# Numerical Simulations of Intergalactic Electromagnetic Cascades with Lorentz Invariance Violation Using CRPropa

Andrey Saveliev

September 26th, 2023

# Lorentz Invariance Violation

Quantum Gravity is still out of reach, however a possible consequence could be the violation of (active) Lorentz Transformations or, in short, Lorentz Invariance Violation (LIV) [Kostelecky 2001]. There are different frameworks to describe that:

- the Robertson-Mansouri-Sexl (RSM) Framework [Mansouri 1977], based on the assumptions of a motion-independent speed of light and a preferred reference frame;
- Doubly Special Relativity (DSR) [Amelino-Camelia 2000, Amelino-Camelia 2000] which extends the validity of the principles of relativity up to the Planck scale, resulting, among others, in a modified dispersion relation;
- Non-Commutative (NC) Field Theory [Carroll 2001, Anisimov 2001] which introduces a commutator algebra $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. LIV is then introduced by replacing the expressions in the Lagrangian with their non-commutative counterparts.

# Minimal Standard Model Extension (SME)

The Minimal Standard Model Extension [Colladay 1996, Colladay 1998] is an effective field theory which is based on the idea to add to the SM Lagrangian LIV-terms $\Delta\mathcal{L}$, i.e.

$$\mathcal{L}_{\mathrm{SME}} = \mathcal{L}_{\mathrm{SM}} + \Delta\mathcal{L} \tag{1}$$

which

- are built from SM fields,
- preserve SU(3)$\times$SU(2)$\times$U(1) gauge invariance and renormalizability,
- might (but do not have to) break CPT symmetry,
- guarantee positive energies and conserve energy and momentum,
- microcausality and hermiticity,

such that SM might be regarded as the low-energy extension of the SME.

# Minimal Standard Model Extension (SME)

| Dim | CPT-odd | CPT-even |
|---|---|---|
| Photon | | |
| 3 | $\times$ | $\times$ |
| 4 | $\times$ | $-\frac{1}{4}(k_F)u_\kappa \eta_{\lambda\mu} u_\nu F^{\kappa\lambda} F^{\mu\nu}$ |
| 5 | $\frac{\xi}{M_{\rm Pl}} u^\mu F_{\mu\nu}(u\cdot\partial)u_\alpha \tilde{F}^{\alpha\nu}$ | $\times$ |
| 6 | ? | $-\frac{1}{2M_{\rm Pl}^2}\beta_\gamma^{(6)} F^{\mu\nu} u_\mu u^\sigma (u\cdot\partial)^2 F_{\sigma\nu}$ |

Table: Stable, non-trivial kinetic LIV operators for photons. Table taken from [Mattingly 2008], where also the corresponding operators for fermions may be found.

# Minimal Standard Model Extension (SME)

In Lorentz gauge, $\partial^\mu A_\mu = 0$, the free field equation of motion for $A_\mu$ in the preferred frame with the dimension six LIV operator is

$$(1 - \frac{\beta^{(6)}}{E_{Pl}^2}\partial_0^2)\Box A_0 = 0 \quad (2)$$

$$(\Box + \frac{\beta^{(6)}}{E_{Pl}^2}\partial_0^4)A_i = 0 \quad (3)$$

where $i = 1, 2, 3$. If LIV is small, we can use the residual gauge freedom of the Lorentz gauge to set $A_0 = 0$ as long as $A_\mu$ is assumed to not contain any Planckian frequencies. For a plane wave $A_\mu = \epsilon_\mu e^{-ik\cdot x}$, there are hence the usual two transverse physical polarizations with dispersion

$$\omega^2 = k^2 + \beta^{(6)}\frac{k^4}{E_{Pl}^2}. \quad (4)$$

# Threshold Calculations

- Assuming energy-momentum conservation we demand that the invariant mass of the incoming particles $s^{\mathrm{in}}$ is equal to the invariant mass of the outgoing particles $s^{\mathrm{out}}$.
- The threshold condition is then given by

$$s^{\mathrm{in}} = \min s^{\mathrm{out}}, \tag{5}$$

  or, parametrized,

$$\max_{0 \le \theta \le \pi} s^{\mathrm{in}}(\theta) = s^{\mathrm{out}}(y), \tag{6}$$

  where $\theta$ is the collision angle and $y$ the inelasticity assuming that the outgoing particles are emitted in parallel.
- The left-hand side of Eq. (6) is maximal for a head-on collision (i.e. $\theta = \pi$). Due to the modified dispersion relation, solving the equation (6) for $y$ is complicated for LIV.

