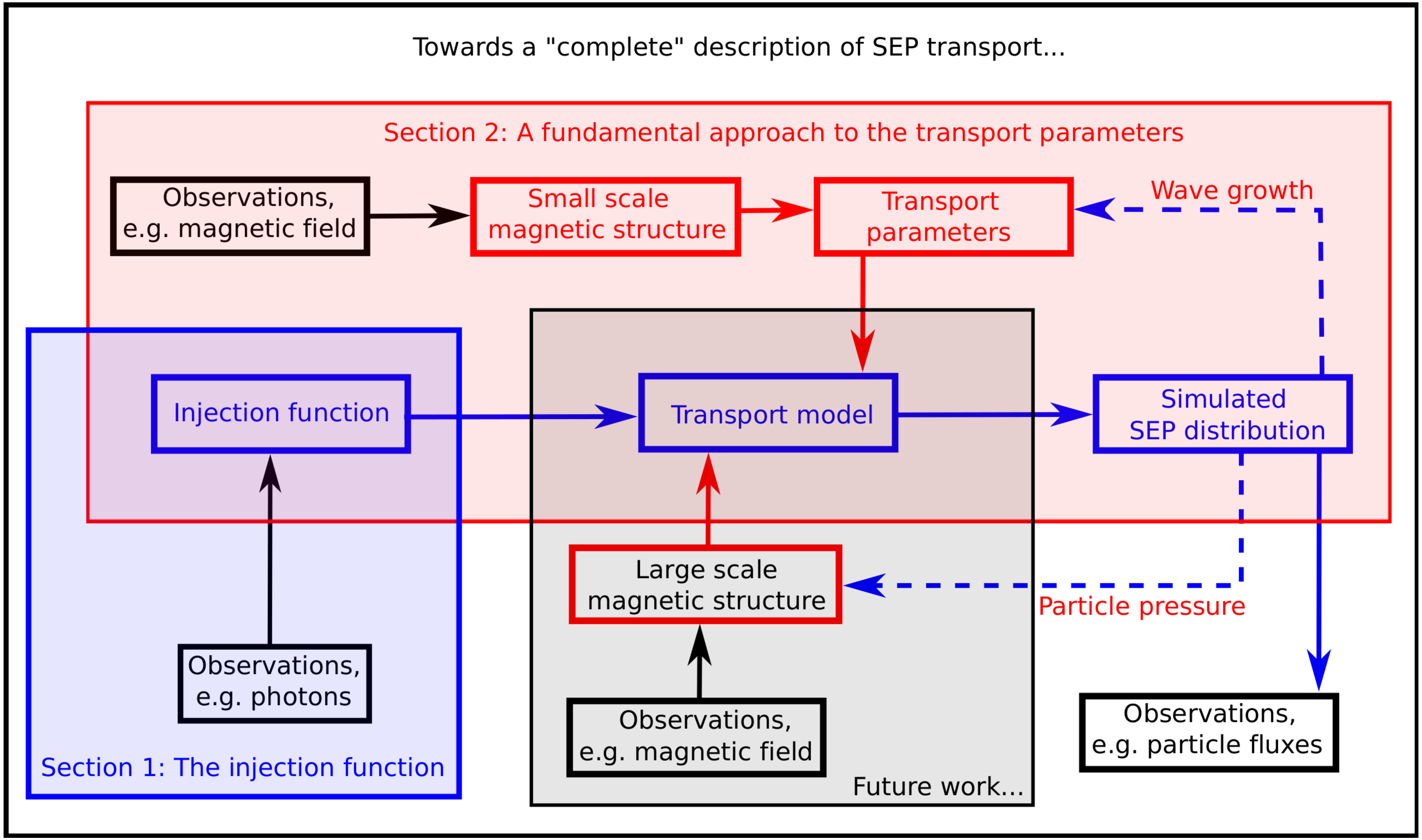
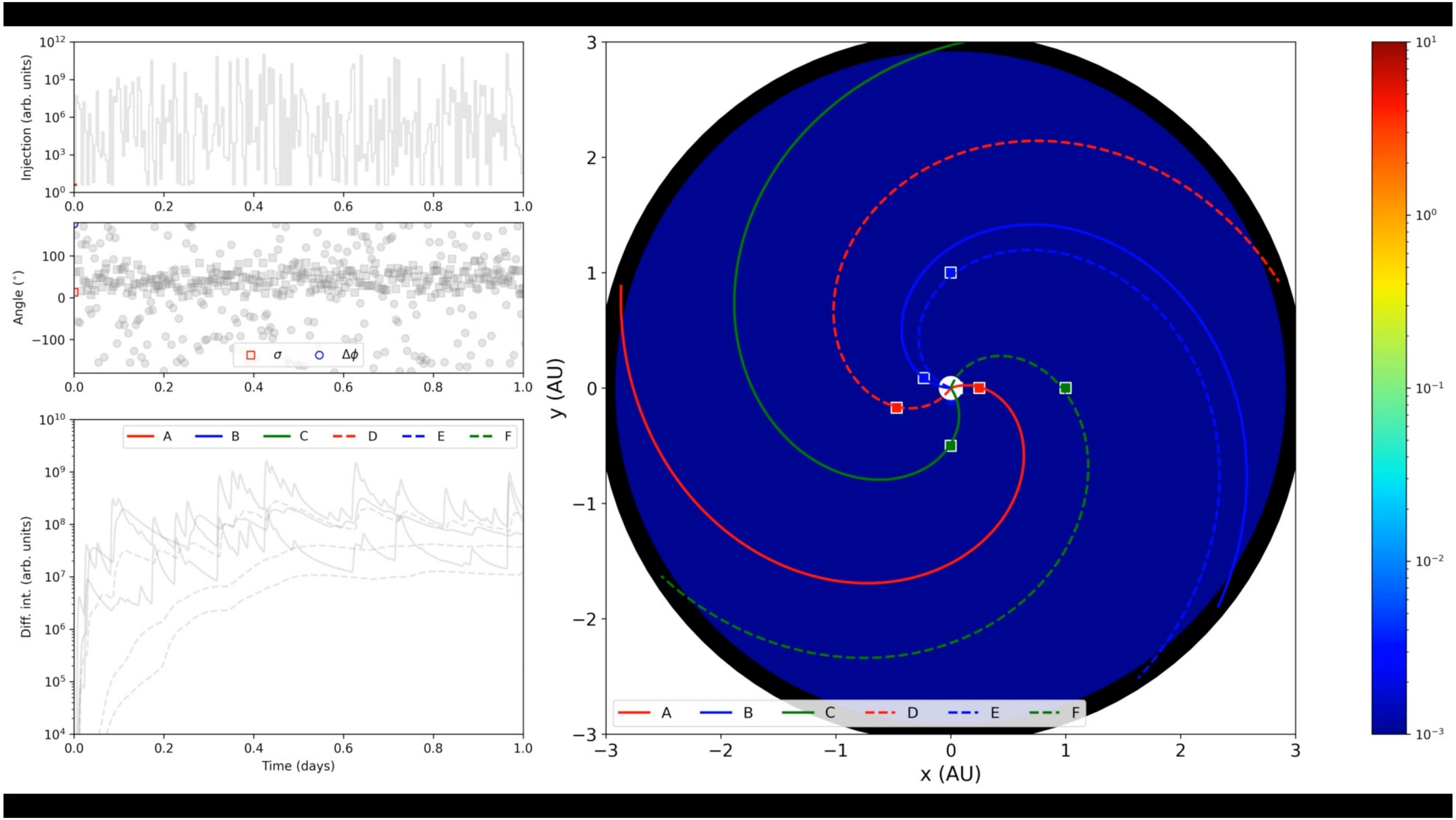


Figure by RDT Strauss

Towards SEP nowcasting

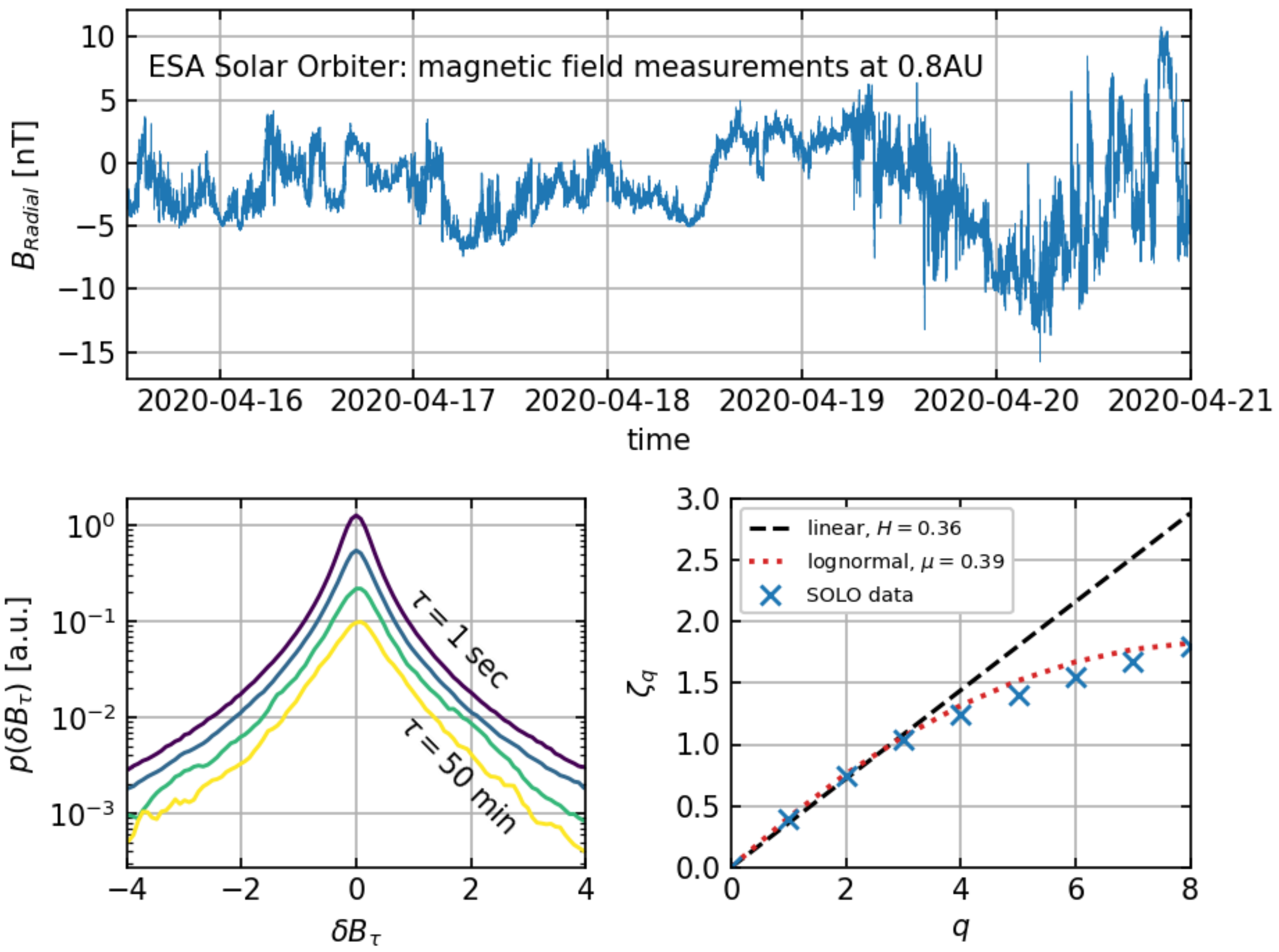
Example of multiple injections (100 keV Electrons)