# Threshold Calculations for IC and PP

For Pair Production (PP) and Inverse Compton (IC) scattering we have

$$(E_{\mathrm{in},e} + \epsilon_{\mathrm{EBL}})^2 - (\mathbf{p}_{\mathrm{in},e} + \mathbf{k}_{\mathrm{EBL}})^2 = (E_{\mathrm{out},e} + E_{\mathrm{out},\gamma})^2 - (\mathbf{p}_{\mathrm{out},e} + \mathbf{k}_{\mathrm{out},\gamma})^2$$
$$(E_{\mathrm{in},\gamma} + \epsilon_{\mathrm{EBL}})^2 - (\mathbf{p}_{\mathrm{in},\gamma} + \mathbf{k}_{\mathrm{EBL}})^2 = (E_{\mathrm{out},e+} + E_{\mathrm{out},e-})^2 - (\mathbf{p}_{\mathrm{out},e+} + \mathbf{p}_{\mathrm{ou}} \tag{7}$$

respectively, which results in

$$\epsilon_{\mathrm{thr}} = \begin{cases} k_\gamma \left[ \left( \frac{m_e}{k_\gamma} \right)^2 + \frac{1}{4} \left( \frac{\chi_n^e}{2^n} - \chi_n^\gamma \right) \left( \frac{k_\gamma}{M_{\mathrm{Pl}}} \right)^n \right] & \text{for PP,} \\ 0 & \text{for IC,} \end{cases} \tag{8}$$

or, equivalently, in the threshold invariant mass, $s_{\mathrm{thr}}$, which may be written as

$$s_{\mathrm{thr}} = \begin{cases} k_\gamma^2 \left[ 4 \left( \frac{m_e}{k_\gamma} \right)^2 + \frac{\chi_n^e}{2^n} \left( \frac{k_\gamma}{M_{\mathrm{Pl}}} \right)^n \right] & \text{for PP,} \\ p_e^2 \left[ \left( \frac{m_e}{p_e} \right)^2 + \chi_n^e \left( \frac{p_e}{M_{\mathrm{Pl}}} \right)^n \right] & \text{for IC.} \end{cases} \tag{9}$$

# Mean Free Path Calculations

Furthermore, also the calculation of the mean free path has to be modified to

$$\lambda_{\mathrm{MFP}} = \frac{1}{2p_{\mathrm{in}}} \int_{s_{\mathrm{thr}}}^{\infty} \mathrm{d}s^* \frac{n_{\mathrm{bp}} \left( \frac{s^* - p_{\mathrm{in}}^2 \left[ \left( \frac{m_{\mathrm{in}}}{p_{\mathrm{in}}} \right)^2 + \chi_n^{\mathrm{in}} \left( \frac{p_{\mathrm{in}}}{M_{\mathrm{Pl}}} \right)^n \right]}{4p_{\mathrm{in}}} \right)}{\left\{ s^* - p_{\mathrm{in}}^2 \left[ \left( \frac{m_{\mathrm{in}}}{p_{\mathrm{in}}} \right)^2 + \chi_n^{\mathrm{in}} \left( \frac{p_{\mathrm{in}}}{M_{\mathrm{Pl}}} \right)^n \right] \right\}^2}$$

$$\times \int_{p_{\mathrm{in}}^2 \left[ \left( \frac{m_{\mathrm{in}}}{p_{\mathrm{in}}} \right)^2 + \chi_n^{\mathrm{in}} \left( \frac{p_{\mathrm{in}}}{M_{\mathrm{Pl}}} \right)^n \right]}^{s^*} \mathrm{d}s \, \sigma(s) \left\{ s - p_{\mathrm{in}}^2 \left[ \left( \frac{m_{\mathrm{in}}}{p_{\mathrm{in}}} \right)^2 + \chi_n^{\mathrm{in}} \left( \frac{p_{\mathrm{in}}}{M_{\mathrm{Pl}}} \right)^n \right] \right\} . \tag{10}$$