Observations of Intermittency in the SW

Turbulence such as in the Solar Wind or Interstellar Medium is highly **intermittent**

Important 2-point statistics: **Increments** $\delta B_\tau = B(t+\tau) - B(t)$, **Structure functions** $\langle \delta B_\tau^q \rangle \propto \tau^{\zeta_q}$



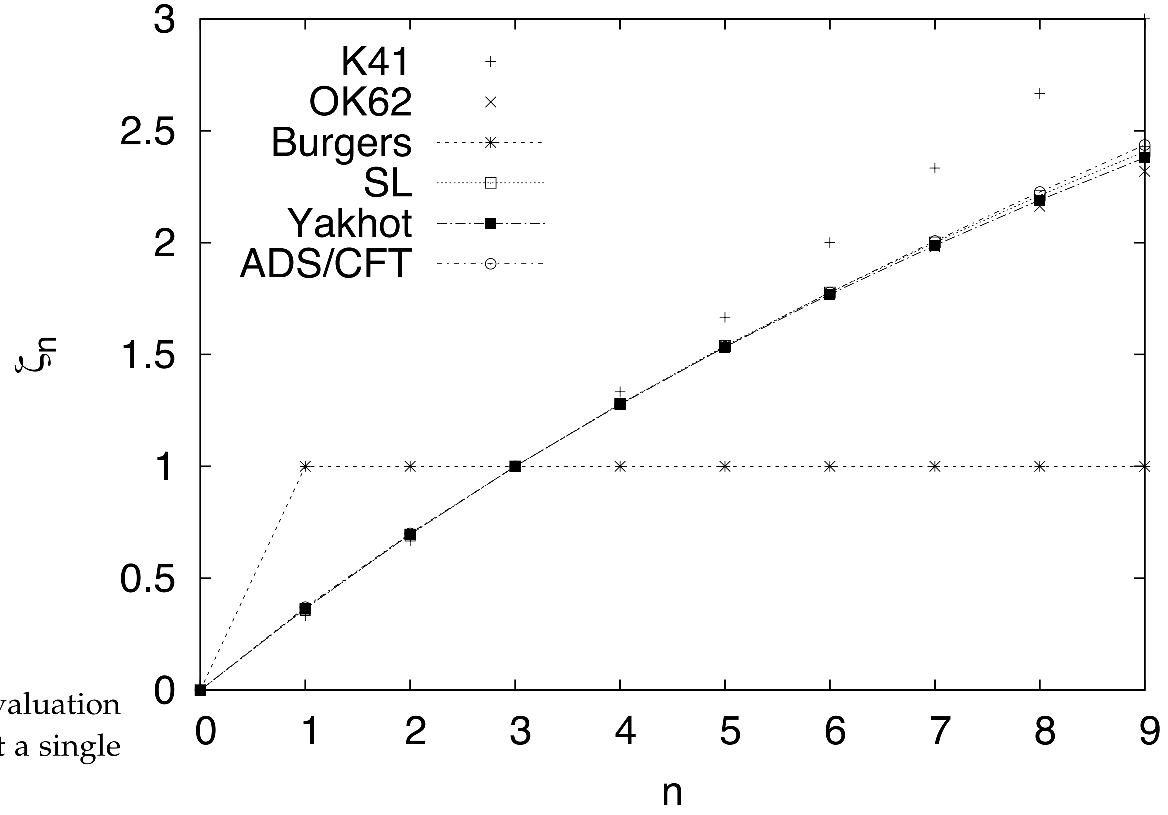
[J. Lübke]

(Hydro) Intermittency Models

Longitudinal velocity increments:

$$\delta_r v(\mathbf{x},t) = (\mathbf{u}(\mathbf{x}+\mathbf{r},t) - \mathbf{u}(\mathbf{x},t)) \cdot \frac{\mathbf{r}}{r}$$

$$\langle (\delta_r v)^n \rangle \sim |r|^{\zeta_n}$$



(i.) Kolmogorov's theory K41:

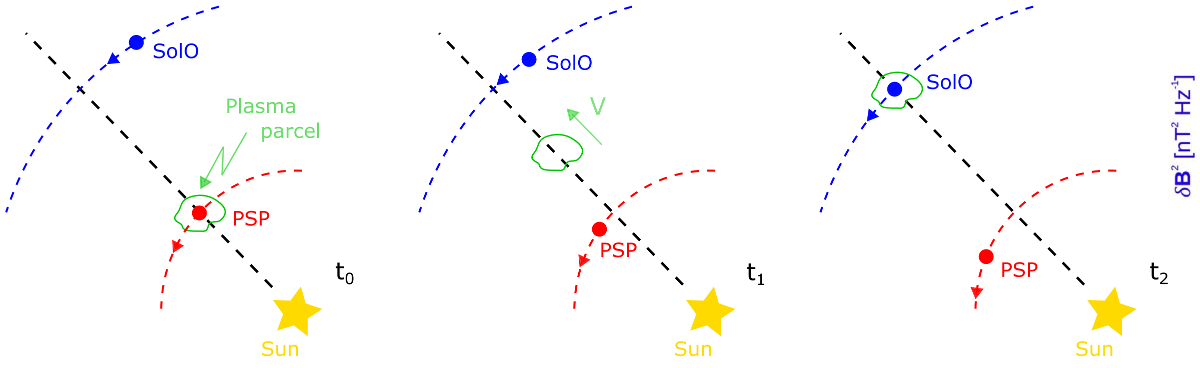
The monofractal K41 phenomenology [3] states that $\langle (\delta_r v)^n \rangle = C_n \langle \varepsilon \rangle^{n/3} r^{n/3}$ and an evaluation of the reduced Kramers–Moyal coefficients (17) suggests that it can be reproduced by just a single Kramers–Moyal coefficient

$$K_n = \begin{cases} 1/3 & \text{for } n \leq 1, \\ 0 & \text{for } n > 1. \end{cases} \tag{19}$$

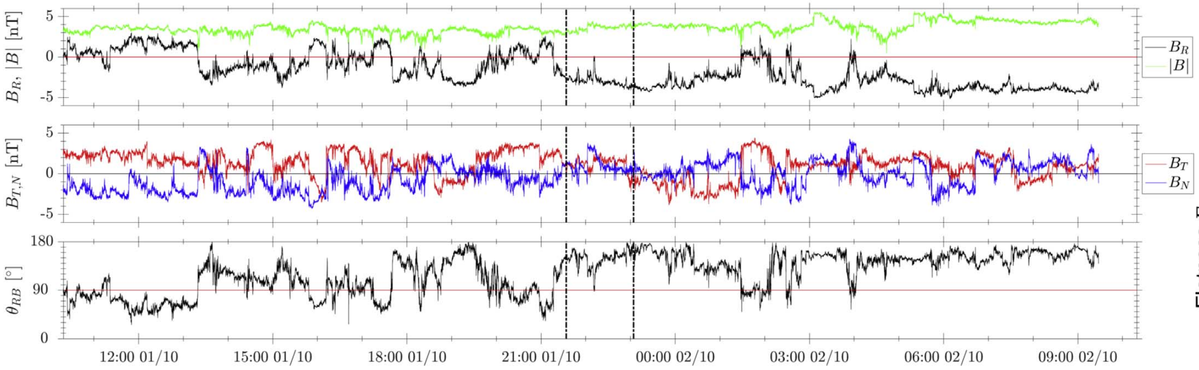
(ii.) Oboukhov–Kolmogorov theory OK62:

$$\langle (\delta_r v)^n \rangle = C_n \langle \varepsilon \rangle^{\frac{n}{3}} r^{\frac{n}{3}} \left(\frac{r}{L}\right)^{-\frac{n(n-3)\mu}{18}}$$

Observations of Intermittency in the SW



SolO Observations



[Telloni et al. 2021]

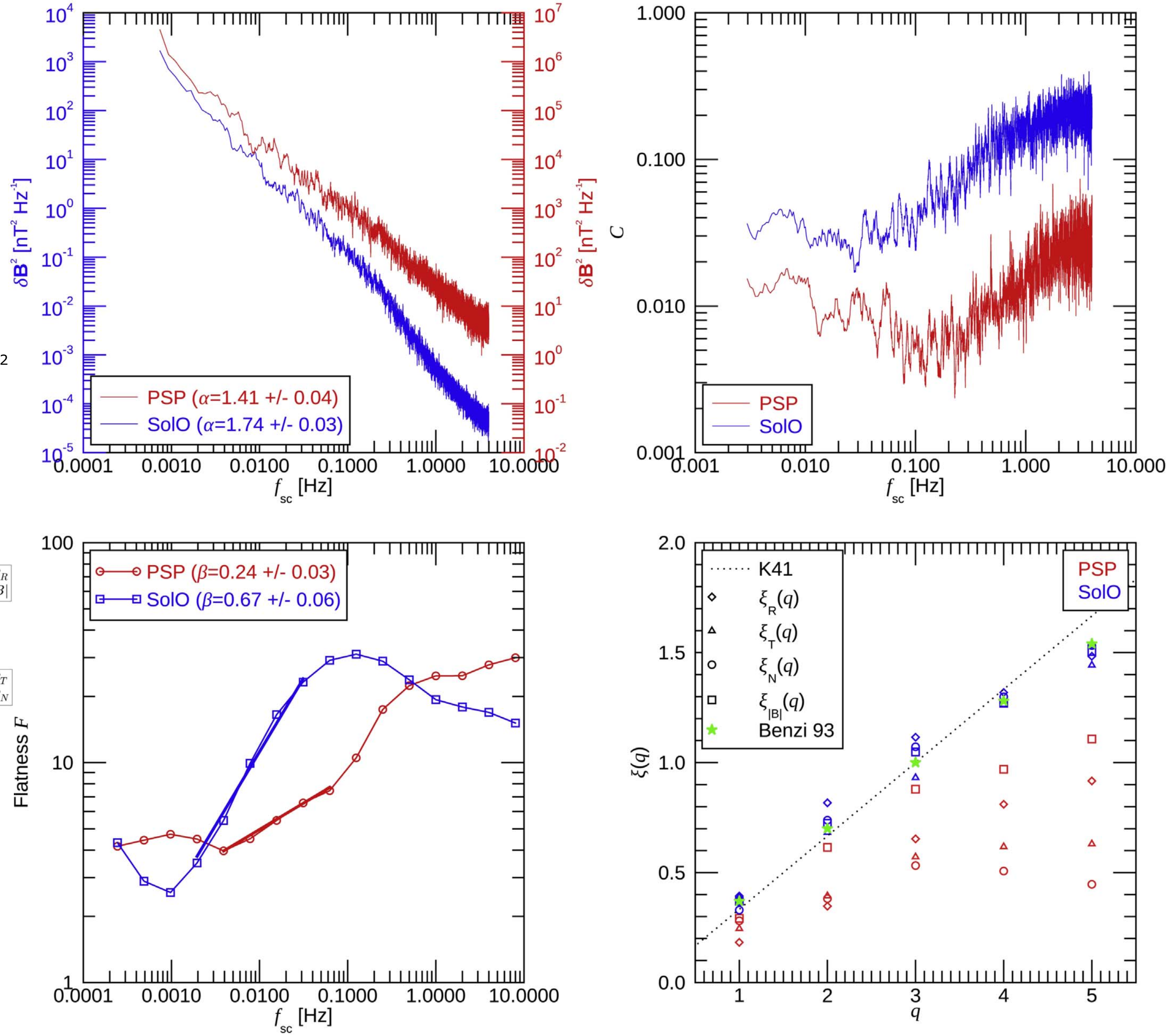


Figure 5. Trace of the magnetic spectral matrix $\delta \boldsymbol{B}^2$ (top left), magnetic compressibility spectrum C (top right), and flatness $\mathcal{F}$ as a function of the spacecraft frequency (bottom left) for PSP (red) and SolO (blue) radially aligned intervals. Power-law fits are displayed as thick lines, while relative scaling exponents are reported in the legends. Bottom right: comparison of the scaling exponents ξ_q of the qth-order structure functions for PSP (red) and SolO (blue) magnetic field RTN components and magnitude (represented by different symbols as reported in the legend). As a comparison, exponents for velocity fluctuations in the inertial range of hydrodynamic turbulence (green stars; Benzi et al. 1993) and the classical K41 ($q/3$) Kolmogorov law (dotted line; Kolmogorov 1941) are also displayed.