# New Reactions due to LIV

Apart from the changes to existing reactions LIV also introduces new ones which were not possible before due to energy-momentum conservation [Jacobson 2002]:

- Vacuum Cherenkov (VC) Effect: $e \to e + \gamma$,
- Photon Decay (PD): $\gamma \to e^- + e^-$,
- Single-photon pair annihilation: $e^- + e^- \to \gamma$ (hardly distinguishable from two-photon pair annihilation),
- Photon splitting: $\gamma \to N\gamma$.

# Constraints on LIV Parameters

There are different types of constraints which may be derived from LIV effects:

- Time-of-flight constraints:With LIV photons with different wavelengths can have different speeds of light, hence providing upper limits for $\chi_n^\gamma$ for a synchronized emission
- Vacuum birefringence constraints: The absence of a measurable difference in the speed of light for different photon polarizations has been used to put constraints on $\chi_0^\gamma$ and $\chi_1^\gamma$
- Threshold constraints: The modification of the threshold for a given reaction can make it (un)accessible for astrophysical observations. This can result in specific signatures which may be used to derive constraints.

# Constraints on LIV Parameters: Example

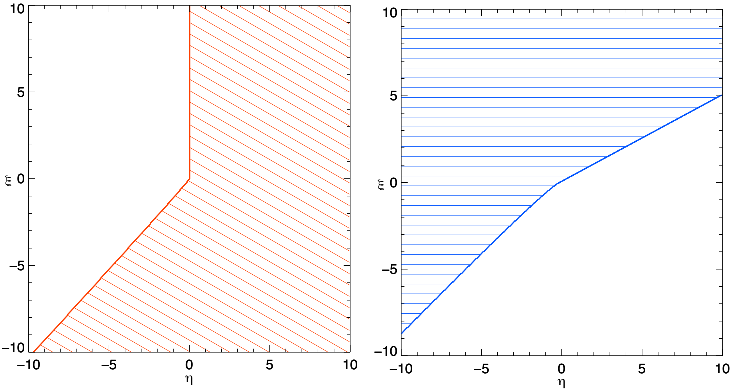


Figure: Constraints for $n = 1$ from the observations of 100 TeV electrons for VC (left) and 50 TeV photons for PD (right). The respective colored region is excluded. Figure taken from [Jacobson 2002].

# LIVPropa

- We have created an advanced simulation tool called `LIVPropa` that seamlessly integrates with the CRPropa code. This plugin enables detailed simulations of gamma-ray propagation while considering LIV.
- One of the key functionalities is the provision of tabulated interaction rate tables tailored for different LIV parameters calculated by dedicated python scripts to ensure versatility and adaptability, we have also included. These scripts facilitate the generation of data for scenarios that may not be readily available within the existing library.
- This flexibility allows studies of a wide range of LIV scenarios and customization of the simulation parameters to suit the user's specific needs.