Full-Orbit Simulation

RUB

Solving the Newton-Lorentz Equations for many charged (Test) Particles

Dimensionless Time: $T = \Omega t$

Dimensionless Rigidity: $\mathbf{R} := \mathbf{v}/(\Omega \ell)$

With gyro frequency Ω and typical length scale (bendover scale) ℓ

$$\frac{d}{dT}\mathbf{R} = \mathbf{R} \times \left(\mathbf{e}_z + \frac{\delta \mathbf{B}(\mathbf{x})}{B_0} \right)$$

$$\frac{d}{dT}\frac{\mathbf{x}}{\ell} = \mathbf{R}.$$

In the literature, often RK methods are used.
We use the Boris Push method for energy conservation:

$$\frac{\mathbf{v}^{n+1/2} - \mathbf{v}^{n-1/2}}{\Delta t} = \frac{q}{m}\left[\mathbf{E} + \frac{\mathbf{v}^{n+1/2} + \mathbf{v}^{n-1/2}}{2} \times \mathbf{B}\right]$$

$$\mathbf{v}^{n-1/2} = \mathbf{v}^{-} - \frac{q\mathbf{E}}{m}\frac{\Delta t}{2} \qquad \mathbf{v}^{n+1/2} = \mathbf{v}^{+} + \frac{q\mathbf{E}}{m}\frac{\Delta t}{2}$$

$$\frac{\mathbf{v}^{+} - \mathbf{v}^{-}}{\Delta t} = \frac{q}{2m}\left(\mathbf{v}^{+} + \mathbf{v}^{-}\right) \times \mathbf{B}$$

Synthetic Turbulence

RUB

Requirements for an Advanced Synthetic Turbulence Model

i) The synthetic magnetic fields have to be divergence free: $\nabla \cdot \vec{B} = 0$.

ii) The synthetic fields need to be homogeneous.

iii) The synthetic fields should reproduce a predefined energy spectrum (Kolmogorov, 1941; Iroshnikov, 1963; Kraichnan, 1965; Boldyrev, 2005).

iv) There should be no restriction other than computational ones for a maximum Reynolds number.

v) The spectrum should be anisotropic, which means that it should have different exponents perpendicular and parallel to a local guide field (Goldreich & Sridhar, 1995; Boldyrev, 2005).

vi) The generation of the synthetic fields must be local and adaptive in space.

vii) The synthetic turbulence should exhibit intermittency, as prescribed by a given intermittency model.

Synthetic Turbulence

Magnetostatic Turbulence (following Shalchi 2020 review)

$$\mathbf{B}(\mathbf{x},t) = B_0\mathbf{e}_z + \delta\mathbf{B}(\mathbf{x},t).$$

$$\delta\mathbf{B}(\mathbf{x}) = \sqrt{2}\delta B\sum_{n=1}^{N} A(k_n)\boldsymbol{\xi}_n\cos[\mathbf{k}_n\cdot\mathbf{x}+\beta_n]$$

$$\mathbf{k}_n = k_n\mathbf{e}_{k,n}$$

$$\mathbf{e}_{k,n} = \begin{pmatrix} \sqrt{1-\eta_n^2}\cos\phi_n \\ \sqrt{1-\eta_n^2}\sin\phi_n \\ \eta_n \end{pmatrix}$$

$$\boldsymbol{\xi}_n = \begin{pmatrix} -\sin\phi_n\cos\alpha_n + \eta_n\cos\phi_n\sin\alpha_n \\ \cos\phi_n\cos\alpha_n + \eta_n\sin\phi_n\sin\alpha_n \\ -\sqrt{1-\eta_n^2}\sin\alpha_n \end{pmatrix}$$

$$A^2(k_n) = G(k_n)\Delta k_n\left(\sum_{m=1}^{N} G(k_m)\Delta k_m\right)^{-1}$$

$$G(k_n) = \frac{k_n^q}{(1+k_n^2)^{(s+q)/2}}.$$

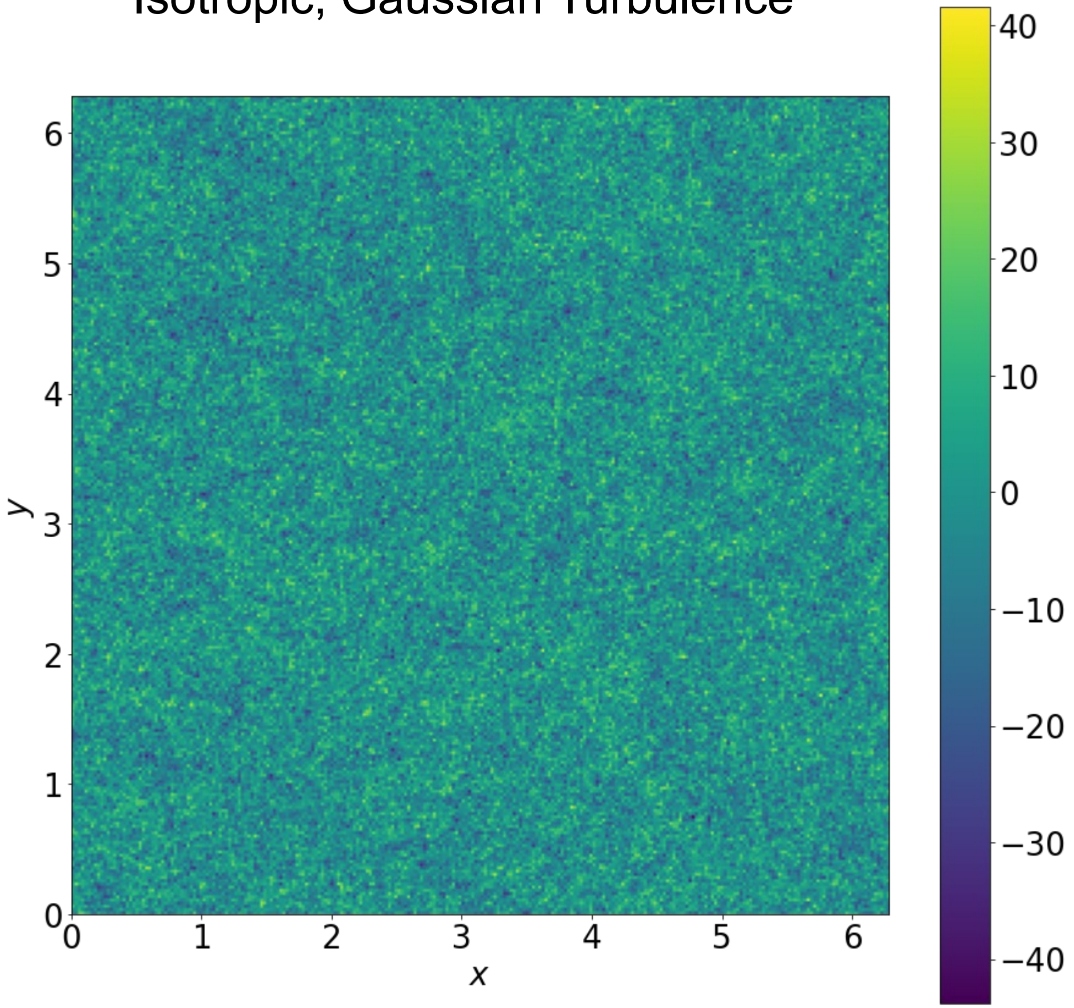
Turbulence model	η_n	α_n	Φ_n	Wave numbers	q
Slab	1	0	Random	$k_n = \ell_\parallel k_\parallel$	0
Two-dimensional	0	0	Random	$k_n = \ell_\perp k_\perp$	2 or 3
Isotropic	Random	Random	Random	$k_n = \ell_0 k$	3
Noisy slab model	0	0	Random	$k_n = \ell_\perp k_\perp$, $k_m = \ell_\parallel k_\parallel$	0
NRMHD	0	0	Random	$k_n = \ell_\perp k_\perp$, $k_m = \ell_\parallel k_\parallel$	3

Synthetic Turbulence

RUB

$$\delta\mathbf{B}(\mathbf{x}) = \sqrt{2}\delta B \sum_{n=1}^{N} A(k_n)\boldsymbol{\xi}_n \cos[\mathbf{k}_n \cdot \mathbf{x} + \beta_n]$$

Isotropic, Gaussian Turbulence



Example Results for the Diffusion Coefficient

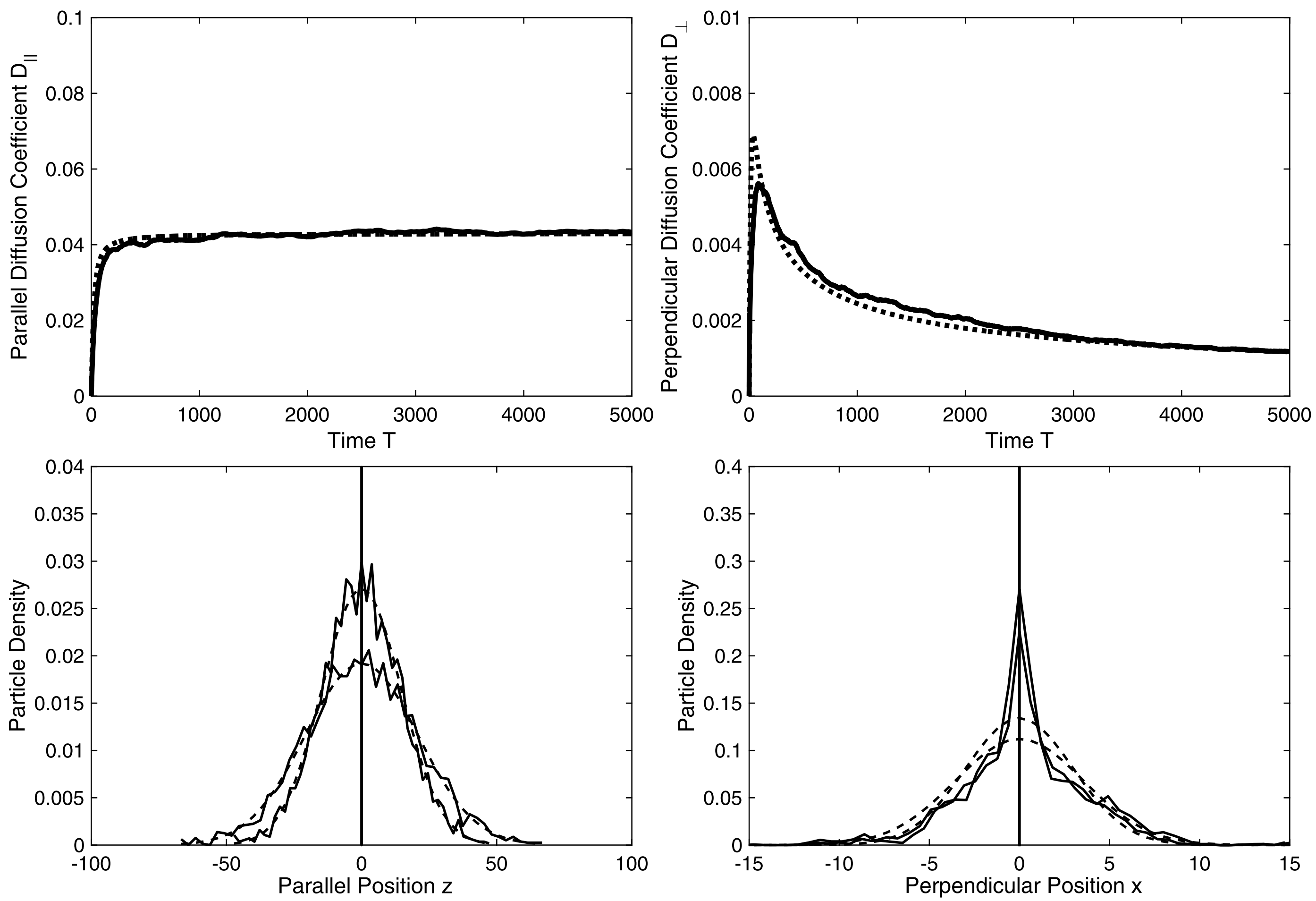


Fig. 16 Diffusion coefficients and distribution functions for pure slab turbulence, a magnetic rigidity of $R = 0.1$, and a magnetic field ratio of $\delta B_{slab}^2/B_0^2 = 1$. The used parameters T, R, $K_\parallel$, and $D_\parallel$ are defined in

Shalchi 2020

Influence of Intermittency? Shukurov 2017

Cosmic Rays in Intermittent Magnetic Fields

Anvar Shukurov[1], Andrew P. Snodin[2], Amit Seta[1], Paul J. Bushby[1], and Toby S. Wood[1]
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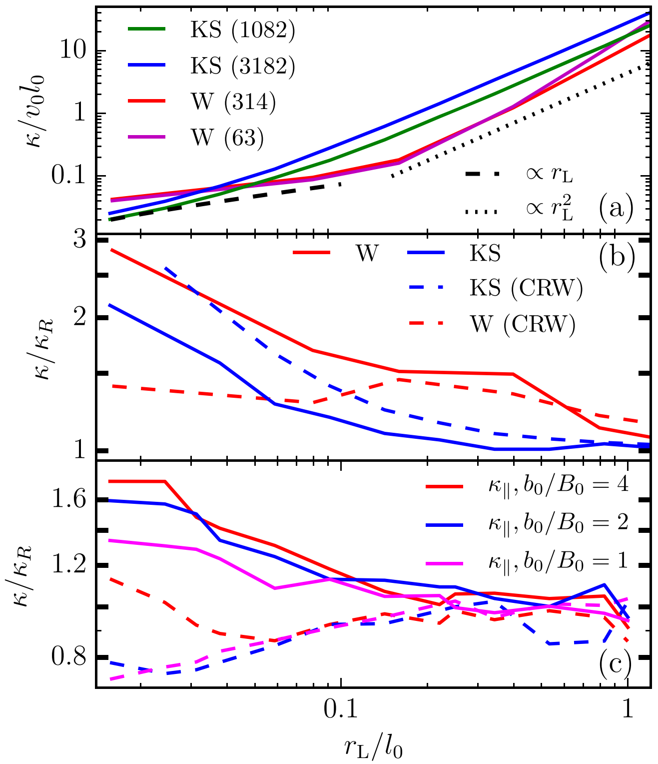
2. Magnetic Field Produced by Dynamo Action

We generate intermittent, statistically isotropic, fully three-dimensional random magnetic fields $\boldsymbol{b}$ by solving the induction equation with a prescribed velocity field $\boldsymbol{u}$:

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b}) + R_{\mathrm{m}}^{-1} \nabla^2 \boldsymbol{b}, \qquad \nabla \cdot \boldsymbol{b} = 0,$$



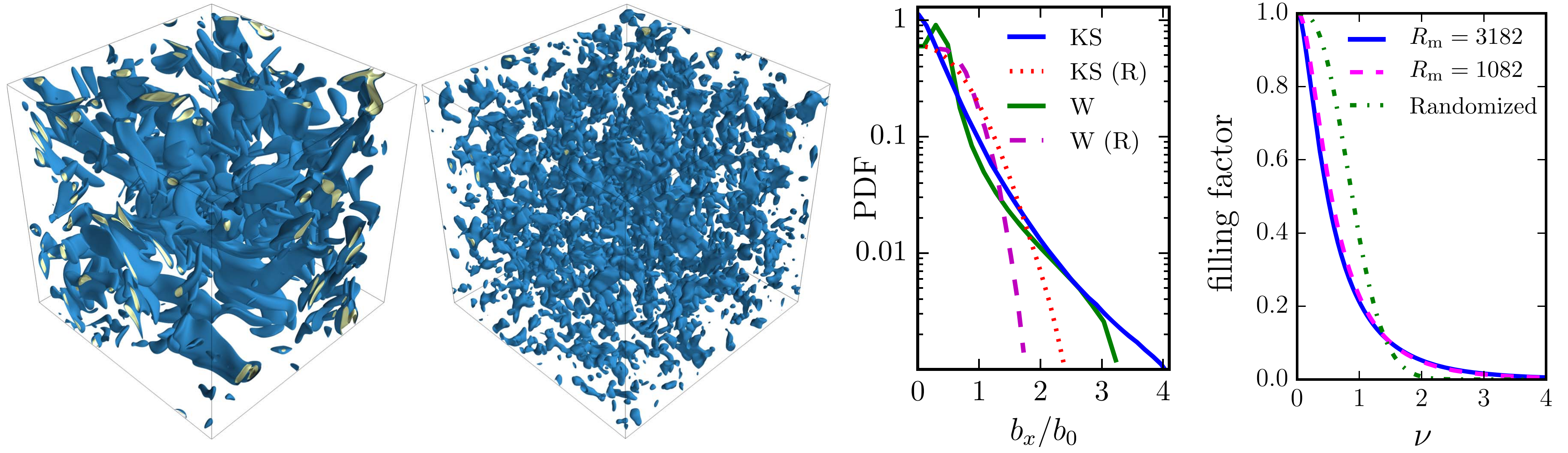


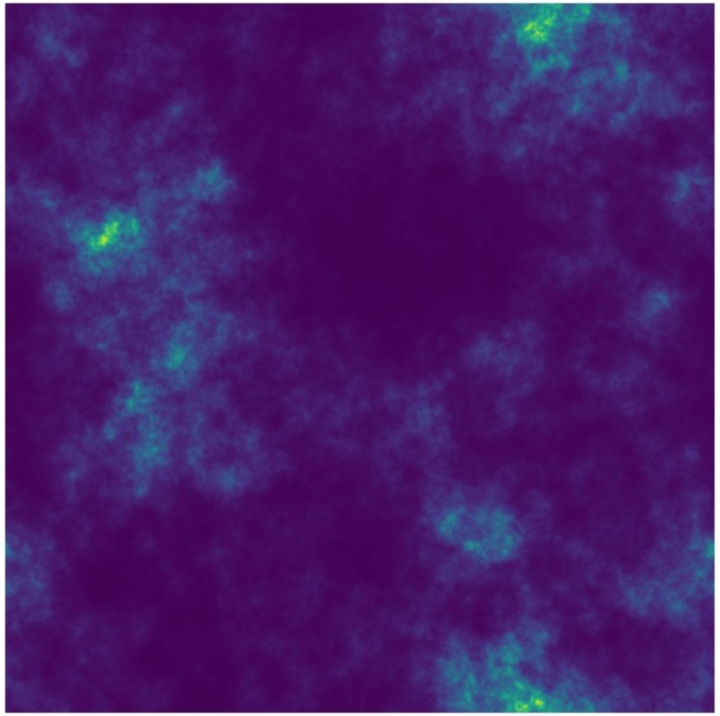
Figure 1. Isosurfaces of magnetic field strength $b^2/b_0^2 = 2.5$ (blue) and $b^2/b_0^2 = 5$ (yellow) with b_0 the rms magnetic field, for magnetic field generated by the KS flow (3) at $R_{\rm m} = 1082$ (left) and for the same magnetic field after Fourier phase randomization as described in the text (second from left). Magnetic field generated by the W flow (2) is similarly affected (not shown). The second from right panel shows the PDFs of a magnetic field component b_x for the original (KS, W: solid) and randomized (KS (R), W (R): dashed) magnetic fields obtained with both velocity fields (only $b_x > 0$ is shown as the PDFs are essentially symmetric about $b_x = 0$). The randomized fields have almost perfectly Gaussian statistics, whereas magnetic intermittency leads to heavy tails. The panel on the right shows the fractional volume within magnetic structures where $b \geqslant \nu b_0$, with b_0 the rms field strength, as a function of ν for the intermittent magnetic field produced by the flow (3) (solid for $R_{\rm m} = 3182$ and dashed for $R_{\rm m} = 1082$) and its Gaussian counterpart (dashed–dotted for $R_{\rm m} = 3142$ and 1082) obtained by Fourier phase randomization; the filling factor of the randomized fields is independent of $R_{\rm m}$.

Synthetic Turbulence with Intermittency

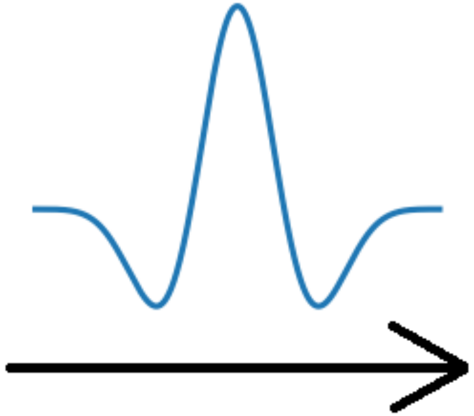
Following Muzy 2019

New approach: $\mathbf{B}(\mathbf{x}) = \nabla \times \int_0^\infty s^{H-2} \left(e^{\omega_s} * \psi_s\right)(\mathbf{x})\, \mathrm{d}s$

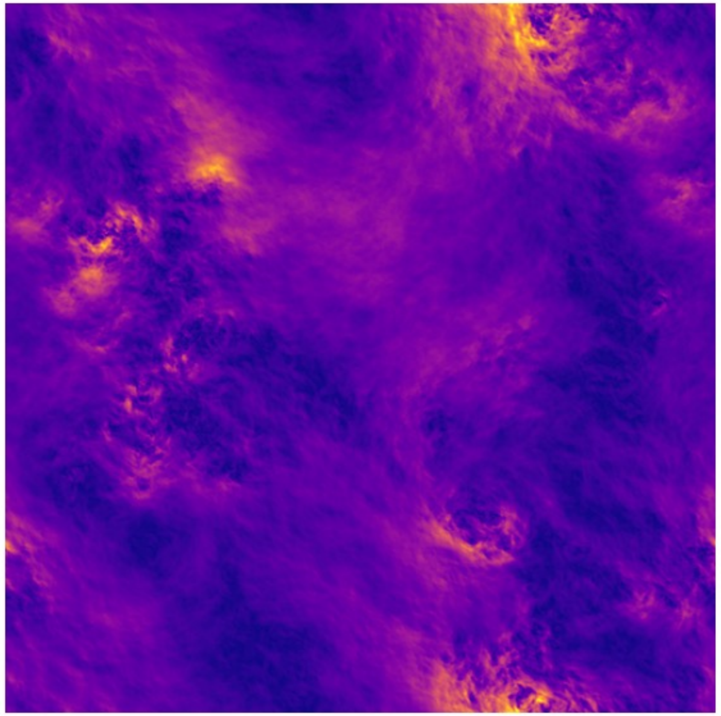
$\omega_s(\mathbf{x})$ is a Gaussian random field resolved at scale s with correlation $\langle \omega_s(\mathbf{x}) \omega_s(\mathbf{y}) \rangle \sim 1/\log \|\mathbf{x} - \mathbf{y}\|$