# LIVPropa – Results

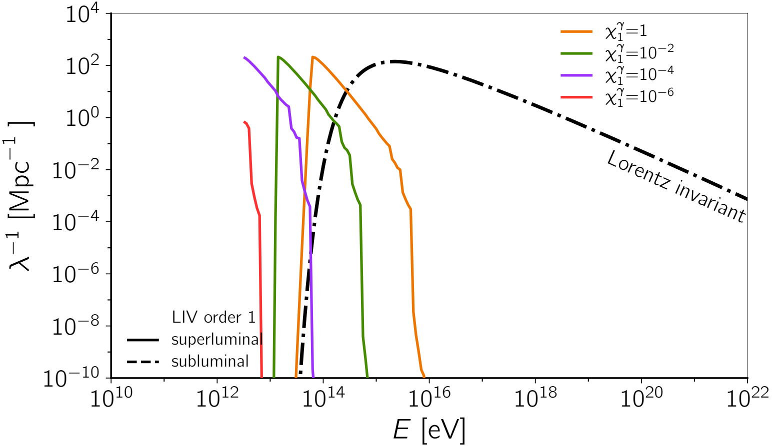


Figure: Inverse mean free path for PP on the CMB

# LIVPropa – Results

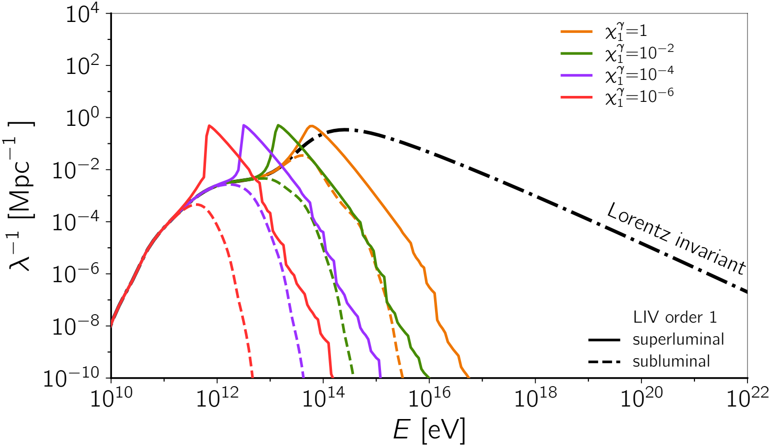


Figure: Inverse mean free path for PP on the IRB [Gilmore 2012]

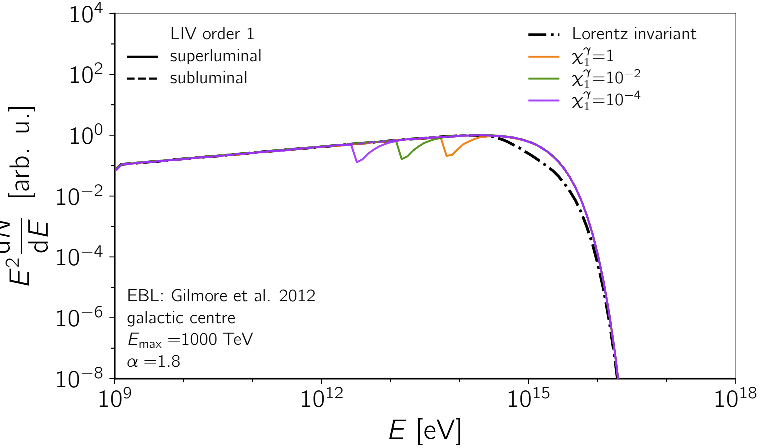


Figure: PP-spectrum for a gamma-ray source at the Galactic center

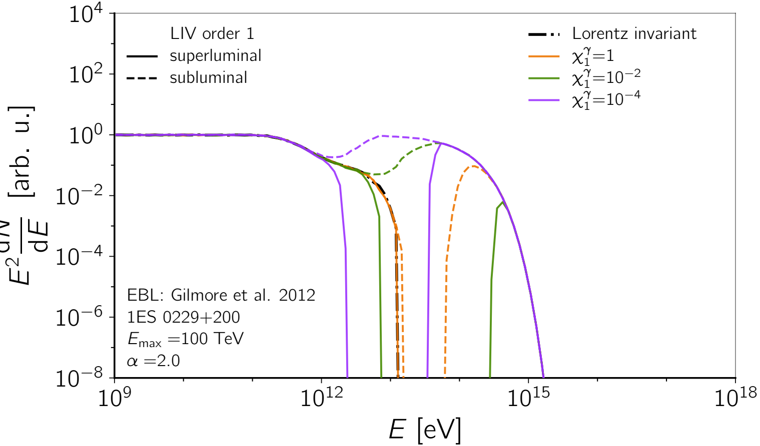


Figure: PP-spectrum for a cosmologically-distant blazar.

# Conclusions and Outlook

- We calculated the modifications of thresholds and propagation lengths for pair production and Inverse Compton Scattering
- These results have then been used to create `LIVPropa`, a plugin for CRPropa which may be used to simulate the development of electromagnetic cascades including LIV
- In the future we will also include new reactions possible only with LIV like the Vacuum Cherenkov effect and Photon Decay
- Furthermore, with `LIVPropa` formalism it will also be possible to simulate the propagation of UHECRs with LIV