$e^{\omega_s(\mathbf{x})}$



$\psi(\mathbf{x}),\ \nabla \times \cdot$

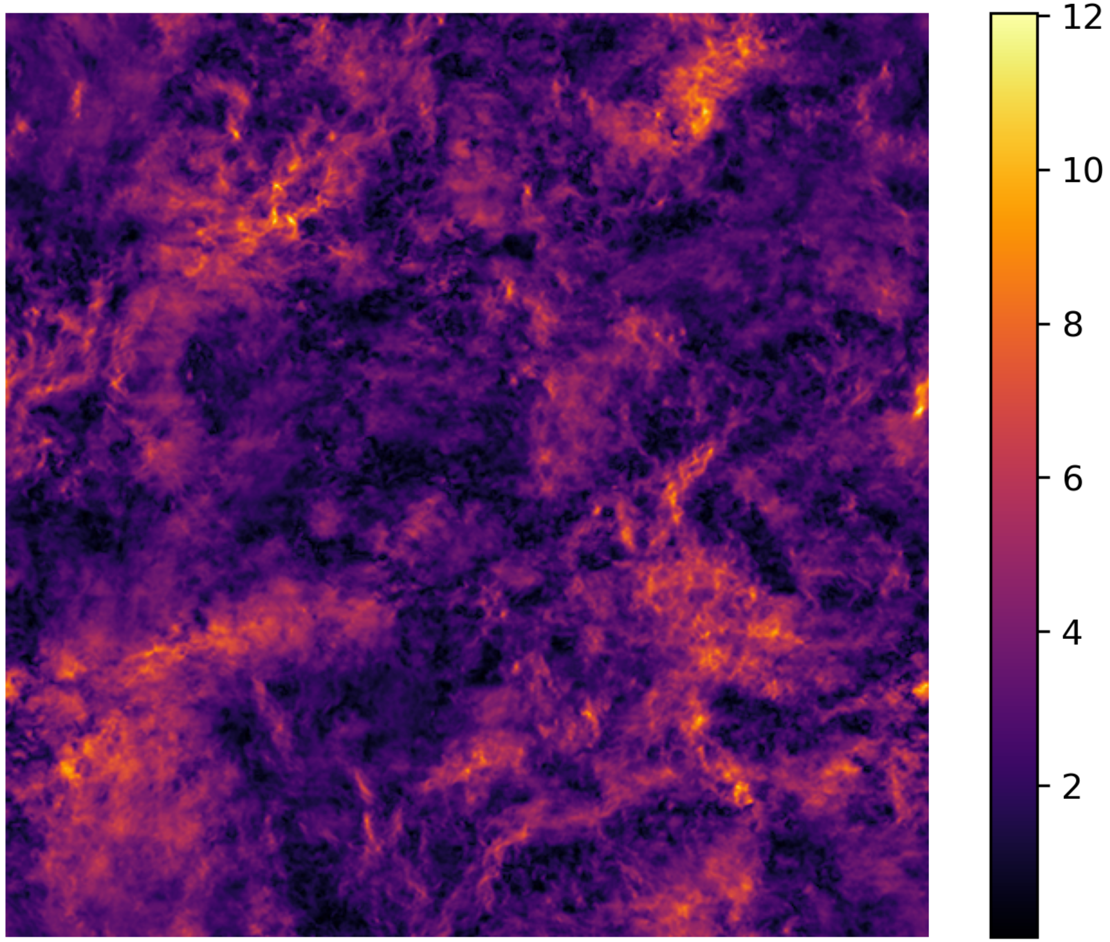


$\|\mathbf{B}(\mathbf{x})\|$

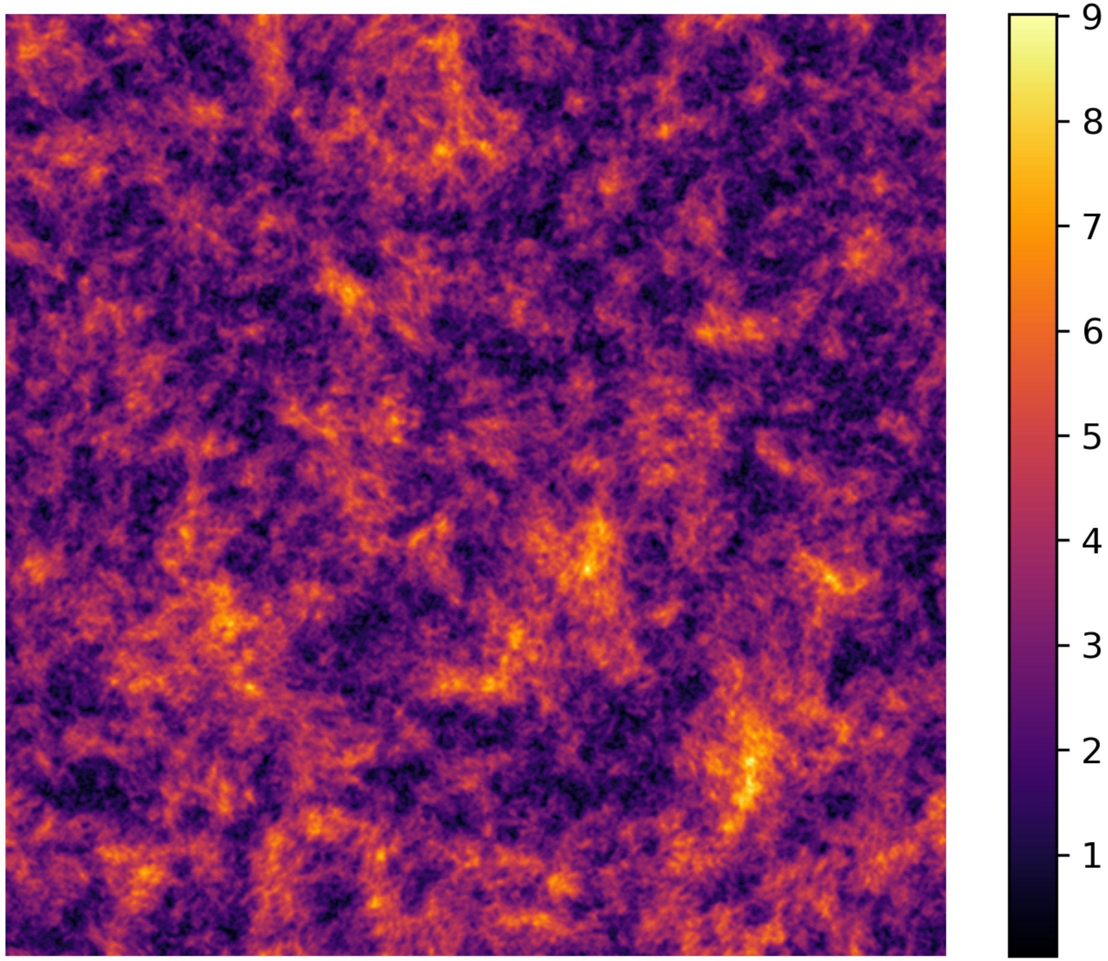
Examples of the New Model

Phase Randomization (Field components are normalized to unit standard deviation)

Intermittent $(\mu = 0.25)$

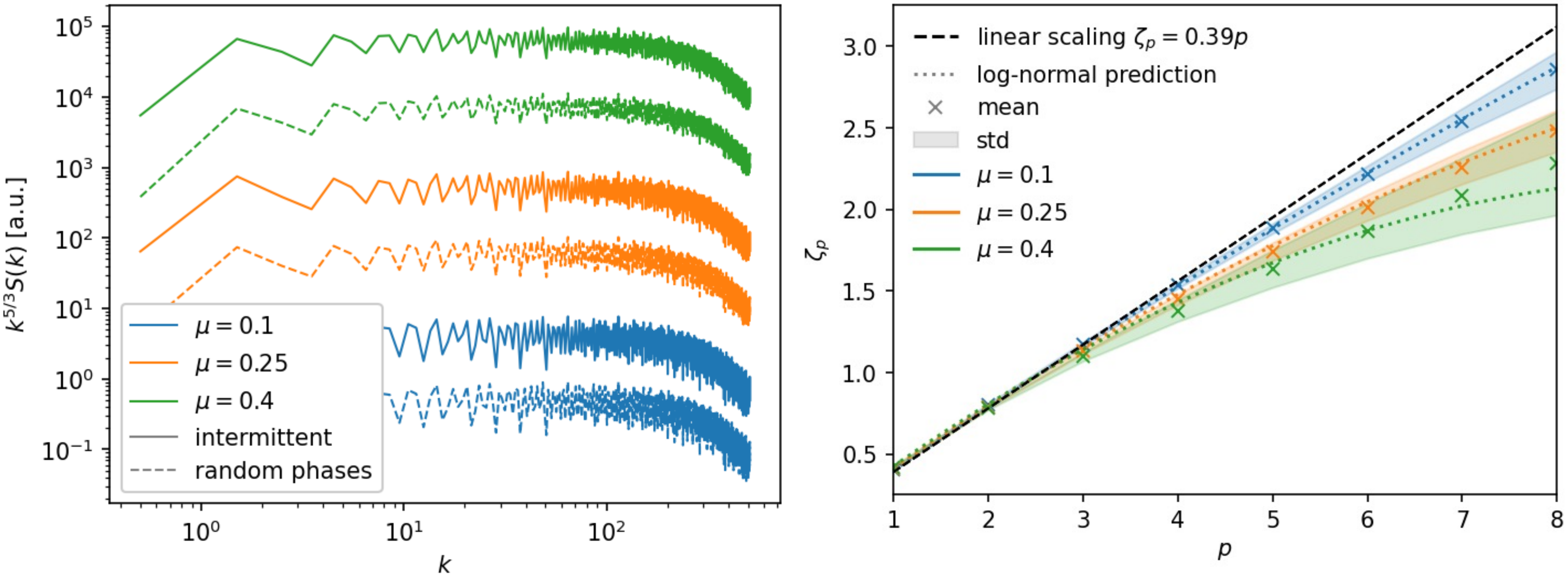


Randomized Phases



Synthetic Turbulence

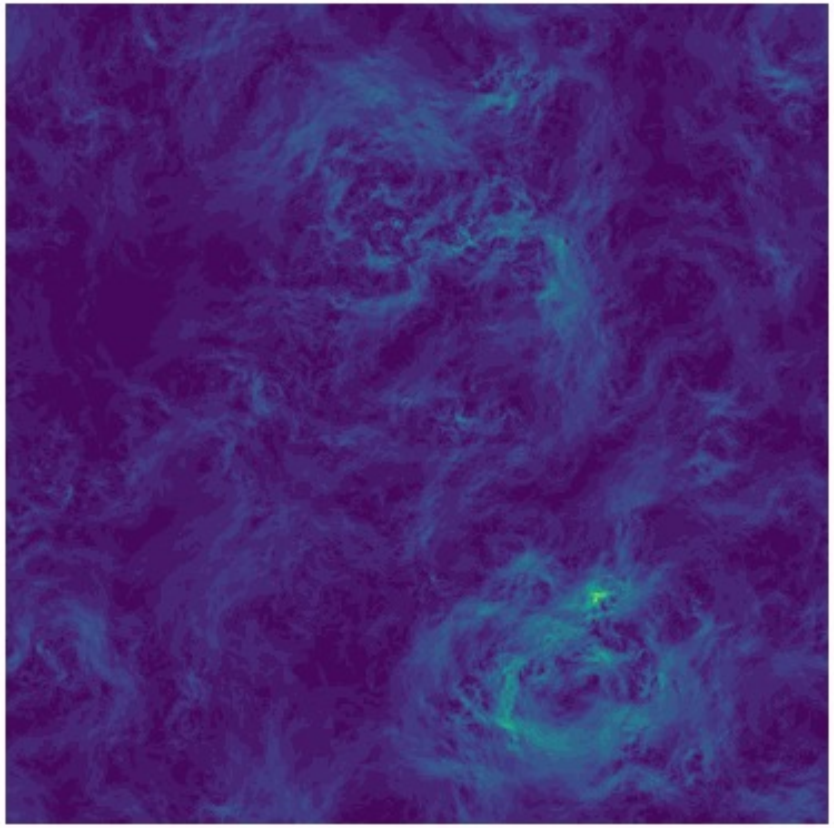
Spectra and structure function exponent scaling



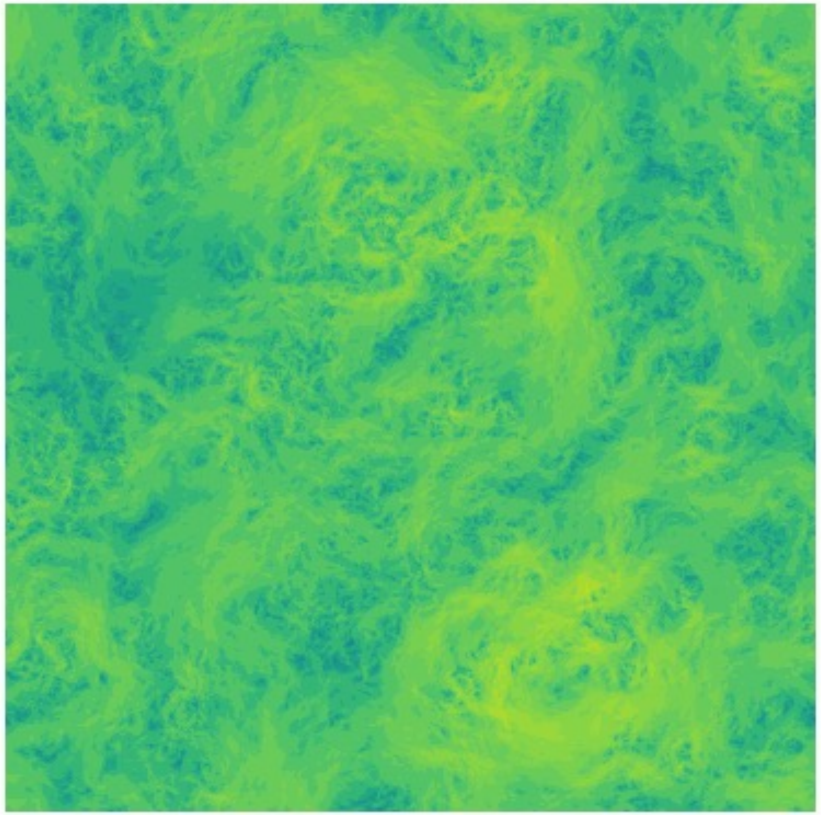
Ideas using 'Minimal Lagrangian Map'

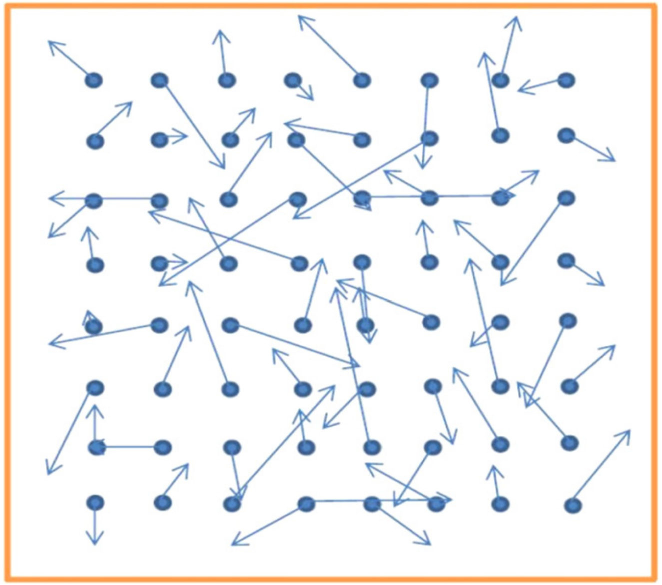
Following Subedi et al. 2014

B



Log E

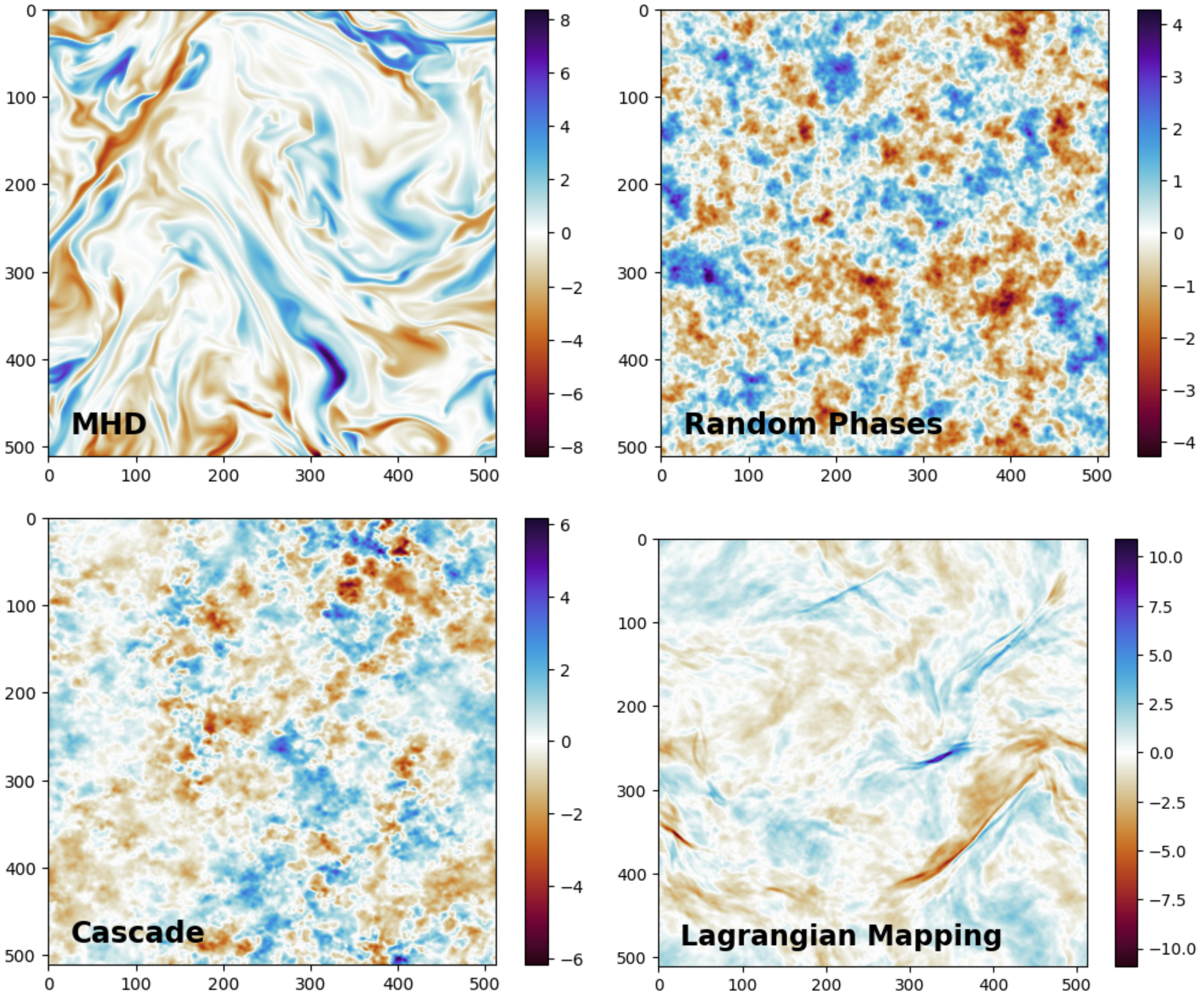




From Subedi et al. 2014

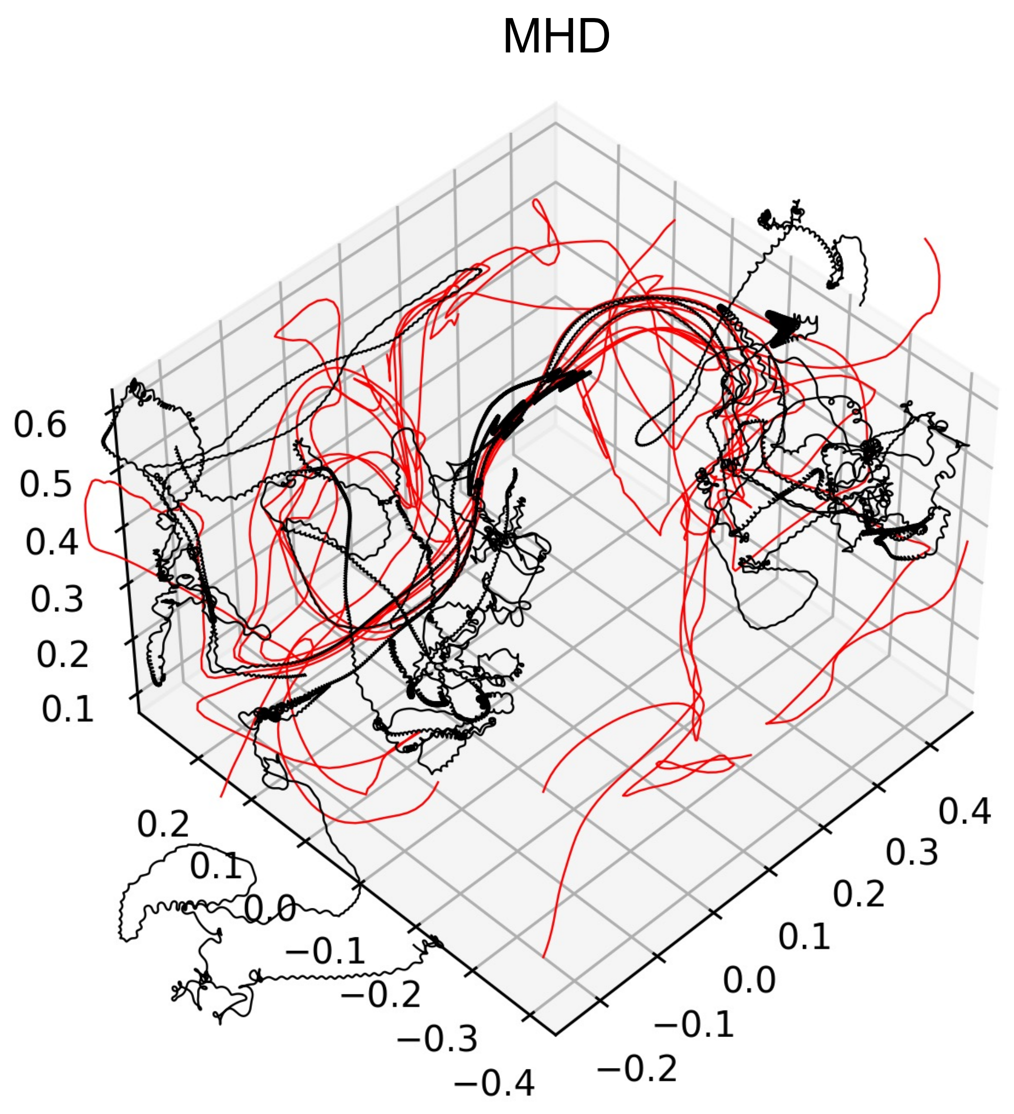
Comparison of Methods

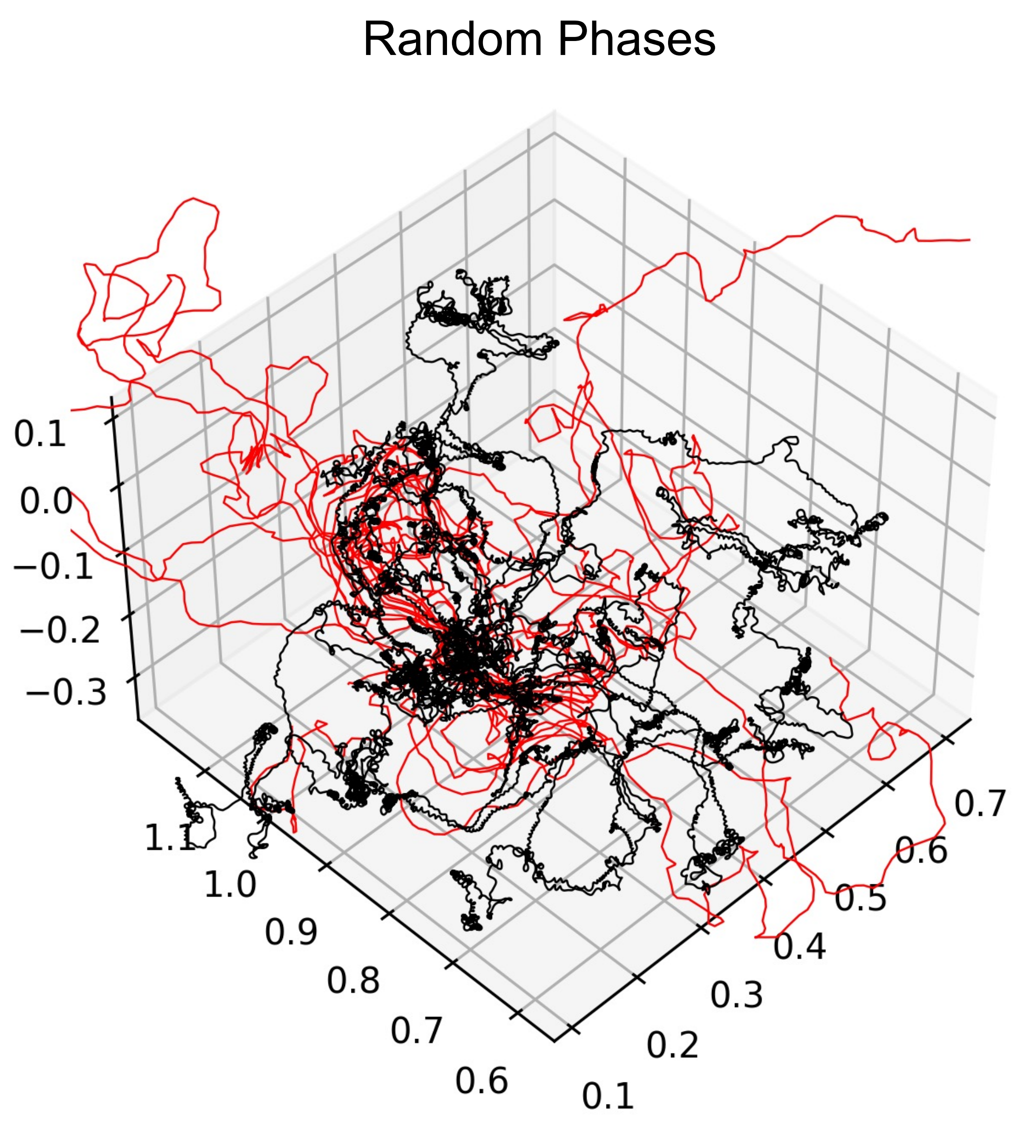
MHD with stochastic driver vs synthetic methods



Comparison of Methods

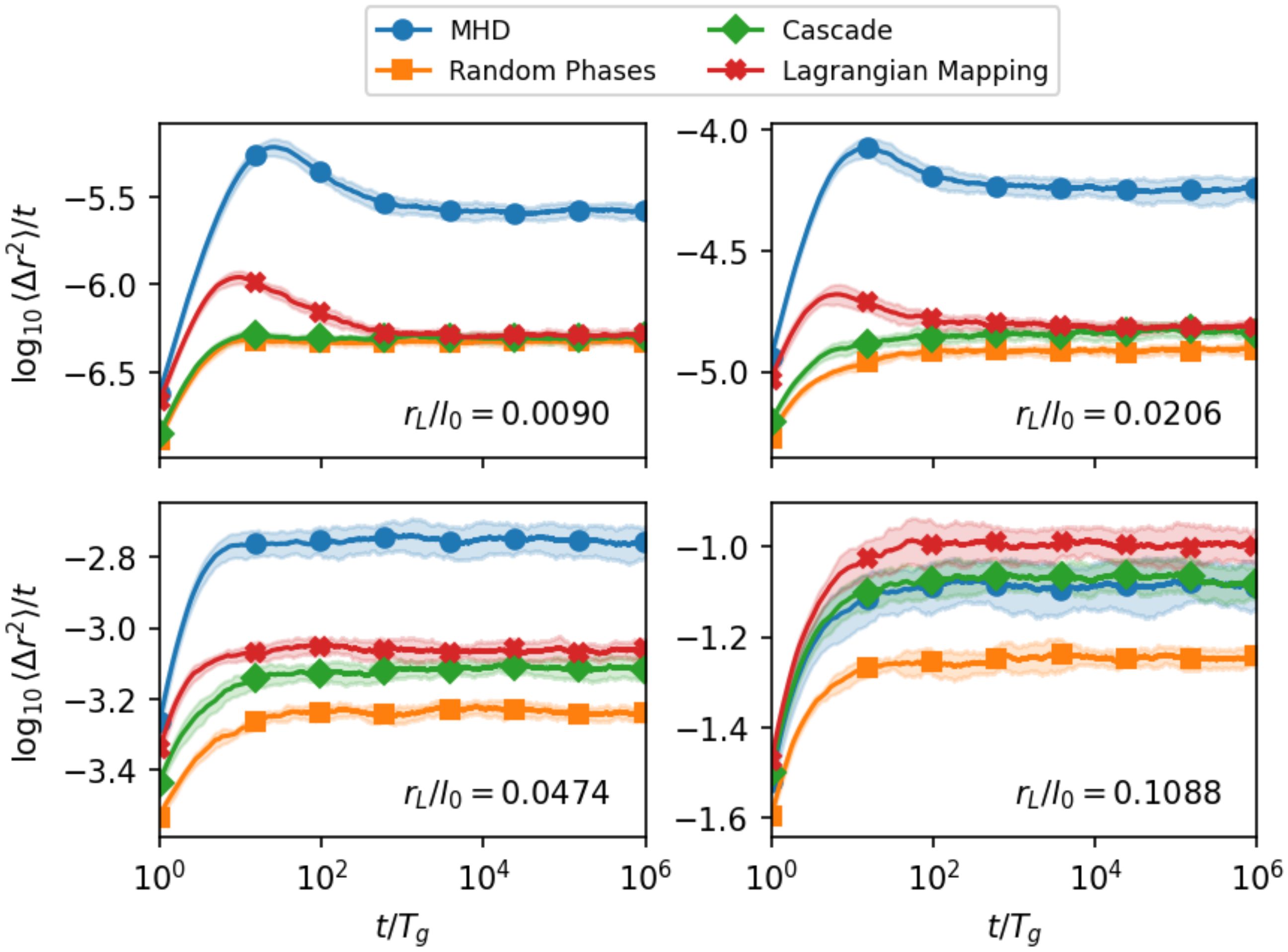
Particle Orbits (black) and Field Lines (red) in MHD and Random Phase Field

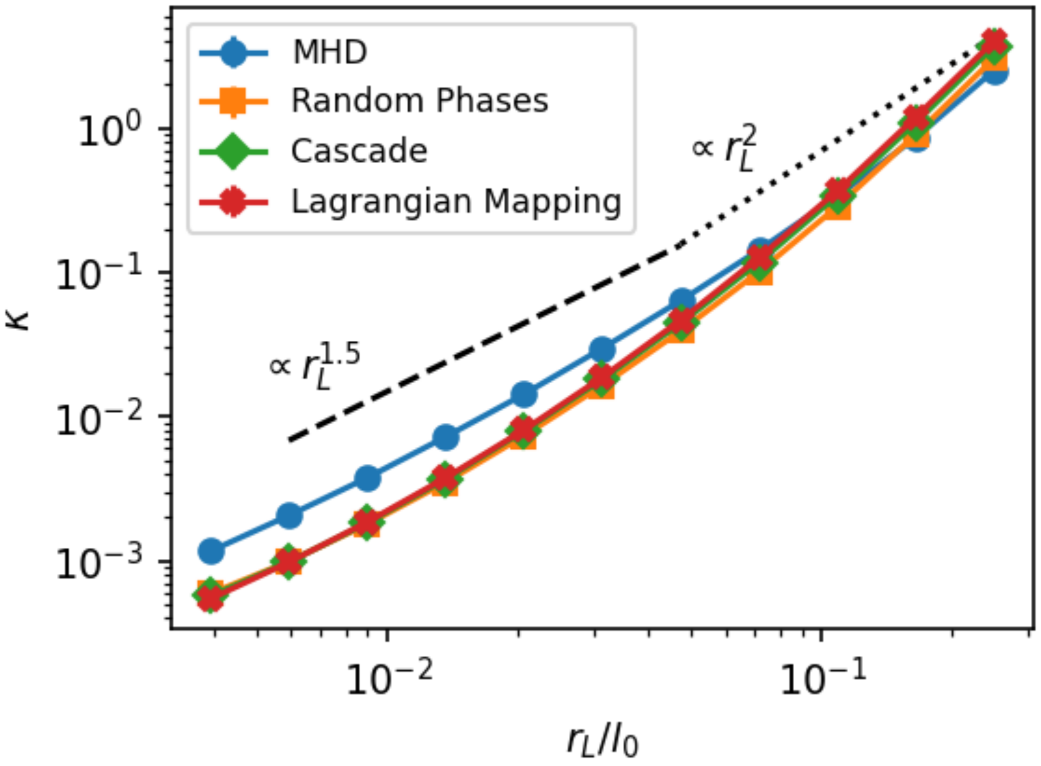




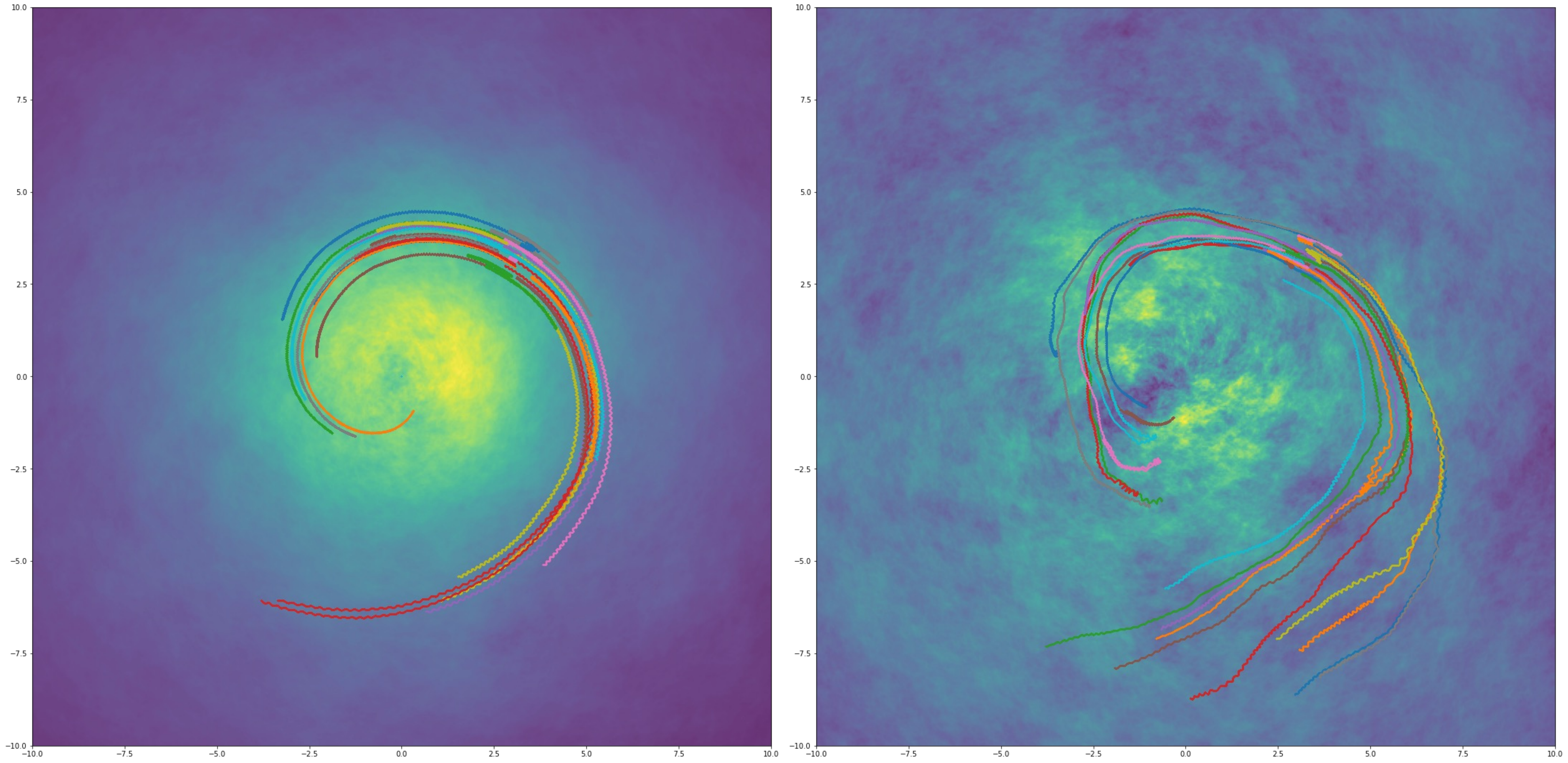
Comparison of Methods

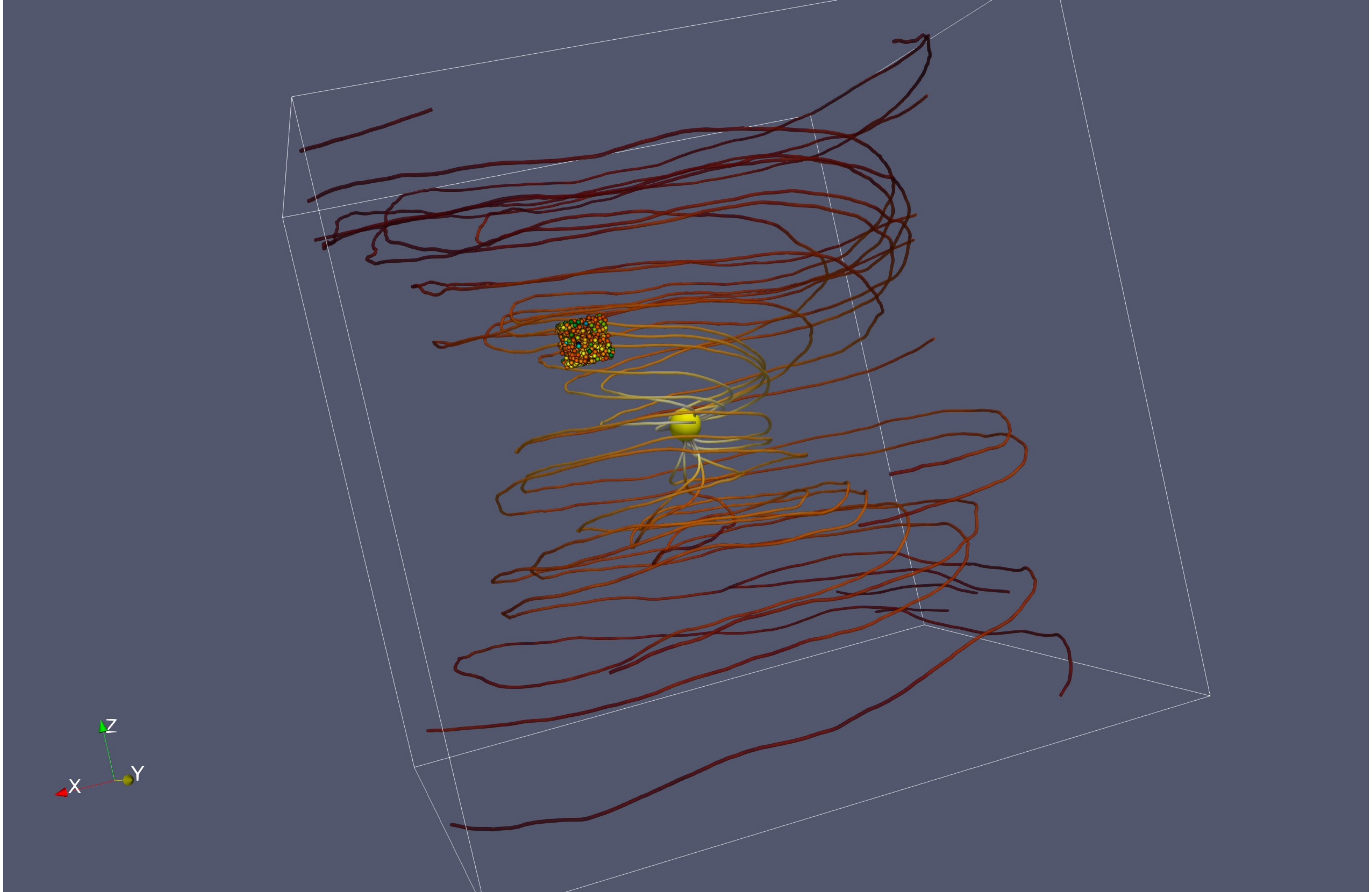
Results for the diffusion coefficients



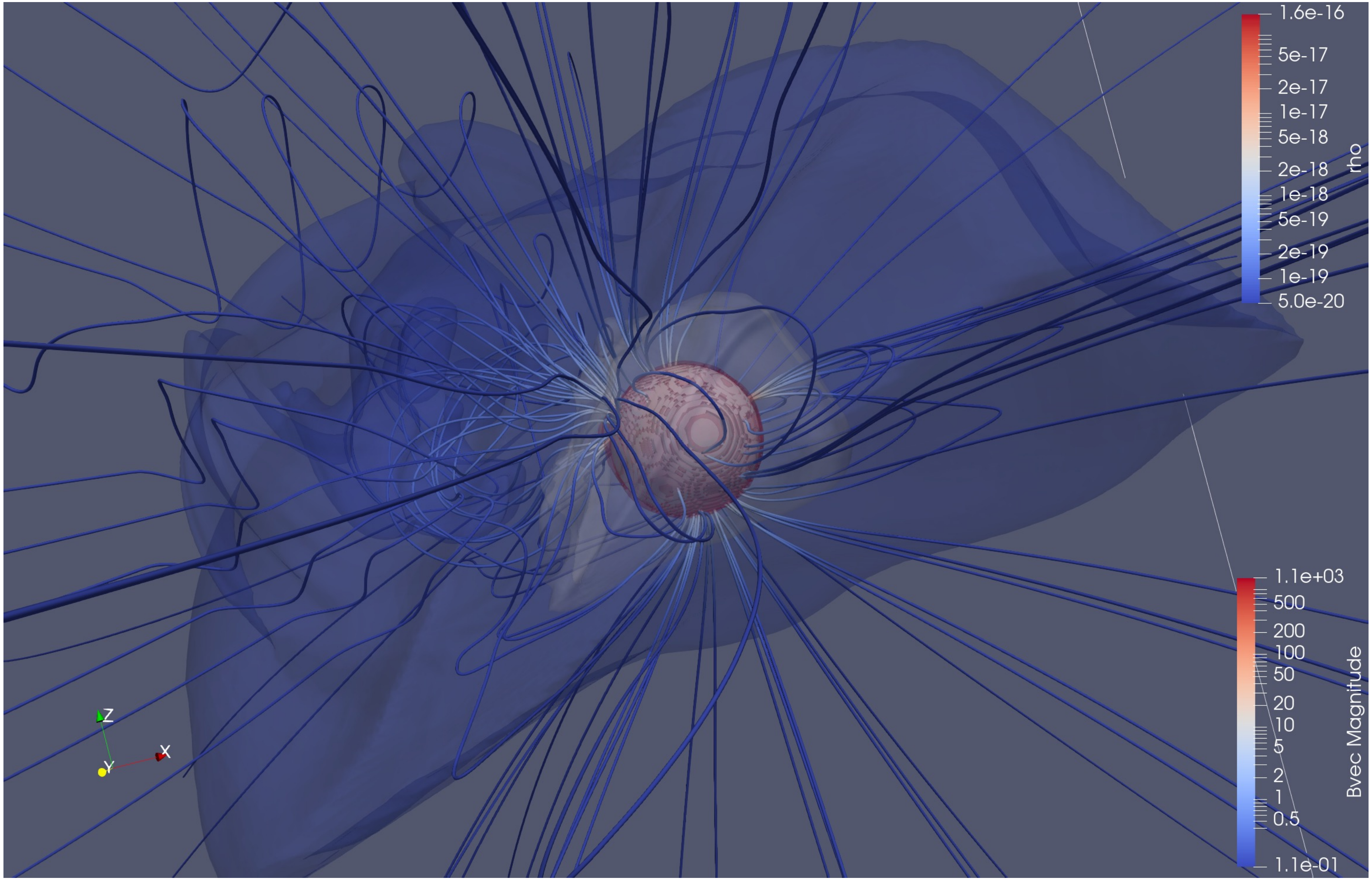


Transport in Parker background field




Z
Y
X

CME Snapshots from SWMF Code (M. Jin)



Summary

Particle transport and (synthetic) turbulence

- **Two approaches to particle transport: Fokker-Planck formulation (diffusion) or full orbit test-particle calculations**
- **Particle transport in synthetic turbulent fields has been studied extensively with numerical methods in the past**
- **Often issues with limited resolution and energy conservation, only periodic boxes**
- **Almost always relying on random phase approximations -> no correlations and intermittency in synthetic turbulence**

Upcoming work

- **Study cases for isotropic and anisotropic synthetic turbulence and MHD turbulence as benchmark**
- **Extend to intermittent fields and study energy dependence of diffusion. Non-diffusive regimes (super- and subdiffusion)?**
- **Work on embedding in large scale background field, e.g. Heliospheric Parker spiral field, Galactic field**